MAT 210 Brief Calculus
Sample Exam 2 (Practice Version C)
Solutions

Instructor	XXXXXXX
Full Name (Print clearly!)	
Student ID	

Important Disclaimer for Students:

- This is a **sample exam**, not the actual in-class exam. Problems are of similar difficulty but may differ in type, numbers, or context.
- Success in the course depends on practicing concepts beyond this sample: lecture notes, Edfinity, Group Work, quizzes, and homework.
- You should complete this exam as if you were in the real exam room: no notes, no internet, no calculators beyond what is allowed, and an approximately 60-minute time limit.
- After finishing, check your answers against the provided solutions and grade yourself.
- If you could not complete within the time limit, practice again and attempt another sample exam.
- Treat every problem in your coursework (notes, quizzes, assignments) as fair game for the actual exam.

Part I - Multiple Choice

1. (5 points) Let f be a function defined on an open interval containing c. Which of the following statements is **true** about critical values?

   A. Every critical value corresponds to a relative maximum or minimum.

   B. A function can have a critical value at a point where it is not continuous.

   C. If $f'(c) = 0$, then c must be a critical value.

   D. A critical value cannot occur where the graph has a sharp corner or cusp.

   E. If f has no critical values, then it must be constant.

   Solution: A critical value is a point c in the domain of f where $f'(c) = 0$ or $f'(c)$ does not exist. Statement C correctly identifies that if $f'(c) = 0$, then c is indeed a critical value. Other options are false or misleading.

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2. (5 points) Suppose $g'(a) = 0$ and g is differentiable at $x = a$. Which of the following must be true?

   A. $x = a$ is a critical value of g

   B. $x = a$ is a relative maximum

   C. $x = a$ is a relative minimum

   D. $g'(x) = 0$ for all values of x

   E. None of the above

   Solution: $g'(a) = 0$ means $x = a$ is a critical number. No info about max or min.

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3. (5 points) Consider the graph of $f(x)$ in Figure 2. Which option lists two correct critical points?

   A. $(1, 2)$ and $(5, 2)$

   B. $(2, 1)$ and $(4, 4)$

   C. $(3, 5)$ and $(0, 3)$

   D. $(5, 2)$ and $(0, 7)$

   E. None of the above

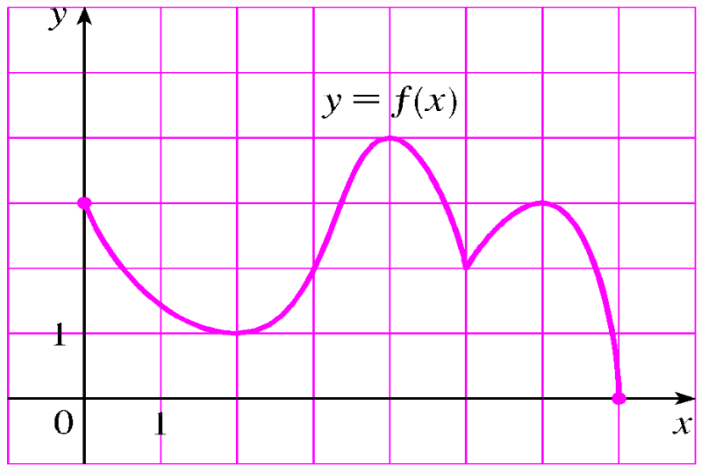

Figure 1: Graph of $f(x)$

Solution: A critical point occurs at a value $x = c$ in the domain of f where $f'(c) = 0$ or $f'(c)$ is undefined. From the graph, the points $(2, 1)$ and $(4, 4)$ correspond to such critical points.

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4. (5 points) A company sells custom water bottles. Market research shows that if the price is p dollars per bottle, the number of bottles sold each week is

$$q(p) = 800 - 40p, \quad R(p) = p \cdot q(p).$$

Find the price that maximizes weekly revenue.

A. $p = 5$

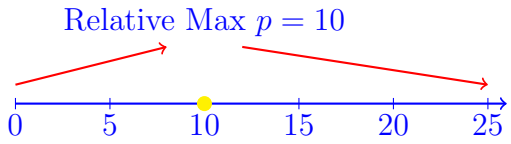
B. $p = 8$

C. $\boxed{p = 10}$

D. $p = 12$

E. None of the above

Solution: Revenue is $R(p) = p(800 - 40p) = 800p - 40p^2$.
The derivative $R'(p) = 800 - 80p$.
Set $R'(p) = 0$: $800 - 80p = 0 \implies p = 10$.



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5. (5 points) Consider $f(x) = \dfrac{\ln x}{x}$. A possible critical value is:

A. $x = 0$

B. $x = \dfrac{1}{e}$

C. $x = 1$

D. $\boxed{x = e}$

E. None of the above

Solution: Compute $f'(x) = \dfrac{1 - \ln x}{x^2}$.
Set $f'(x) = 0$: $1 - \ln x = 0 \implies \ln x = 1 \implies x = e$.
So $x = e$ is a critical value.

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6. (5 points) A startup tracks the number of customer reviews it receives per week:

$$r(t) = 8t^2 + 5t \quad \text{(reviews per week)},$$

and the average helpfulness score per review grows exponentially:

$$h(t) = e^{0.1t} \quad \text{(points per review)}.$$

The total helpfulness points of all reviews is

$$H(t) = r(t) \cdot h(t).$$

How fast is the total helpfulness score changing at **week 6**? (Round to nearest point per week.)

A. Increasing at 220 points per week
B. Increasing at 242 points per week
C. Increasing at 260 points per week
D. Decreasing at 242 points per week
E. Decreasing at 220 points per week

Solution: Use the product rule:

$$H'(t) = r'(t)h(t) + r(t)h'(t), \quad r'(t) = 16t + 5, \quad h'(t) = 0.1e^{0.1t}.$$

At $t = 6$:

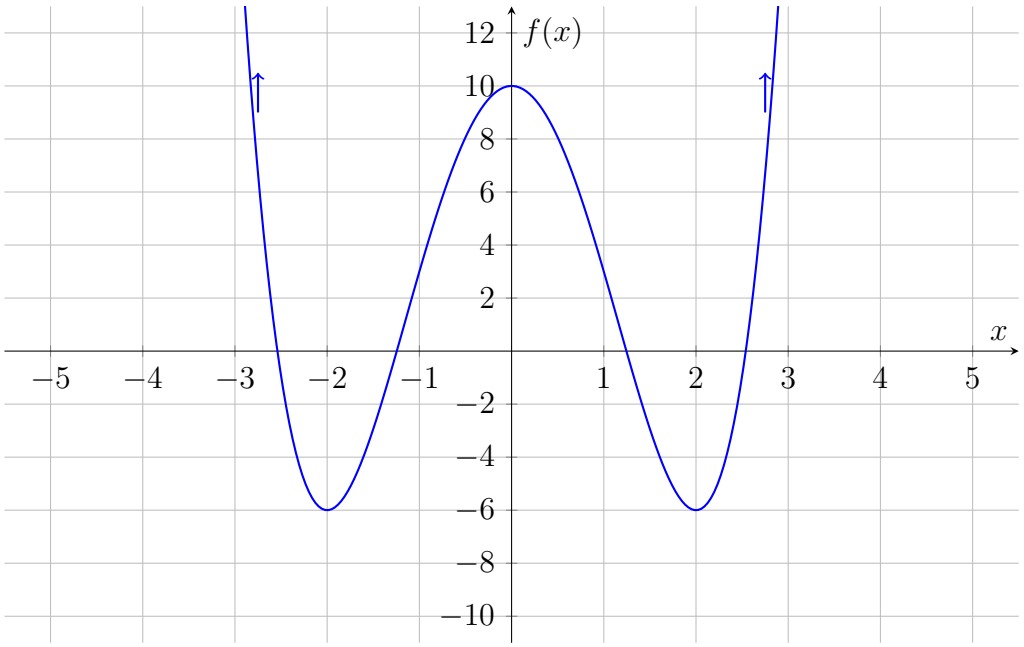
$$r'(6) = 101, \quad r(6) = 318, \quad h(6) = e^{0.6} \approx 1.822, \quad h'(6) = 0.1822$$

$$H'(6) \approx 101(1.822) + 318(0.1822) = 242 \text{ points per week.}$$

So total helpfulness is increasing at about 242 points per week.

Part II - Short Answer

7. (12 points) Examine the graph of the function $f(x)$ below. Use it to answer all the questions in the table.



Graph of $f(x)$

Instructions: Fill in the table below by carefully analyzing the graph. For each entry:

- Provide exact x-values where possible. Separate multiple answers using commas.
- For increasing/decreasing intervals, write them in interval notation.
- Use the graph to estimate values if necessary.

Find all relative minima.	$x = -2, 2$
Find all relative maxima.	$x = 0$
Find all absolute extrema.	Absolute minimum: $f(-2) = f(2) = -6$, Absolute maximum: does not exist
Where is $f'(x) = 0$?	$x = -2, 0, 2$
Intervals where f is deccreasing?	$(-\infty, -2), (0, 2)$
Intervals where f is increasing?	$(-2, 0), (2, \infty)$

Part III - Free Response

8. (15 points) Let the function $f(x) = 2x^3 - 15x^2 + 24x - 5$ be defined on the interval $[-1, 6]$.

a) Find $f'(x)$.

$$f'(x) = 6x^2 - 30x + 24$$

b) Find the critical point(s) of f algebraically. Write your answer as a list of ordered pairs.

$$\text{Set } f' = 0$$
$$x^2 - 5x + 4 = 0$$
$$(x-1)(x-4) = 0$$
$$x = 1, 4$$
$$f(1) = 6, \quad f(4) = -21$$

Critical points:

$$(1, 6),\ (4, -21)$$

c) Find the value of the function at the endpoints of the interval $[-1, 6]$.

$$f(-1) = -46$$

$$f(6) = 31$$

Endpoints:

$$(-1, -46),\ (6, 31)$$

d) Fill in the table below with the x and $y = f(x)$ coordinates of the points that you found in parts (b) and (c). *Leave any unused boxes blank.*

x	-1	1	4	6		
$y = f(x)$	-46	6	-21	31		

e) Use part (d) to find the location of the **absolute** extrema of f on $[-1, 6]$. State each extremum as an ordered pair.
Compare values from table above: $-46, 6, -21, 31$
Absolute minimum: $(-1, -46)$
Absolute maximum: $(6, 31)$

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9. (6 points) A company is evaluating how changes in its daily production level affect its profit from selling a new product. The company uses x to represent the change in production, measured in hundreds of units, relative to its current production level. Thus, negative values of x represent producing fewer units than the current level, while positive values represent producing more.

   The company's daily profit, measured in hundreds of dollars, is modeled by

$$f(x) = 3x^3 - 18x^2 - 45x - 20$$

   where x is measured in hundreds of units and $f(x)$ is the profit in hundreds of dollars. For operational reasons, the company considers production changes only within the interval $[-2, 9]$

   1. (3 points) Find all critical points of the profit function. Write your answers in the form $(x, f(x))$.

$$f'(x) = 9x^2 - 36x - 45 = 9(x^2 - 4x - 5) = 9(x - 5)(x + 1)$$

      Set $f'(x) = 0$:

$$9(x - 5)(x + 1) = 0$$

      Critical values:

$$x = -1,\ x = 5$$

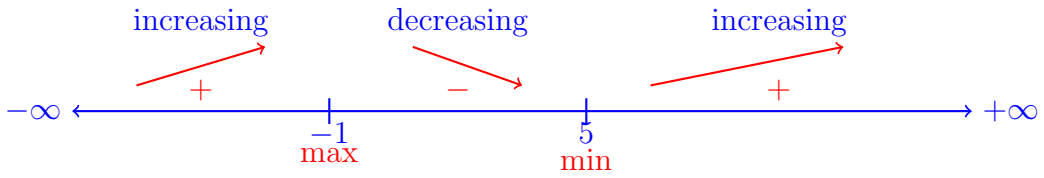
      Critical points:

$$(-1, 4),\ (5, -320)$$

   2. (3 points) Determine the production levels where the company experiences a relative maximum or minimum profit. Write your answers in the form $(x, f(x))$. (**Hint:** Use the First Derivative Test)
      Identify the relative extrema. Since

$$f'(x) = 9(x - 5)(x + 1),$$

      we analyze the sign of $f'(x)$.



$$(+) \to (-) \text{ at } x = -1 \Rightarrow \text{relative maximum}$$
$$(-) \to (+) \text{ at } x = 5 \Rightarrow \text{relative minimum}$$

      Relative maximum:

$$(-1, 4)$$

      Relative minimum:

$$(5, -320)$$

      Interpretation: Since $f(x)$ represents profit in hundreds of dollars, positive values correspond to a profit and negative values correspond to a loss.
      At $x = -1$, the company achieves a relative maximum profit of \$400.
      At $x = 5$, the company experiences a relative minimum profit of -320 hundred dollars, which represents a loss of \$32,000.

10. (15 points) Find the derivatives. Do not simplify.

a) $h(x) = \dfrac{4x^3 - 5\sqrt{x}}{e^{x^2}}$

Solution:

$$f = 4x^3 - 5x^{1/2} \qquad g = e^{x^2}$$
$$f' = 12x^2 - \frac{5}{2}x^{-1/2} \qquad g' = e^{x^2}(2x)$$
$$h'(x) = \frac{f'g - fg'}{g^2}$$
$$h'(x) = \frac{\left(12x^2 - \frac{5}{2}x^{-1/2}\right)e^{x^2} - (4x^3 - 5x^{1/2}) \cdot e^{x^2}(2x)}{(e^{x^2})^2}$$

b) $k(x) = 5\left(3x^2 - \frac{4}{x}\right)^{-6}$

Solution:

$$k'(x) = 5(-6)\left(3x^2 - \frac{4}{x}\right)^{-7}\left(6x + \frac{4}{x^2}\right)$$
$$= (-30)\left(3x^2 - \frac{4}{x}\right)^{-7}\left(6x + \frac{4}{x^2}\right)$$

c) $f(x) = \ln(5x^4 - 7x + 2)$

Solution:

$$f'(x) = \frac{1}{5x^4 - 7x + 2}(20x^3 - 7)$$

d) $p(x) = \dfrac{2}{x^4}(\sqrt{x^2 + 4})$

Solution:

$$f = \frac{2}{x^4} = 2x^{-4} \qquad g = (x^2 + 4)^{1/2}$$
$$f' = -8x^{-5} \qquad g' = \frac{1}{2}(x^2 + 4)^{-1/2}(2x)$$
$$p'(x) = f'g + fg'$$
$$p'(x) = (-8x^{-5})(x^2 + 4)^{1/2} + \frac{2}{x^4}\left[\frac{1}{2}(x^2 + 4)^{-1/2}(2x)\right]$$

e) $r(x) = \left(4\sqrt{x} + \frac{2}{x^3}\right)^2 \ln(x^2 + 1)$

Solution:

$$f = \left(4x^{1/2} + 2x^{-3}\right)^2 \qquad g = \ln(x^2 + 1)$$
$$f' = 2\left(4x^{1/2} + 2x^{-3}\right)\left(2x^{-1/2} - 6x^{-4}\right) \qquad g' = \frac{1}{x^2 + 1}(2x)$$
$$r'(x) = f'g + fg'$$
$$r'(x) = 2\left(4x^{1/2} + 2x^{-3}\right)\left(2x^{-1/2} - 6x^{-4}\right)\ln(x^2 + 1) + \left(4x^{1/2} + 2x^{-3}\right)^2 \frac{1}{x^2 + 1}(2x)$$

11. (12 points) A delivery company has a total cost (in dollars) to ship n packages given by

$$C(n) = 300 + 5n + 0.1n^2.$$

a) Find the average cost per package $AC(n)$.
Solution:

$$AC(n) = \frac{C(n)}{n} = \frac{300 + 5n + 0.1n^2}{n}$$

$$AC(n) = \frac{300}{n} + 5 + 0.1n$$

b) Find the number of packages n that minimizes the average cost.
Solution: Find critical points

$$AC'(n) = -\frac{300}{n^2} + 0.1$$

Set $AC'(n) = 0$:

$$-\frac{300}{n^2} + 0.1 = 0$$

$$0.1 = \frac{300}{n^2}$$

$$0.1n^2 = 300$$

$$n^2 = 3000$$

$$n = \sqrt{3000} \approx 55$$

Since $AC'(n) < 0$ for $n < 55$ and $AC'(n) > 0$ for $n > 55$, the First Derivative Test confirms that $AC(n)$ has a minimum at $n = \sqrt{3000} \approx 55$ items.
Minimum average cost occurs at $n = \sqrt{3000} \approx 55$ items.

c) Find the minimum average cost and explain what it means in the context of shipping.

$$AC(\sqrt{3000}) = \frac{300}{\sqrt{3000}} + 5 + 0.1\sqrt{3000}$$

$$AC(\sqrt{3000}) \approx 15.95$$

The minimum average cost is approximately \$15.95 per package.

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12. (8 points) A small town has a population of 2,500, and it is growing at a monthly rate of r (as a decimal). The population after 8 years is modeled by

$$P(r) = 2500\left(1 + \frac{r}{12}\right)^{96}.$$

a) (4 points) Find the derivative $P'(r)$.

$$P'(r) = 2500(96)\left(1+\frac{r}{12}\right)^{95}\left(\frac{1}{12}\right)$$
$$= 20000\left(1+\frac{r}{12}\right)^{95}$$

b) (4 points) Find the marginal rate of change of the population when $r = 2.4\%$. Include units.
Note that $2.4\% = 0.024$,

$$P'(0.024) = 20000\left(1+\frac{0.024}{12}\right)^{95}$$
$$\approx 24180$$

The population is increasing at a rate of approximately 24,180 people per unit increase in r.
In other words, increasing the monthly growth rate r by 1 unit increases the population after 8 years by approximately 24,180 people.

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13. (12 points) A technology company is launching a new model of wireless headphones. Market analysis shows that in order to sell x units, the price per unit (in dollars) must be

$$p(x) = 180 - 3x,$$

and the total cost (in dollars) of producing x units is

$$C(x) = 1500 + 12x.$$

a) (5 points) Write the profit function $P(x)$ in terms of x.

$$R(x) = x\,p(x) = x(180-3x) = 180x - 3x^2$$
$$P(x) = R(x) - C(x)$$
$$P(x) = (180x - 3x^2) - (1500 + 12x)$$
$$P(x) = -3x^2 + 168x - 1500$$

b) (2 points) Find $P'(x)$.

$$P'(x) = -6x + 168$$

c) (5 points) Determine the price per unit that must be charged to maximize profit.
Set $P'(x) = 0$:

$$-6x + 168 = 0$$
$$x = 28$$
$$p(28) = 180 - 3(28) = 96$$

Since $P'(x) > 0$ for $x < 28$ and $P'(x) < 0$ for $x > 28$, the First Derivative Test confirms that profit is maximized at $x = 28$.
Price that maximizes profit: \$96