MAT 210 Brief Calculus
Sample Exam 2 (Practice Version B)
Solutions

Instructor	XXXXXXX
Full Name (Print clearly!)	
Student ID	

Important Disclaimer for Students:

- This is a **sample exam**, not the actual in-class exam. Problems are of similar difficulty but may differ in type, numbers, or context.
- Success in the course depends on practicing concepts beyond this sample: lecture notes, Edfinity, Group Work, quizzes, and homework.
- You should complete this exam as if you were in the real exam room: no notes, no internet, no calculators beyond what is allowed, and an approximately 60-minute time limit.
- After finishing, check your answers against the provided solutions and grade yourself.
- If you could not complete within the time limit, practice again and attempt another sample exam.
- Treat every problem in your coursework (notes, quizzes, assignments) as fair game for the actual exam.

Part I - Multiple Choice: Circle the response that best completes the statement or answers the question. Then write the letter of your answer in the table after Problem 7.

1. (5 points) Which of the following conditions must be true for $x = c$ to be a critical value of the function $g(x)$?

   A. $g'(c)$ exists and equals 0

   B. $\boxed{g(c) \text{ is defined, and either } g'(c) = 0 \text{ or } g'(c) \text{ does not exist.}}$

   C. $x = c$ must correspond to a local maximum or minimum of $g(x)$

   D. $g'(c)$ is positive at $x = c$

   E. None of the above

Solution: A critical point occurs where the derivative of the function is zero or does not exist, provided the function itself is defined at that point. Therefore, option B is correct.

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2. (5 points) If $h(2)$ exists, what can we conclude if $h'(2)$ does not exist?

   A. $x = 2$ cannot be a critical value since the derivative does not exist.

   B. $x = 2$ must be a relative maximum or a relative minimum.

   C. $\boxed{x = 2 \text{ is a critical value.}}$

   D. $x = 2$ cannot be a relative extrema value.

   E. Nothing can be concluded without more information about the function.

Solution: A critical point occurs where the derivative is zero or does not exist, provided the function is defined at that point. Since $h(2)$ exists and $h'(2)$ does not exist, $x = 2$ is a critical value.

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3. (5 points) Consider the graph of the function $g(x)$ in Figure 2. The graph has four critical values. Which option correctly identifies **two** of the critical points (*as ordered pairs of* x *and* $g(x)$)?

A. $\boxed{(0,-30) \text{ and } (-3,0)}$

B. $(0,0)$ and $(4,10)$

C. $(-3,-30)$ and $(4,0)$

D. $(0,-10)$ and $(-4,0)$

E. None of the above

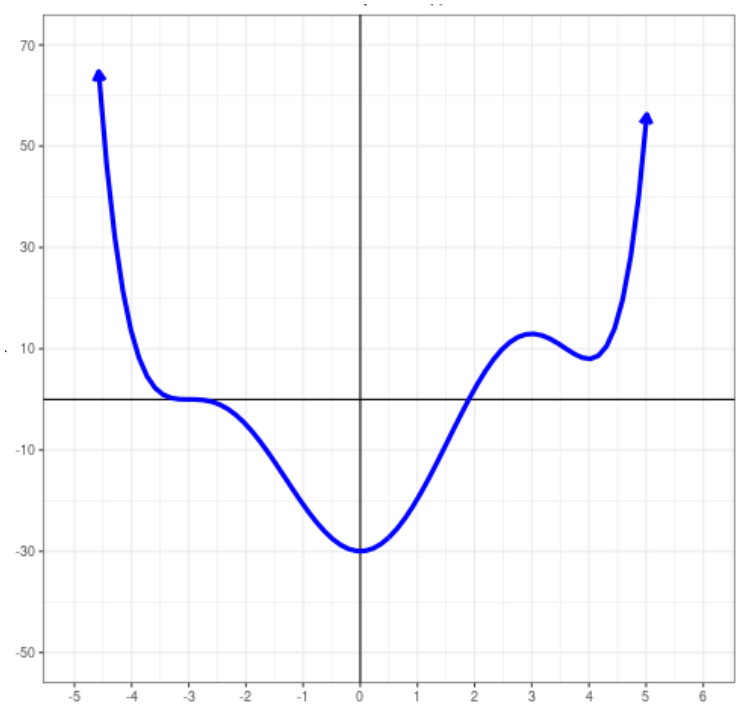

Figure 1: Graph of $g(x)$

Solution: Critical points occur where the derivative is zero or does not exist. Observing the graph, at $x = 0$ and $x = -3$, the tangent lines are horizontal (slope 0). Therefore, these points are critical points.

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4. (5 points) A small business models revenue with a nonlinear demand function:

$$q(p) = 750 - 10p^2, \quad R(p) = p \cdot q(p)$$

where p is the price per unit (in dollars) and $q(p)$ is the number of units sold.

Determine the price that maximizes revenue.

A. $p = 4$

B. $\boxed{p = 5}$

C. $p = 6$

D. $p = 7$

E. None of the above

Solution: To maximize revenue, find $R'(p)$ and set it to zero:

$$R(p) = p(750 - 10p^2) = 750p - 10p^3, \quad R'(p) = 750 - 30p^2.$$

Set $R'(p) = 0$: $750 - 30p^2 = 0 \implies p^2 = 25 \implies p = 5$ (positive price).
Since $R'(p) > 0$ for $p < 5$ and $R'(p) < 0$ for $p > 5$, the First Derivative Test confirms that $p = 5$ gives a maximum. Thus, the revenue is maximized at \$5 per unit.

5. (5 points) Consider the function

$$f(x) = xe^{2x}.$$

Which of the following is a **possible critical value** of $f(x)$ on $(-\infty, \infty)$?

A. $x = 0$

B. $x = 1$

C. $\boxed{x = -\frac{1}{2}}$

D. $x = 2$

E. None of the above

Solution: To find critical points, compute the derivative:

$$f'(x) = \frac{d}{dx}\left(xe^{2x}\right) = e^{2x} + 2xe^{2x} = e^{2x}(1 + 2x).$$

Set $f'(x) = 0$: $e^{2x}(1 + 2x) = 0$. Since $e^{2x} \neq 0$ for all x, we get $1 + 2x = 0 \implies x = -\frac{1}{2}$. The correct critical value is $x = -\frac{1}{2}$, so option C is correct.

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6. (5 points) The number of hats made by a manufacturer t days after the start of the month and the cost per hat (in dollars) are modeled as

$$n(t) = 16t^2 + 2t, \quad c(t) = \ln(230t),$$

so that the total cost of the hats is

$$v(t) = n(t) \cdot c(t).$$

Determine how the total cost is changing on the 16th day. (Round your answer to the nearest cent.)

A. $\boxed{\text{The total cost is increasing at a rate of 4478.28 dollars per day on the 16th day.}}$

B. The total cost is increasing at a rate of 4320.50 dollars per day on the 16th day.

C. The total cost is decreasing at a rate of 4600.75 dollars per day on the 16th day.

D. The total cost is decreasing at a rate of 4200.00 dollars per day on the 16th day.

E. None of the above.

Solution: The total cost is given by $v(t) = n(t) \cdot c(t) = (16t^2 + 2t)\ln(230t)$. To find how the cost is changing, compute the derivative using the product rule:

$$v'(t) = n'(t)c(t) + n(t)c'(t).$$

Step 1: Compute derivatives of $n(t)$ and $c(t)$:

$$n'(t) = 32t + 2, \quad c'(t) = \frac{1}{t}.$$

Step 2: Substitute $t = 16$:

$$v'(16) = 4478.28 \textit{dollars per day.}$$

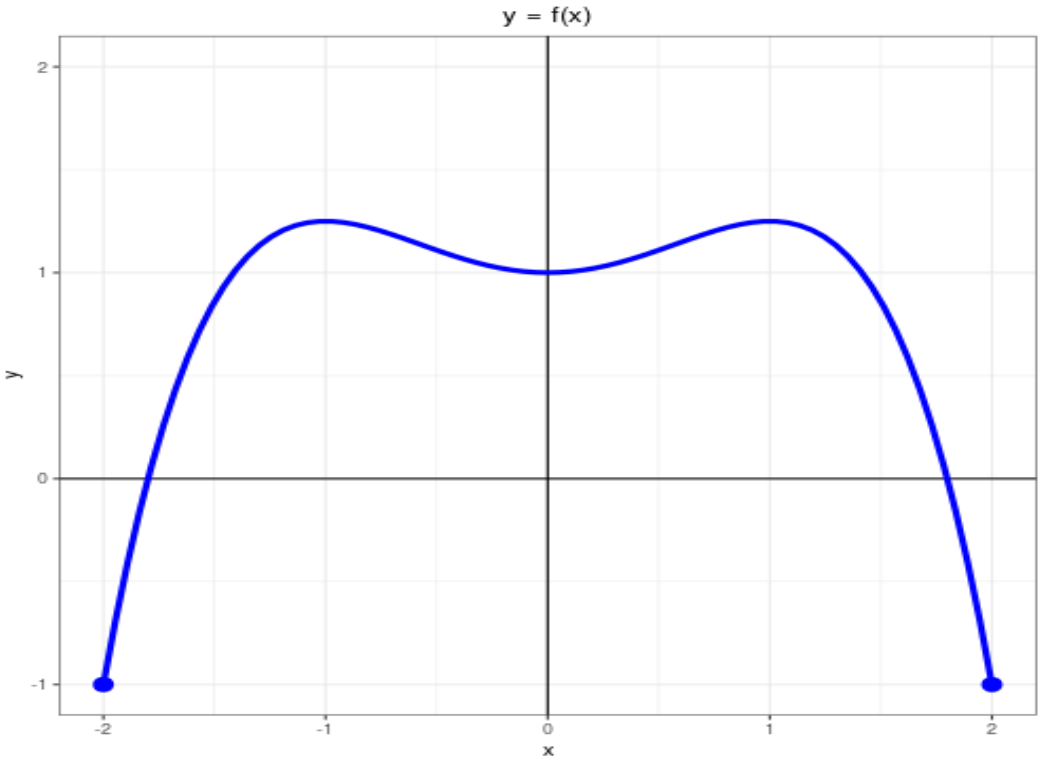
Answer: Option A. The total cost is increasing at a rate of 4478.28 dollars per day on the 16th day.

PART I Answers: Please write the letter of your selected answers to multiple choice questions 1 through 6 in the following table.

Problem	1	2	3	4	5	6
Answer	B	C	A	B	C	A

Part II - Short Answer question.

7. (12 points) The function $f(x)$ is graphed below. Use the graph to answer the following questions. Show all answers in the spaces provided.



Question	Answer
1. Find the value(s) of x where $f(x)$ has a relative minimum.	0
2. Find the value(s) of x where $f(x)$ has a relative maximum.	-1 and 1
3. Find the value(s) of x where $f(x)$ has an absolute minimum.	-2 and 2
4. Find the value(s) of x where $f'(x) = 0$.	-1, 0, and 1
5. Find the interval(s) where $f'(x) > 0$.	(-2,-1) and (0,1)
6. Find the interval(s) where $f'(x) < 0$.	(-1,0) and (1,2)

Solution: Observe the graph to determine the absolute and relative extrema based on where the function increases or decreases and where the tangent lines are horizontal.

Part III - Free Response: You must show your work in the space provided to receive full credit. Correct final answers without sufficient supporting work may receive zero credit.

Please put a box around your final answers.

8. (15 points) Let the function $f(x) = 3x^3 - 18x^2 + 27x - 10$ be defined on the interval $[-2, 5]$.

a) Find $f'(x)$.

Solution: $f'(x) = 9x^2 - 36x + 27 = 9(x^2 - 4x + 3) = 9(x - 1)(x - 3)$

b) Find the critical point(s) of f algebraically. Write your answer as a list of ordered pairs.

Solution: Critical points occur when $f'(x) = 0$.
Solve $9(x - 1)(x - 3) = 0$ to get $x = 1$ and $x = 3$.
Compute $f(1) = 3 - 18 + 27 - 10 = 2$, $f(3) = 81 - 162 + 81 - 10 = -10$.
Thus, critical points: $(1, 2)$ and $(3, -10)$.

c) Find the value of the function at the endpoints of the interval $[-2, 5]$.

Solution: $f(-2) = -24 - 72 - 54 - 10 = -160$,
$f(5) = 375 - 450 + 135 - 10 = 50$.
Endpoints: $(-2, -160)$, $(5, 50)$.

d) Fill in the table below with the x and $y = f(x)$ coordinates of the points that you found in parts (b) and (c). *Leave any unused boxes blank.*

x	1	3	-2	5		
$y = f(x)$	2	-10	-160	50		

e) Use part (d) to find the location of the **absolute** and **relative** extrema of f on $[-2, 5]$. State each extremum as an ordered pair and label whether it is absolute or relative.

Solution:

- Relative maximum: $(1, 2)$
- Relative minimum: $(3, -10)$
- Absolute maximum: $(5, 50)$
- Absolute minimum: $(-2, -160)$

(Use the first derivative test:
the derivative changes from positive to negative at $x = 1 \rightarrow$ relative max
-negative to positive at $x = 3 \rightarrow$ relative min.
Compare critical points and endpoints for absolute extrema.)

9. (6 points) Consider the function $f(x)$ and answer the following questions.

$$f(x) = 2x^3 - 9x^2 - 24x + 20$$

a) (3 points) Find all critical points of the function. Write your answers in the form of $(x, f(x))$.

Solution:

$$f'(x) = 6x^2 - 18x - 24 = 6(x^2 - 3x - 4) = 6(x-4)(x+1)$$

Set $f'(x) = 0$:

$$x - 4 = 0 \implies x = 4, \quad x + 1 = 0 \implies x = -1$$

Evaluate $f(x)$ at these points:

$$f(-1) = 2(-1)^3 - 9(-1)^2 - 24(-1) + 20 = -2 - 9 + 24 + 20 = 33$$

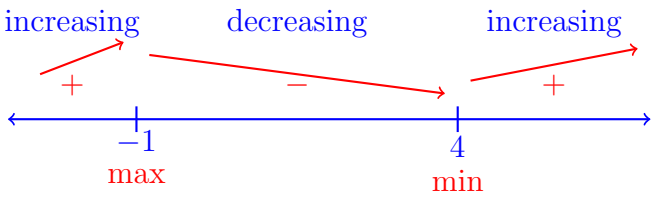
$$f(4) = 2(64) - 9(16) - 24(4) + 20 = 128 - 144 - 96 + 20 = -92$$

Critical points:

$$(-1, 33), \quad (4, -92)$$

b) (3 points) Identify all relative extrema. Write your answers in the form $(x, f(x))$. (**Hint:** Use the First Derivative Test.)

c) **Sign chart of $f'(x) = 6(x+1)(x-4)$:**



Relative maximum:

$$(-1, 33)$$

Relative minimum:

$$(4, -92)$$

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10. (15 points) Determine the derivative of the following functions. Show your work. You **do NOT need** to simplify your results.

a) $h(x) = \dfrac{5x^2 - 6\sqrt{x}}{e^{2x^3}}$

Solution: Use the quotient rule:

$$h'(x) = \frac{(10x - 3x^{-1/2})e^{2x^3} - (5x^2 - 6\sqrt{x})(6x^2 e^{2x^3})}{(e^{2x^3})^2}.$$

b) $g(x) = 3\left(2x^4 - \frac{6}{x}\right)^{-8}$

Solution: Use the chain rule:

$$g'(x) = 3 \cdot (-8)\left(2x^4 - \frac{6}{x}\right)^{-9} \cdot \left(8x^3 + \frac{6}{x^2}\right) = -24\left(2x^4 - \frac{6}{x}\right)^{-9}\left(8x^3 + \frac{6}{x^2}\right).$$

c) $k(x) = \ln(2x^5 - 10x + 3)$

Solution: Use the derivative of ln(u):

$$k'(x) = \frac{10x^4 - 10}{2x^5 - 10x + 3}.$$

d) $m(x) = \frac{3}{2x^5} \cdot \sqrt{x^2 + 1}$

Solution: Use the product rule: Let $u = \frac{3}{2x^5}$, $v = (x^2+1)^{1/2}$. Then

$$m'(x) = u'v + uv' = \left(-\frac{15}{2}x^{-6}\right)\sqrt{x^2+1} + \frac{3}{2x^5} \cdot \frac{x}{\sqrt{x^2+1}}.$$

e) $f(x) = \left(7 + \frac{3}{x^2}\right)^3 \ln(x^3 + 2x)$

Solution: Let

$$u = \left(7 + \frac{3}{x^2}\right)^3 \quad \text{and} \quad v = \ln(x^3 + 2x).$$

Then by the product rule,

$$f'(x) = u'v + uv'.$$

We have

$$u' = 3\left(7 + \frac{3}{x^2}\right)^2 \cdot \frac{d}{dx}\left(7 + \frac{3}{x^2}\right) = 3\left(7 + \frac{3}{x^2}\right)^2\left(-\frac{6}{x^3}\right),$$

and

$$v' = \frac{d}{dx}\ln(x^3 + 2x) = \frac{3x^2 + 2}{x^3 + 2x}.$$

Substituting into the product rule,

$$f'(x) = 3\left(7 + \frac{3}{x^2}\right)^2\left(-\frac{6}{x^3}\right)\ln(x^3 + 2x) + \left(7 + \frac{3}{x^2}\right)^3 \cdot \frac{3x^2 + 2}{x^3 + 2x}.$$

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11. (12 points) An audio products company is deciding on the retail price for a new pair of wireless earbuds. Market research suggests that at a price of $\$p$ (dollars) the estimated number of units sold is

$$q(p) = 150000 - 2500p.$$

a) (8 points) Find the price p (in dollars) that maximizes the revenue $R(p)$. Show your work and include units.

Solution: The revenue function is

$$R(p) = p \cdot q(p) = p(150000 - 2500p) = 150000p - 2500p^2.$$

To maximize revenue, take the derivative and set it equal to zero:

$$R'(p) = 150000 - 5000p = 0 \implies p = 30.$$

Therefore, the price that maximizes revenue is $\boxed{p = \$30}$.

b) (4 points) What is the maximum revenue? Include units and interpret your answer in a sentence.

Solution: Substitute $p = 30$ into the revenue function:

$$R(30) = 30(150000 - 2500 \cdot 30) = 30(150000 - 75000) = 30 \cdot 75000 = 2,250,000.$$

Since $R'(p) > 0$ for $p < 30$ and $R'(p) < 0$ for $p > 30$, the First Derivative Test confirms that $p = 30$ gives a maximum.
The maximum revenue is $\boxed{\$2,250,000}$. This means if the company charges \$30 per pair, the total sales revenue will be maximized at \$2,250,000.

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12. (8 points) If \$1000 are invested at an annual interest rate r compounded monthly, the amount in the account at the end of 5 years is given by

$$A(r) = 1000\left(1 + \frac{r}{12}\right)^{60}.$$

a) (4 points) Find the derivative $A'(r)$.

Solution: Using the chain rule:

$$A'(r) = 1000 \cdot 60\left(1 + \frac{r}{12}\right)^{59} \cdot \frac{1}{12} = 5000\left(1 + \frac{r}{12}\right)^{59}.$$

b) (4 points) Find the marginal rate of change of the amount when $r = 3.95\%$. Include units and interpret your answer in context.

Solution: Substitute $r = 0.0395$:

$$A'(0.0395) = 5000\left(1 + \frac{0.0395}{12}\right)^{59} \approx 6069.81.$$

Thus, the amount is increasing at approximately \$6069.81 per unit of interest rate.
Interpretation: When the interest rate is (3.95%), an increase of 1 unit in the interest rate would increase the account balance by approximately \$6069.81.

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13. (8 points) The volume (in liters) of water in a tank changes according to

$$V(t) = t^3 e^{0.02t^2}, \quad t \geq 0 \text{ hours.}$$

a) Find $V'(t)$.

Solution: Use the product rule:

$$V'(t) = f'g + g'f = 3t^2 e^{0.02t^2} + t^3 \cdot e^{0.02t^2} \cdot 0.04t = e^{0.02t^2}(3t^2 + 0.04t^4).$$

b) Find the **marginal rate of change of the water volume** at $t = 5$ hours. Interpret your answer in context and include appropriate units.

Solution: Substitute $t = 5$:

$$V'(5) = e^{0.02(25)}\,[3(25) + 0.04(625)] = e^{0.5}(75 + 25) = 100e^{0.5} \approx 164.87 \text{ liters/hour.}$$

Interpretation: At $t = 5$ hours, the volume of water is increasing at approximately 164.87 liters per hour.
In other words, adding an additional hour, the volume will increase by approximately 164.87 liters.