MAT 272

SPRING 2015

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Test 2

SoMSS, ASU

Directions:

1. **There are 7 questions worth a total of 100 points.**
2. **Read all the questions carefully.**
3. **You must show all work in order to receive credit for the free response questions!!**
4. **When possible, box your answer, which must be complete, organized, and exact unless otherwise directed.**
5. **Always indicate how a calculator was used (i.e. sketch graph, etc. ...).**
6. **No calculators with QWERTY keyboards or ones like TI-89 or TI-92 that do symbolic algebra may be used.**

Honor Statement:

By signing below you confirm that you have neither given nor received any unauthorized assistance on this exam. This includes any use of a graphing calculator beyond those uses specifically authorized by the Mathematics Department and your instructor. Furthermore, you agree not to discuss this exam with anyone until the exam testing period is over. In addition, your calculator's program memory and menus may be checked at any time and cleared by any testing center proctor or Mathematics Department instructor.

____________________________ ________________

Signature Date

PRINT NAME: Solution - Platte

RECITATION (Tuesday or Thursday): ____________________

1.

a. Find the line of intersection of the planes

$$9x - 4y + 6z = -74 \text{ and } 6x + 4y + 3z = -28 \qquad [8pts]$$

$n_1 = \langle 9, -4, 6\rangle$

$n_2 = \langle 6, 4, 3\rangle$

$n_1 \times n_2 = \langle -12-24, 36-27, 36+24\rangle$

$= \langle -36, 9, 60\rangle$ ← vector parallel to line

Point on the line:

$\begin{cases} 9x-4y+6z=-74 \\ 6x+4y+3z=-28 \end{cases} \Rightarrow$ $-4y+6z=-74$, $4y+3z=-28$

let $x=0$: $0+9z=-102 \Rightarrow z=\frac{-102}{9} \Rightarrow 4y=\frac{102}{3}-28$

$y=\frac{51}{6}-7$

$y=\frac{51-42}{6}$

$y=\frac{9}{6}$

$\boxed{y=\frac{3}{2}}$

$\boxed{z=\frac{-34}{3}}$

Point on the line $(0, \frac{3}{2}, \frac{-34}{3})$

Equation of line:

$\vec{r}(t) = \langle -36t, \frac{3}{2}+9t, \frac{-34}{3}+60t\rangle$

b. Find the equation of the plane passing through the point (1, 2, 3) and orthogonal to the line of intersection obtained in part a. of the problem. [6pts]

$n = \langle -36, 9, 60\rangle$

Point $(1, 2, 3)$

Equation of plane

$$\boxed{-36(x-1)+9(y-2)+60(z-3)=0}$$

or

$$-36x+9y+60z = -36+18+180 = 162$$

2. Let $f(x, y) = 3x^2y + y^3 - 3x^2 - 3y^2 + 2$.

a. Find all the critical points of f. [8 pts]

$f_x = 6xy - 6x = 0 \Rightarrow 6x(y-1) = 0$

$f_y = 3x^2 + 3y^2 - 6y = 0$

$\boxed{x = 0 \text{ or } y = 1}$

If $x = 0 \Rightarrow 3y^2 - 6y = 0 \Rightarrow 3y(y-2) = 0 \Rightarrow \boxed{y = 0 \text{ or } y = 2}$

If $y = 1 \Rightarrow 3x^2 + 3 - 6 = 0 \Rightarrow x^2 = 1 \Rightarrow \boxed{x = \pm 1}$

Critical points: $(0,0)$, $(0,2)$, $(1,1)$, $(-1,1)$

b. Apply the second derivative test to identify the critical point (0,0) of f as a maximum, minimum or saddle points. [8 pts]

$f_{xx} = 6y - 6$ @ $(0,0) \Rightarrow f_{xx}(0,0) = -6$

$f_{yy} = 6y - 6$ $\quad f_{yy}(0,0) = -6$

$f_{xy} = 6x$ $\quad f_{xy}(0,0) = 0$

$\Rightarrow D = (-6)^2 = 36$

$\boxed{D > 0 \text{ and } f_{xx} < 0} \Rightarrow \boxed{\text{local max}}$

3. [14pts]

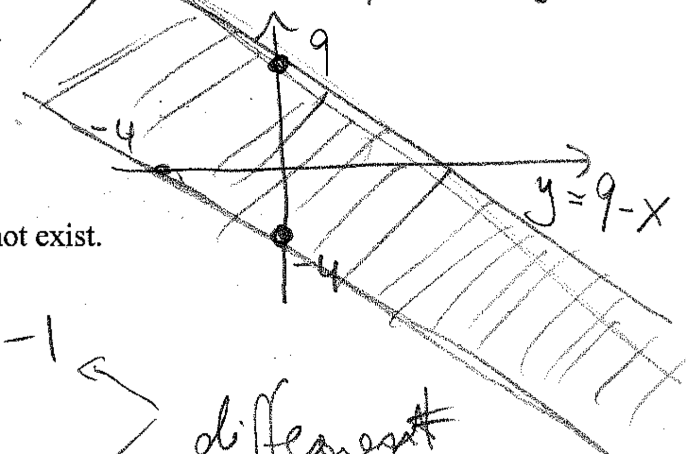
   a. Find and sketch the domain of the function $f(x,y)=\sqrt{x+y+4}+\sqrt{9-x-y}$

$x+y+4 \geq 0$ | $9-x-y \geq 0$

$x+y \geq -4$ | $y \leq 9-x$

$y \geq -4-x$

$D: \{(x,y) \mid -4-x \leq y \leq 9-x\}$



$y=-4-x$

   b. Show that $\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{x^2-y^2}$ does not exist.

let $x=0 \Rightarrow \lim_{y\to 0} \frac{y^2}{-y^2} = -1$

let $y=0 \Rightarrow \lim_{x\to 0} \frac{x^2}{x^2} = 1$

different values

$\Rightarrow$ limit does not exist

4. A company makes metal cylinders with a nominal radius of 5cm and a nominal height of 10cm. Use <u>differentials</u> to estimate the deviation of a cylinder's <u>volume</u> from the nominal value if the radius can be off by as much as 0.1cm, and the height can be off by as much as 0.2 cm. [10pts]

$V=\pi r^2 h \Rightarrow dV = \pi 2 r h\, dr + \pi r^2 dh$

$dV = \pi\, 2\times 5\times 10\times 0.1 + \pi (5)^2\, 0.2$

$dV = 10\pi + 5\pi$

$\boxed{dV = 15\pi} \approx 47.124\ldots$

5. Let $f(x,y) = xy^2 - x^2y$.

   a. Find the $\nabla f(-1,2)$. [5pts]

   $\nabla f = \langle y^2 - 2xy, 2xy - x^2 \rangle$ $\quad \nabla f(-1,2) = \langle 4 + \cancel{4}, -4 - 1 \rangle$

   $\boxed{\nabla f(-1,2) = \langle 8, -5 \rangle}$

   b. Determine the directional derivative of f in the direction $\langle 3,4 \rangle$. [5pts]

   $u = \frac{1}{5}\langle 3, 4 \rangle$

   $D_u f(-1,2) = \langle 8, -5 \rangle \cdot \frac{1}{5}\langle 3, 4 \rangle$

   $= \frac{24 - 20}{5} = \boxed{\frac{4}{5}}$

   c. In which direction is the directional derivative of f maximum? [5pts]

   in the direction of the gradient;

   $\langle 8, -5 \rangle$

   d. What is the maximum directional derivative of f? [5pts]

   magnitude of the gradient: $|\langle 8, -5 \rangle| = \sqrt{64 + 25}$

   $= \boxed{\sqrt{89}}$

6. Find an equation of the tangent plane to the surface at the given point. [10pts]

$$z = xy - x - y, (-5, 3, -13)$$

$z_x = y - 1$, $z_y = x - 1$

$z_x(-5,3) = 2$ $\quad z_y(-5,3) = -6$

Equation of plane:

$$\boxed{z = 2(x+5) - 6(y-3) - 13}$$

7. **[16 pts]** State whether the following statements are TRUE or FALSE. If FALSE, correct the statement.

   a. The equation $x^2 - y^2 - z^2 = -1$ represents a cone.

False. Equation of a cone is $x^2+y^2=z^2$

   b. If f is a differentiable function with $f(1,1) = 1$ and $f_x(1,1) = 5$ and $f_y(1,1) = 2$, then the linear approximation of $f(4,2)$ is $L(4,2) = 18$.

$$\mathcal{L}(x,y) = 5(x-1) + 2(x-1) + 1$$
$$\Rightarrow \mathcal{L}(4,2) = 5(4-1) + 2(2-1) + 1 = 15+2+1 = \boxed{18}$$

True.

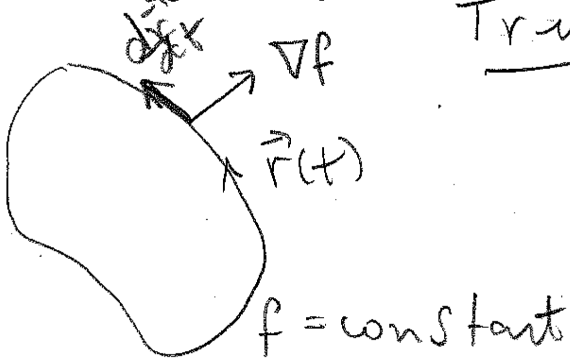
   c. If f is a differentiable function of two variables and $f_x(1,2) = 3$, then as x changes from $x = 1$ to $x = 2$, the value of f changes by 3 units.

False. The derivative gives the approximate change.

$$f_x(1,2) = \lim_{\Delta x \to 0} \frac{f(1+\Delta x, 2) - f(1,2)}{\Delta x}$$

   d. If $\vec{r}(t) = < x(t), y(t) >$ represents a level curve of a differentiable function of two variables, f, then at every point $(x(t), y(t))$, $\frac{d}{dt}\vec{r}(t)$ must be orthogonal to the gradient of f.

True.



$\frac{d\vec{r}}{dt}$ is parallel to $\vec{r}$

∇f is orthogonal to a level curve.