MAT 210 Brief Calculus
Sample Exam 2 (Practice Version A)
Solutions

Instructor	XXXXXXX
Full Name (Print clearly!)	
Student ID	

Important Disclaimer for Students:

- This is a **sample exam**, not the actual in-class exam. Problems are of similar difficulty but may differ in type, numbers, or context.
- Success in the course depends on practicing concepts beyond this sample: lecture notes, Edfinity, Group Work, quizzes, and homework.
- You should complete this exam as if you were in the real exam room: no notes, no internet, no calculators beyond what is allowed, and an approximately 60-minute time limit.
- After finishing, check your answers against the provided solutions and grade yourself.
- If you could not complete within the time limit, practice again and attempt another sample exam.
- Treat every problem in your coursework (notes, quizzes, assignments) as fair game for the actual exam.

Part I - Multiple Choice: Circle the response that best completes the statement or answers the question. Then write the letter of your answer in the table after Problem 6.

1. (5 points) The population of a certain bacterial culture (*in hundreds*) t hours after being placed in a nutrient solution is modeled by

$$P(t) = \sqrt{500t + 2800}.$$

Find the rate at which the population is growing *(in organisms per hour)* after 6 hours. (Round your answer to the nearest whole organism.)

A. After 6 hours the population is increasing by 250 organisms per hour

B. **After 6 hours the population is increasing by 328 organisms per hour**

C. After 6 hours the population is increasing by 500 organisms per hour

D. After 6 hours the population is decreasing by 250 organisms per hour

E. After 6 hours the population is decreasing by 500 organisms per hour

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Solution: We are given $P(t) = \sqrt{500t + 2800}$ (in hundreds).

$$P'(t) = \frac{1}{2}(500t + 2800)^{-1/2}(500) = \frac{250}{\sqrt{500t + 2800}}.$$

At $t = 6$:

$$P'(6) = \frac{250}{\sqrt{500(6) + 2800}} = \frac{250}{\sqrt{5800}} \approx 3.28.$$

Since $P(t)$ is in hundreds, the actual rate is

$$100 \times 3.28 = 328 \text{ organisms per hour.}$$

Therefore, the population is increasing at approximately 328 organisms per hour.

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2. (5 points) Refer to the graph of $f(x)$ in Figure 1. Fill in the blanks to correctly describe where $f(x)$ is increasing and decreasing.

A. $f(x)$ is increasing on $(2, 5)$; decreasing on $(0, 2)$.

B. $f(x)$ is increasing on $(0, 5)$ and decreasing on $(-\infty, 0)$ and $(2, \infty)$.

C. $f(x)$ is increasing only on $(2, \infty)$ and decreasing elsewhere.

D. $f(x)$ is decreasing over two intervals as shown in the graph.

E. None of the above

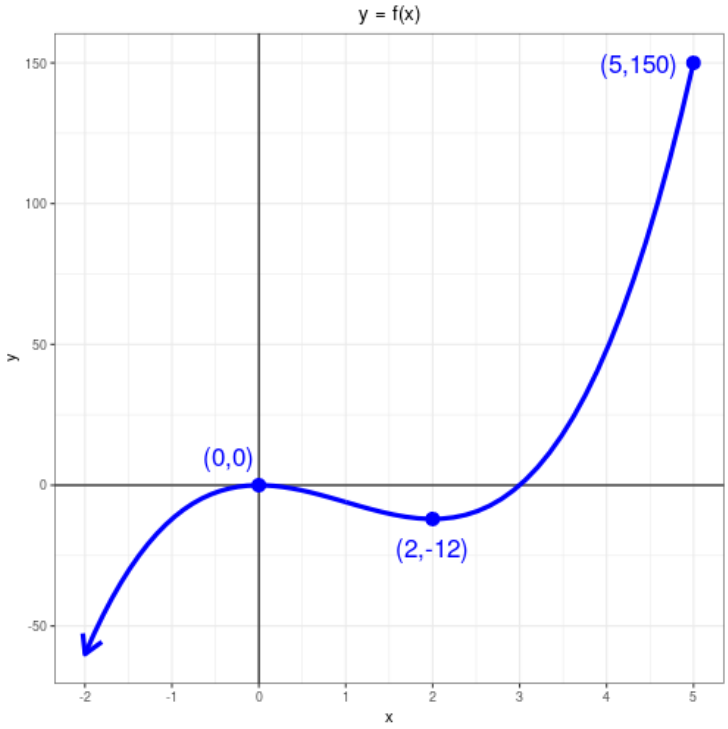

Figure 1: Graph of $f(x)$

Solution: Observe the slopes of the tangents on the graph. Where tangent slopes are negative the function is decreasing, and where tangent slopes are positive the function is increasing. From the graph the tangent slopes are positive on $(-\infty, 0)$ and $(2, 5)$ and negative on $(0, 2)$, so option **A** is correct.

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3. (5 points) Find the derivative of $f(x) = x^2 e^{3x^2} - (2x^2 - x + 1)^5$.

A. $2xe^{3x^2} + 6x^3 e^{3x^2} - 5(4x + 1)(2x^2 - x + 1)^4$

B. $e^{3x^2} - 5(2x^2 - x + 1)^4$

C. $2xe^{3x^2} + 6x^3 e^{3x^2} - 5(4x - 1)(2x^2 - x + 1)^4$

D. $2xe^{3x^2} + 6x^3 e^{3x^2} - 5(2x^2 - x + 1)^4$

E. None of the above

Solution: Use the product rule for the first term and the chain rule for the second.

$$\frac{d}{dx}\left(x^2 e^{3x^2}\right) = (2x)e^{3x^2} + x^2 \cdot e^{3x^2} \cdot (6x) = 2xe^{3x^2} + 6x^3 e^{3x^2}.$$

$$\frac{d}{dx}\left((2x^2 - x + 1)^5\right) = 5(2x^2 - x + 1)^4 \cdot (4x - 1).$$

Putting the pieces together (and remembering the minus sign):

$$f'(x) = 2xe^{3x^2} + 6x^3 e^{3x^2} - 5(4x - 1)(2x^2 - x + 1)^4,$$

so option **C** is correct.

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4. (5 points) Let

$$h(x) = \frac{4x^3 + 2}{\ln(x + 1)}, \qquad (x > -1).$$

Which of the following correctly represents $h'(x)$?

A. $h'(x) = \dfrac{12x^2}{\ln(x + 1)} - \dfrac{4x^3 + 2}{x + 1}$

B. $h'(x) = \dfrac{12x^2 \ln(x + 1) + \frac{4x^3+2}{x+1}}{[\ln(x + 1)]^2}$

C. $h'(x) = \dfrac{12x^2(\ln(x + 1))^2 - (4x^3 + 2)}{[\ln(x + 1)]^3}$

D. $h'(x) = \dfrac{12x^2 \ln(x + 1) - \dfrac{4x^3 + 2}{x + 1}}{[\ln(x + 1)]^2}$

E. None of the above

Solution: Use the quotient rule. For $h(x) = \dfrac{u(x)}{v(x)}$ with $u(x) = 4x^3 + 2$ and $v(x) = \ln(x+1)$,

$$h'(x) = \frac{u'(x)v(x) - u(x)v'(x)}{[v(x)]^2}.$$

Here $u'(x) = 12x^2$ and $v'(x) = \dfrac{1}{x+1}$. Hence

$$h'(x) = \frac{12x^2 \ln(x+1) - \left(4x^3+2\right)\dfrac{1}{x+1}}{[\ln(x+1)]^2},$$

which matches option **D**.

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5. (5 points) Which of the following functions has a **critical value** at $x = 0$? Be careful to consider the domain of each function. Assume all functions have domain $(-1, \infty)$.

A. $f(x) = x^3 - 3x^2 + 2$

B. $g(x) = x^4 - 4x^3 + 6x$

C. $h(x) = \ln(x + 10)$

D. $k(x) = e^x - x^2$

E. None of the above

Solution: A critical value occurs where $f'(x) = 0$ or $f'(x)$ does not exist, provided $f(x)$ is defined.

$$g'(x) = 4x^3 - 12x^2 + 6.$$

At $x = 0$, $g'(0) = 6 \neq 0$, so check others:

$$f'(x) = 3x^2 - 6x, \quad f'(0) = 0.$$

Do not forget to check this; $f'(0) = 0$ gives a critical point! But let's check domain — all real numbers are valid, so indeed $f(x)$ has a critical value at $x = 0$. None of the other derivatives $h'(x)$ or $k'(x)$ become 0 or undefined at $x = 0$.

Thus, the correct choice is **A**.

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6. (5 points) Suppose f is continuous at $x = c$ and its derivative $f'(x)$ changes from positive to negative at $x = c$. What can we conclude about f?

A. f has a relative minimum at $x = c$.

B. f has a relative maximum at $x = c$.

C. f is increasing on both sides of $x = c$.

D. f is decreasing on both sides of $x = c$.

E. None of the above.

Solution: The first derivative test tells us that if $f'(x)$ changes from positive to negative at $x = c$, the function is increasing before c and decreasing after c. Therefore, f reaches a relative maximum at $x = c$. Hence, the correct answer is **B**.

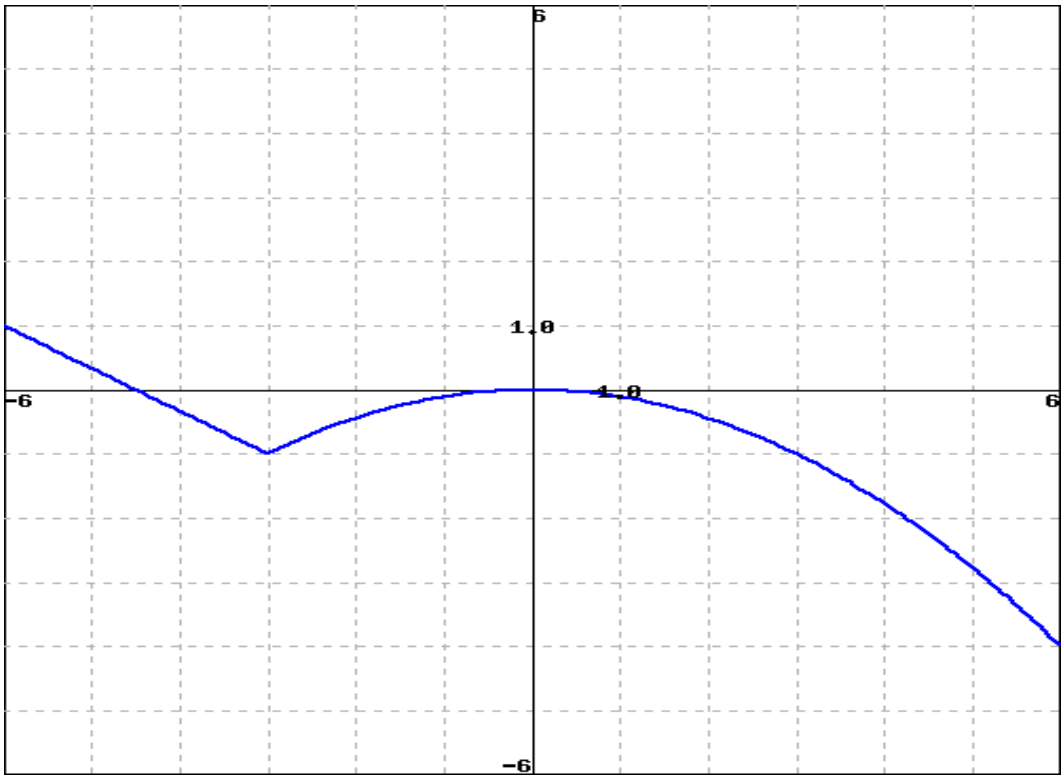
PART I Answers: Please write the letter of your selected answers to multiple choice questions 1 through 6 in the following table.

Problem	1	2	3	4	5	6
Answer	B	A	C	D	A	B

Part II - Short Answer question.

7. (6 points) The function $y = f(x)$ is graphed below. It has domain $(-\infty, \infty)$.

   Find the **absolute maximum** and **absolute minimum** y-values of $f(x)$ on the given intervals. If a value does not exist, write DNE. Round your answers to 4 decimal places if necessary.



Note: Below are **three** different intervals to consider. Examine the graph carefully for each interval before determining the absolute maximum and minimum.

a) For the interval $(-\infty, \infty)$, **Absolute Maximum:** DNE **Absolute Minimum:** DNE

b) For the interval $[-4, 3]$, **Absolute Maximum:**0 **Absolute Minimum:**-1

c) For the interval $[-4, 0)$, **Absolute Maximum:**DNE **Absolute Minimum:**-1

Explanation: Absolute maximum and minimum values are determined by looking at the highest and lowest points of the graph over the given interval.
- For $(-\infty, \infty)$, the graph extends indefinitely, so we cannot see the entire behavior of the function; therefore, the absolute maximum and minimum do not exist (DNE).
- For $[-4, 3]$, the highest point visible on the graph is 0 and the lowest is -1.
- For $[-4, 0)$, we cannot see the behavior as x approaches 0 from the left, so the absolute maximum is DNE, but the lowest point in this interval is -1.
This illustrates that if the full behavior of the graph is not visible on the interval, the absolute extrema may be undefined.

Part III - Free Response: You must show your work in the space provided to receive full credit. Correct final answers without sufficient supporting work may receive zero credit.

Please put a box around your final answers.

8. (15 points) Let the function $f(x) = 2x^3 - 9x^2 + 12x - 4$ be defined on the interval $[0, 4]$.

a) Find $f'(x)$.

Solution: Differentiate term by term:

$$f'(x) = 6x^2 - 18x + 12.$$

b) Find the critical point(s) of f algebraically. Write your answer as a list of ordered pairs.

Solution: Critical points occur where $f'(x) = 0$:

$$6x^2 - 18x + 12 = 0 \quad \Rightarrow \quad x^2 - 3x + 2 = 0 \quad \Rightarrow \quad (x-1)(x-2) = 0$$

So $x = 1$ and $x = 2$. Find corresponding y-values:

$$f(1) = 2 - 9 + 12 - 4 = 1, \quad f(2) = 16 - 36 + 24 - 4 = 0$$

$\Rightarrow$ Critical points: $(1, 1), (2, 0)$.

c) Find the value of the function at the endpoints of the interval $[0, 4]$.

Solution:

$$f(0) = -4, \quad f(4) = 128 - 144 + 48 - 4 = 28$$

$\Rightarrow$ Endpoints: $(0, -4), (4, 28)$.

d) Fill in the table below with the x and $y = f(x)$ coordinates of the points that you found in parts (b) and (c). *Leave any unused boxes blank.*

x	0	1	2	4		
$y = f(x)$	-4	1	0	28		

Solution: Coordinates from parts (b) and (c) are filled in the table as shown.

e) Use part (d) to find the location of the **absolute** extrema of f on $[0, 4]$. State each extremum as an ordered pair and label whether it is absolute max or min.

Solution: Compare all y-values from critical points and endpoints:

$$f(0) = -4, \ f(1) = 1, \ f(2) = 0, \ f(4) = 28$$

- Absolute maximum: $(4, 28)$
- Absolute minimum: $(0, -4)$

9. (6 points) A wildlife reserve is monitoring the population of a species of fish in a lake. The population (in thousands of fish) t years after monitoring begins is modeled by

$$P(t) = t^3 - 6t^2 + 9t + 5$$

where $t \geq 0$ is measured in years and $P(t)$ is the population in thousands. The model accounts for seasonal fluctuations and natural environmental effects over the first several years of monitoring.

a) (3 points) Find all critical points of the population function. Write your answers in the form $(t, P(t))$.
Solution: Find the critical points.
Population function:

$$P(t) = t^3 - 6t^2 + 9t + 5$$

$$P'(t) = 3t^2 - 12t + 9 = 3(t-1)(t-3)$$

Set derivative equal to zero:

$$3(t-1)(t-3) = 0 \implies t = 1,\ t = 3$$

Evaluate population at these points:

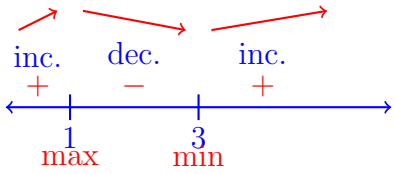
$$P(1) = 1 - 6 + 9 + 5 = 9$$

$$P(3) = 27 - 54 + 27 + 5 = 5$$

Critical points:

$$(1, 9), \quad (3, 5)$$

b) (3 points) Determine the times when the population experiences a relative maximum or relative minimum. Interpret what these points mean in the context of the fish population.
(**Hint:** Use the First Derivative Test)
Determine relative extrema and interpret.
Sign chart of $P'(t) = 3(t-1)(t-3)$:



Relative maximum:

$$(1, 9)$$

Relative minimum:

$$(3, 5)$$

Interpretation:
- At $t = 1$ year, the fish population reaches a **relative maximum** of 9 thousand.
- At $t = 3$ years, the population drops to a **relative minimum** of 5 thousand.

10. (15 points) Determine the derivative of the following functions. Show your work. You **do NOT need** to simplify your results.

a) $h(x) = \dfrac{4x^3 - 5\sqrt{x}}{e^{x^4}}$

Solution: Use the quotient rule:

$$h'(x) = \frac{(12x^2 - \frac{5}{2}x^{-1/2})e^{x^4} - (4x^3 - 5\sqrt{x})(4x^3 e^{x^4})}{(e^{x^4})^2}$$

b) $g(x) = 2\left(3x^3 - \frac{8}{x}\right)^{-7}$

Solution: Use the chain rule:

$$g'(x) = 2 \cdot (-7)\left(3x^3 - \frac{8}{x}\right)^{-8} \cdot \left(9x^2 + \frac{8}{x^2}\right) = -14\left(3x^3 - \frac{8}{x}\right)^{-8}\left(9x^2 + \frac{8}{x^2}\right)$$

c) $k(x) = \ln(3x^4 - 8x + 5)$

Solution: Use the derivative of $\ln(u)$:

$$k'(x) = \frac{d}{dx}[3x^4 - 8x + 5] \cdot \frac{1}{3x^4 - 8x + 5} = \frac{12x^3 - 8}{3x^4 - 8x + 5}$$

d) $m(x) = \frac{5}{3x^4} \cdot \sqrt{x^2 + 4}$

Solution: Use the product rule: Let $u = \frac{5}{3x^4}$, $v = (x^2 + 4)^{1/2}$. Then

$$u' = -\frac{20}{3}x^{-5}, \quad v' = \frac{x}{\sqrt{x^2 + 4}}$$

$$m'(x) = u'v + uv' = \left(-\frac{20}{3}x^{-5}\right)(\sqrt{x^2 + 4}) + \left(\frac{5}{3x^4}\right)\left(\frac{x}{\sqrt{x^2 + 4}}\right)$$

e) $f(x) = \left(5\sqrt{x} + \frac{4}{x^3}\right)^4 \ln(2x^2 + 3x)$

Solution: Use the product rule: Let $u = \left(5\sqrt{x} + \frac{4}{x^3}\right)^4$, $v = \ln(2x^2 + 3x)$. Then

$$u' = 4\left(5\sqrt{x} + \frac{4}{x^3}\right)^3\left(\frac{5}{2}x^{-1/2} - \frac{12}{x^4}\right), \quad v' = \frac{4x + 3}{2x^2 + 3x}$$

$$f'(x) = u'v + uv' = 4\left(5\sqrt{x} + \frac{4}{x^3}\right)^3\left(\frac{5}{2}x^{-1/2} - \frac{12}{x^4}\right)\ln(2x^2 + 3x) + \left(5\sqrt{x} + \frac{4}{x^3}\right)^4 \frac{4x + 3}{2x^2 + 3x}$$

11. (12 points) A savings account grows according to the function:

$$A(t) = 150\ e^{0.008t^2+1.2t}$$

where $A(t)$ is the value of the account (in dollars) after t years since the account was opened.

a) Find $A'(t)$.

Solution: Use the chain rule:

$$A'(t) = 150 \cdot e^{0.008t^2+1.2t} \cdot \frac{d}{dt}(0.008t^2 + 1.2t) = 150\ e^{0.008t^2+1.2t}(0.016t + 1.2)$$

b) Determine the marginal rate of change of the amount after 3 years. Include units, and round your answer to two decimal places.

Solution: Substitute $t = 3$ into $A'(t)$:

$$A'(3) = 150\ e^{0.008(3)^2+1.2(3)}(0.016(3) + 1.2) = 150\ e^{0.072+3.6}(1.248) \approx 150 \cdot e^{3.672} \cdot 1.248$$

$$A'(3) \approx 150 \cdot 39.36 \cdot 1.248 \approx 7362.59$$

$\Rightarrow$ The account is increasing at approximately \$7,362.59 per year after 3 years.

c) Interpret your result from part (b).

Solution: After 3 years, the account balance is increasing at a marginal rate of approximately \$7,362.59 per year.
In other words, an additional year of investment at $t = 3$ years will increase the account balance by approximately \$7,362.59.

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12. (10 points) A bakery produces x loaves of bread each day. The total cost (in dollars) to produce x loaves and the total revenue (in dollars) from selling x loaves are modeled by

$$C(x) = 150 + 2x, \quad R(x) = 3x - 0.002x^2.$$

a) Write the profit function $P(x)$.

Solution: Profit is revenue minus cost:

$$P(x) = R(x) - C(x) = \left(3x - 0.002x^2\right) - (150 + 2x) = -0.002x^2 + x - 150$$

b) Find the number of loaves, x, that will maximize the profit function.

Solution: To maximize profit, set $P'(x) = 0$:

$$P'(x) = -0.004x + 1$$

Set derivative to zero:

$$-0.004x + 1 = 0 \quad \Rightarrow \quad x = \frac{1}{0.004} = 250$$

Using the First Derivative Test, $P'(x) > 0$ for $x < 250$ and $P'(x) < 0$ for $x > 250$, so $P(x)$ changes from increasing to decreasing at $x = 250$. Therefore, the maximum profit occurs at 250 loaves.

c) Interpret your result from part (b).

Solution: The bakery maximizes its profit by producing and selling 250 loaves of bread per day. Producing more or fewer loaves would result in lower profit.

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13. (12 points) A coffee machine company determines that in order to sell x units of a new coffee maker, the price per unit (in dollars) is given by $u(x) = 210-2x$, and the cost of producing x units is given by $c(x) = 1000+10x$.

a) Write the profit function $P(x)$ in terms of x.

Solution: Revenue is $R(x) = u(x) \cdot x = (210 - 2x)x = 210x - 2x^2$.
Profit is revenue minus cost:

$$P(x) = R(x) - c(x) = (210x - 2x^2) - (1000 + 10x) = -2x^2 + 200x - 1000$$

b) Find $P'(x)$.

Solution:

$$P'(x) = \frac{d}{dx}(-2x^2 + 200x - 1000) = -4x + 200$$

c) Determine the price per unit that must be charged to maximize profit.

Solution: Set $P'(x) = 0$ to find the maximizing number of units:

$$-4x + 200 = 0 \quad \Rightarrow \quad x = 50$$

Using the First Derivative Test, $P'(x) > 0$ for $x < 50$ and $P'(x) < 0$ for $x > 50$, so $P(x)$ changes from increasing to decreasing at $x = 50$. Therefore, $x = 50$ maximizes the profit. The price per unit is then:

$$u(50) = 210 - 2(50) = 210 - 100 = 110$$

Conclusion: To maximize profit, the company should charge \$110 per coffee machine.