PRACTICE FINAL

Printed Name: _____ **Class Start Time:** _____

- Record answers to Section 1 in the grid below, or you will receive a score of 0 for Section 1.
- Graphing Calculators without CAS are allowed. All other devices strictly prohibited.
- Only the first sheet of paper will be marked, and the exam will not be returned.

SECTION 1: Multiple Choice: Record One Clear Letter Answer Here (5 Points Each)

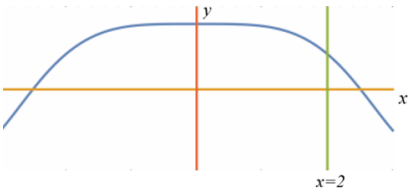
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

SECTION 2: Free Response (20 Points Total)

Show complete work. No credit for answers without justification.

Question 17 (10 Points): Cylindrical Shells

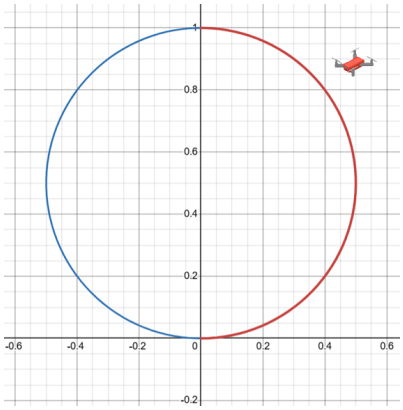
Using the method of cylindrical shells, find the volume of the solid obtained by rotating the region bounded by $y = \cos(\frac{x^2}{4})$ and $y = 0$, from $x = 0$ to $x = 2$ about the y-axis.
Show all steps of integration.



Exact Answer: _____

Question 18 (10 Points): Arc Length Representations

The drone is flying west. Its path is given below, in three different forms we learned in this class. Its path is also graphed in red (in quadrant 1) starting from the origin. What arc length expressions represent the distance traveled by the drone? Set up all three integrals and then solve any **one of them**.



a) Cartesian: $x^2 = -y^2 + y$ from $y = 0$ to $y = 1$

b) Parametric: $x = \frac{1}{2}\sin(2t), y = \sin^2(t)$ from $t = 0$ to $t = \pi/2$

c) Polar: $r = \sin(\theta)$ from $\theta = 0$ to $\theta = \pi/2$

Solution:

a) Cartesian setup: (calculate derivatives)

b) Parametric setup: (calculate derivatives)

c) Polar setup: (calculate derivatives)

Solve one of the integrals:

Exact Answer: _____

SECTION 1: Multiple Choice Questions

Select the single best answer and write it in the grid on Page 1.

1. Find the indefinite integral: $\displaystyle\int \frac{(\tan^{-1} x)^7}{1+x^2}\,dx$

A. $\frac{1}{7}(\tan^{-1} x)^6 + C$

B. $\frac{1}{8}(\tan^{-1} x)^8(x + \frac{1}{3}x^3) + C$

C. $\frac{1}{8}\left(\frac{1}{1+x^2}\right)^8 + C$

D. $(\tan^{-1} x)^8 + C$

E. $\frac{1}{8}(\tan^{-1} x)^8 + C$

F. $\frac{1}{7}(\tan^{-1} x)^8 + C$

2. Find the following indefinite integral: $\displaystyle\int 4x\sin(4x)\,dx$

A. $-x\cos(4x) - \frac{1}{4}\sin(4x) + C$

B. $-x\cos(4x) + \frac{1}{4}\sin(4x) + C$

C. $-4x\cos(4x) + \sin(4x) + C$

D. $x\cos(4x) + \frac{1}{4}\sin(4x) + C$

E. $-x\sin(4x) + \frac{1}{4}\cos(4x) + C$

F. $-x\cos(4x) + 4\sin(4x) + C$

3. Find the following indefinite integral: $\displaystyle\int 12\cos^2(3x)\,dx$

A. $6x + \sin(6x) + C$

B. $6x - \sin(6x) + C$

C. $12x + \sin(6x) + C$

D. $6x + \frac{1}{2}\sin(6x) + C$

E. $4\cos^3(3x) + C$

F. $6x + 6\sin(6x) + C$

4. Find the form of the partial fraction decomposition of: $\dfrac{2x^4 + 13x^3 - 8x + 1}{(x+4)^3(x^2+x+5)}$

A. $\dfrac{A}{(x+4)^3} + \dfrac{Bx+C}{x^2+x+5}$

B. $\dfrac{A}{x+4} + \dfrac{B}{(x+4)^2} + \dfrac{C}{(x+4)^3} + \dfrac{Dx+E}{x^2+x+5}$

C. $\dfrac{A}{x+4} + \dfrac{B}{(x+4)^2} + \dfrac{C}{(x+4)^3} + \dfrac{D}{x^2+x+5}$

D. $\dfrac{A}{x+4} + \dfrac{B}{x^2+x+5}$

E. $\dfrac{A}{(x+4)^3} + \dfrac{D}{x^2+x+5}$

F. $\dfrac{A}{x+4} + \dfrac{Bx+C}{(x+4)^2} + \dfrac{D}{(x+4)^3} + \dfrac{Ex+F}{x^2+x+5}$

5. Using the method of washers, which expression represents the volume of the solid obtained by rotating the region bounded by $y = x/2$ and $y - \sqrt{x} = 0$ about the line $x = -1$?

A. $\pi \int_0^1 \left[(2y+1)^2 - (y^2+1)^2\right]\, dy$

B. $\pi \int_0^2 \left[(y^2+1)^2 - (2y+1)^2\right]\, dy$

C. $\pi \int_0^2 \left[(2y+1)^2 - (y^2+1)^2\right]\, dy$

D. $\pi \int_0^2 \left[(2y-1)^2 - (y^2-1)^2\right]\, dy$

E. $\pi \int_0^2 \left[(2y)^2 - (y^2)^2\right]\, dy$

F. $\pi \int_0^2 \left[(y^2-1)^2 - (2y-1)^2\right]\, dy$

6. Algebraically determine the behavior of $\displaystyle\int_{-\infty}^{0} \frac{1}{81+x^2}\, dx$.

A. Diverges

B. Converges to $-\frac{\pi}{18}$

C. Converges to $\frac{\pi}{9}$

D. Converges to $\frac{\pi}{18}$

E. Converges to $\frac{\pi}{81}$

F. Converges to 0

7. Use **Simpson's Rule** to approximate $\displaystyle\int_0^2 \frac{1}{x^4+3}\, dx$ with $n = 4$. The choices are rounded to 5 decimals

A. 0.44804

B. 0.45028

C. 0.44859

D. 0.48273

E. 0.50500

F. 0.48000

8. Set up the integral for the volume of the solid generated by rotating the region bounded by $y = x^2$ and $y = 8 - x^2$ (in the first quadrant) about the y-axis.

A. $\displaystyle\int_0^2 2\pi x \left(x^2 - (8-x^2)\right)\, dx$

B. $\displaystyle\int_0^8 2\pi x \left(8 - 2x^2\right)\, dx$

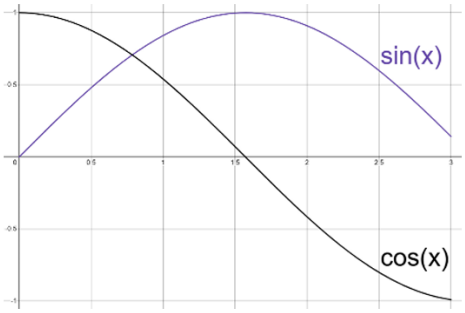
C. $\displaystyle\int_0^2 \pi \left((8-x^2)^2 - (x^2)^2\right)\, dx$

D. $\displaystyle\int_0^2 2\pi x \left((8-x^2) - x^2\right)\, dx$

E. $\displaystyle\int_0^2 2\pi \left(8 - 2x^2\right)\, dx$

F. $\displaystyle\int_0^2 2\pi \left((8-x^2)^2 - x^4\right)\, dx$

9. Find the area between $y = \sin(x)$ and $y = \cos(x)$ and from $x = 0$ to $x = 3$



A. $\displaystyle\int_0^3 (\cos(x) - \sin(x))\ dx$

B. $\displaystyle\int_0^{\pi/4} (\cos(x) - \sin(x))\ dx$

C. $\displaystyle\int_0^2 (\cos(x) - \sin(x))\ dx$

D. $\displaystyle\int_0^{\pi/4} (\cos(x) - \sin(x))\ dx + \int_{\pi/4}^3 (\sin(x) - \cos(x))\ dx$

E. $\displaystyle\int_0^3 (\sin(x) - \cos(x))\ dx$

F. $\displaystyle\int_0^{\pi/2} (\cos(x) - \sin(x))\ dx + \int_{\pi/2}^3 (\sin(x) - \cos(x))\ dx$

10. Find the first three non-zero terms of the Taylor series for $f(x) = 3e^{2x} - 4x$, centered at $a = 1$.

A. $3e^2 + 6e^2(x-1) + 6e^2(x-1)^2$

B. $(3e^2 - 4) + 6e^2(x-1) + 6e^2(x-1)^2$

C. $(3e^2 - 4) + (6e^2 - 4)(x-1) + 6e^2(x-1)^2$

D. $3e^2 + (6e^2 - 4)(x-1) + 6e^2(x-1)^2$

E. $(3e^2 - 4) + (6e^2 - 4)(x-1) + 12e^2(x-1)^2$

F. $(3e^2 - 4) + 6e^2(x-1) + 12e^2(x-1)^2$

11. Given $\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$, obtain the Maclaurin series for $f(x) = \frac{x^2}{2}\cos x$.

A. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+2}}{2(2n)!}$

B. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2(2n)!}$

C. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+2}}{2(2n+1)!}$

D. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n}}{2(2n)!}$

E. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+2}}{(4n)!}$

F. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+2}}{(2n)!}$

12. For the parametric curve defined by $x = 3 + \sin(t), \quad y = 4 - \sin(3t)$, algebraically find the equation of the line tangent to the curve at the point corresponding to $t = 0$.

A. $y = -3x + 4$

B. $y = -3x + 13$

C. $y = 3x - 5$

D. $y = -x + 7$

E. $y = 3x - 13$

F. $y = 3x + 13$

13. Set up the integral to find the exact area enclosed by one loop of the polar curve $r = \sin(8\theta)$.

A. $\displaystyle\int_0^{\pi} \frac{1}{2}(\sin(8\theta))^2\,d\theta$

B. $\displaystyle\int_0^{\pi/8} (\sin(8\theta))^2\,d\theta$

C. $\displaystyle\int_0^{\pi/4} \frac{1}{2}(\sin(8\theta))^2\,d\theta$

D. $\displaystyle\int_0^{\pi/8} (8\cos(8\theta))^2\,d\theta$

E. $\displaystyle\int_0^{\pi/8} \frac{1}{2}(\sin(8\theta))^2\,d\theta$

F. $\displaystyle\int_0^{\pi} (\sin(8\theta))^2\,d\theta$

14. Find a Cartesian equation for the curve described by the polar equation: $r = \dfrac{3}{\cos\theta + 2\sin\theta}$

A. $x^2 + 4y^2 = 9$

B. $(x-3)^2 + (y-3/2)^2 = 9$

C. $x - 2y = 3$

D. $x + 2y = 3$

E. $(x-3)^2 + 2(y-3/2)^2 = 3$

F. $x^2 + y^2 = 9$

15. Find the radius of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{(2n-1)}{9^n n!}(x-6)^n$.

A. 0

B. 6

C. 9

D. ∞

E. 1/9

F. 1

16. Find the open interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{(2n-1)}{9^n n!}(x-6)^n$.

A. $(5, 7)$

B. $(-3, 15)$

C. $(-\infty, \infty)$

D. $(-9, 9)$

E. $(-15, 3)$

F. Only at $x = 6$

Solutions

MC Answers

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
E	B	A	B	C	D	A	D	D	C	A	B	E	D	D	C

Question 17 (10 Points): Cylindrical Shells

Rotate the region bounded by $y = \cos(\frac{x^2}{4})$ and $y = 0$, from $x = 0$ to $x = 2$ about the y-axis.

Shell Setup: $\boxed{V = \int_a^b 2\pi(r)(h)\,dx}$

$$V = \int_0^2 2\pi x \cos\left(\frac{x^2}{4}\right)\,dx$$

Use u-substitution: Let $u = \frac{x^2}{4}$, so $du = \frac{x}{2}\,dx \implies 2du = x\,dx$.
Change bounds: $x = 0 \implies u = 0$ and $x = 2 \implies u = 1$.

$$V = 2\pi \int_0^1 \cos(u)(2\,du) = 4\pi \int_0^1 \cos(u)\,du$$
$$= 4\pi\Big[\sin(u)\Big]_0^1 = 4\pi(\sin(1) - \sin(0)) = \boxed{4\pi\sin(1)}$$

Question 18 (10 Points): Arc Length Representations

a) Cartesian: $x^2 = -y^2 + y \implies x = \sqrt{y - y^2}$.

$x'(y) = \frac{1-2y}{2\sqrt{y-y^2}}$. Integral: $\boxed{L = \int_0^1 \sqrt{1 + \frac{(1-2y)^2}{4(y-y^2)}}\,dy}$

b) Parametric: $x = \frac{1}{2}\sin(2t), y = \sin^2(t)$.

$x'(t) = \cos(2t)$, $y'(t) = 2\sin(t)\cos(t) = \sin(2t)$. Integral: $\boxed{L = \int_0^{\pi/2} \sqrt{\cos^2(2t) + \sin^2(2t)}\,dt}$

c) Polar: $r = \sin(\theta)$.

$r'(\theta) = \cos(\theta)$. Integral: $\boxed{L = \int_0^{\pi/2} \sqrt{\sin^2(\theta) + \cos^2(\theta)}\,d\theta}$

Evaluation (using Polar/Parametric identity $\sin^2 + \cos^2 = 1$):

$$L = \int_0^{\pi/2} \sqrt{1}\,d\theta = \Big[\theta\Big]_0^{\pi/2} = \boxed{\frac{\pi}{2}}$$