PRACTICE EXAM 2

Printed Name: _____ **Class Start Time:** _____

- Record answers to Section 1 in the grid below, or you will receive a score of 0 for Section 1.
- Graphing Calculators without CAS are allowed. All other devices strictly prohibited.
- Only the first sheet of paper will be marked, and only returned if you come to office hours.

SECTION 1: Multiple Choice: Record One Clear Letter Answer Here (6 Points Each)

1	2	3	4	5	6	7	8	9	10

SECTION 2: Free Response (40 Points Total)

Show complete work. No credit for answers without justification.

Question 11 (16 Points): Volume of Revolution

Consider the region bounded by $y = x$, and $y = 2x^2$. Compute the volume of the solid of revolution generated by revolving this region about the **y-axis**. Set up integrals for BOTH Disk/Washer AND Shell methods, and then finish integrating one of them. Show each step.

a) Sketch the 2D graph, indicate all points of intersection

b) Set up the integral for the Disk/Washer method

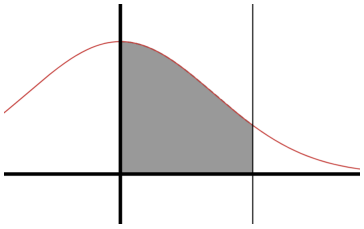
c) Set up the integral for the Shell method

d) Solve one of the two integrals to find the **exact** volume.

Exact Answer: _____

Question 12 (14 Points): Shell Method

Set up and solve the integral for the volume generated by rotating the region bounded by $y = e^{-x^2}$, $y = 0$, $x = 0$, and $x = 1$ about the y-axis. (No credit will be given for using the wrong method)



a) Set up the integral for the Shell method

b) Show all steps for the integration to give the **exact** volume.

Exact Answer: _____

Question 13 (10 Points): SET UP

The flight path of an insect is recorded as $y^2 = (7x + 2)^3$. Assuming $y \geq 0$, set up the integral for the distance flown from $x = 0$ to $x = 1$.

Show steps of setting up the integral. **SET UP ONLY, DO NOT integrate!**

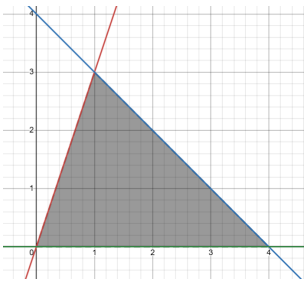
SECTION 1: Multiple Choice Questions

Select the single best answer and write it in the grid on Page 1.

1. Algebraically determine the behavior of $\displaystyle\int_{-\infty}^{0} \frac{1}{36+x^2}\,dx$.

A. Diverges
B. Converges to $-\frac{\pi}{12}$
C. Converges to $\frac{\pi}{6}$
D. Converges to 0
E. Converges to $\frac{\pi}{12}$
F. Converges to $\frac{\pi}{4}$

2. Set up the integral with respect to x for the area of the region bounded by $y = 3x$, $y = 4 - x$, and $y = 0$.



A. $\displaystyle\int_0^1 3x\,dx + \int_1^4 (4-x)\,dx$
B. $\displaystyle\int_0^4 (4-4x)\,dx$
C. $\displaystyle\int_0^1 (4-4x)\,dx$
D. $\displaystyle\int_0^1 (4-x)\,dx + \int_1^4 3x\,dx$
E. $\displaystyle\int_0^4 ((4-x)-3x)\,dx$
F. $\displaystyle\int_0^1 3x\,dx - \int_1^4 (4-x)\,dx$

3. Using the method of washers, which expression represents the volume of the solid obtained by rotating the region bounded by $y = x$ and $y = x^2$ about the line $y = -1$?

A. $\pi\displaystyle\int_0^1 \left[(x-1)^2 - (x^2-1)^2\right]\,dx$
B. $\pi\displaystyle\int_0^1 \left[(x+1)^2 - (x^2+1)^2\right]\,dx$
C. $\pi\displaystyle\int_0^1 \left[(x^2+1)^2 - (x+1)^2\right]\,dx$
D. $\pi\displaystyle\int_0^1 \left[x^2 - (x^2)^2\right]\,dx$
E. $\pi\displaystyle\int_{-1}^0 \left[(y+1)^2 - (y^2+1)^2\right]\,dy$
F. $\pi\displaystyle\int_0^1 \left[(x+1)^2 + (x^2+1)^2\right]\,dx$

4. Calculate the work done by the force $F(x) = 10\cos(\pi x)$ moving an object from $x = 0$ to $x = 1/2$.

A. 10 Joules
B. $5/\pi$ Joules
C. 0 Joules
D. $10/\pi$ Joules
E. 5 Joules
F. $-10/\pi$ Joules

5. Set up the integral for the volume of the solid generated by rotating the region bounded by $y = x^2 + 2$ and $y = 10 - x^2$ (in the first quadrant) about the y-axis.

A. $\displaystyle\int_0^2 2\pi x\left(2x^2 - 8\right)\,dx$

B. $\displaystyle\int_0^2 2\pi x\left(8 - 2x^2\right)\,dx$

C. $\displaystyle\int_0^{\sqrt{8}} 2\pi x\left(8 - 2x^2\right)\,dx$

D. $\displaystyle\int_0^2 \pi\left((10 - x^2)^2 - (x^2 + 2)^2\right)\,dx$

E. $\displaystyle\int_0^2 2\pi\left(8 - 2x^2\right)\,dx$

F. $\displaystyle\int_0^2 2\pi x\left(x^2 + 2\right)\,dx$

6. Consider a cylindrical tank that is standing on its circular base with a radius of 7 meters and a height of 14 meters. Using g m/s^2 for the acceleration due to gravity and ρ kg/m^3 as the density of water, set up the integral for the work required to pump the water out of the tank if the tank is **half full** of water and it is being pumped out of a 1-meter long vertical **spout** at the top of the tank.

A. $\displaystyle\int_0^{14} g \cdot \rho \cdot 49\pi(15 - y)\,dy$

B. $\displaystyle\int_0^{7} g \cdot \rho \cdot 49\pi(14 - y)\,dy$

C. $\displaystyle\int_0^{7} g \cdot \rho \cdot 49\pi(15 - y)\,dy$

D. $\displaystyle\int_7^{14} g \cdot \rho \cdot 49\pi(15 - y)\,dy$

E. $\displaystyle\int_0^{7} g \cdot \rho \cdot 14\pi(15 - y)\,dy$

F. $\displaystyle\int_0^{7} g \cdot \rho \cdot 49\pi(7 - y)\,dy$

7. Describe the behavior of the sequence $a_n = \dfrac{9^{n+1}}{11^n}$.

A. Converges to 0

B. Converges to 9/11

C. Diverges

D. Converges to 9

E. Converges to 11/9

F. Converges to 1

8. Describe the behavior of the sequence $a_n = \dfrac{(1 + 7n)^2(2n + 7)}{2n^2 + n}$.

A. Converges to 7

B. Converges to 49

C. Converges to 0

D. Converges to $\frac{7}{2}$

E. Diverges

F. Converges to 98

9. Describe the behavior of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(2n+7)n}{(4n+1)^2}$

A. Converges to 1/8 **B.** Converges to 1/2 **C.** Converges to 0

D. Converges to 7 **E.** Diverges **F.** Converges to 1/4

10. Describe the behavior of the series $\displaystyle\sum_{n=0}^{\infty} 4 \cdot \left(\frac{1}{3}\right)^{n+1}$

A. Converges to 4/3 **B.** Diverges **C.** Converges to 4/9

D. Converges to 2 **E.** Converges to 4 **F.** Converges to 6

Solutions

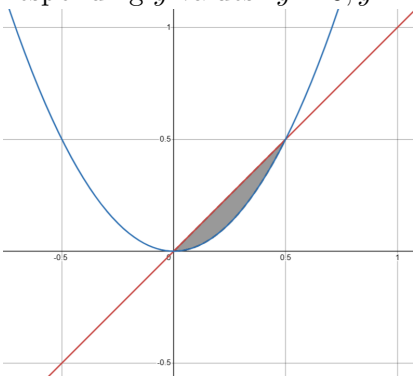
Section 1: Multiple Choice

1	2	3	4	5	6	7	8	9	10
E	A	B	D	B	C	A	E	E	D

Question 11: Volume of Revolution

Intersection: $x = 2x^2 \implies 2x^2 - x = 0 \implies x(2x-1) = 0 \implies x = 0, x = 1/2$.

Corresponding y-values: $y = 0, y = 1/2$.



b) Washer Setup: $\boxed{V = \pi \int_c^d (R^2 - r^2)\, dy}$

$$V = \pi \int_0^{1/2} \left[\left(\sqrt{\frac{y}{2}} \right)^2 - (y)^2 \right] dy = \pi \int_0^{1/2} \left(\frac{y}{2} - y^2 \right) dy$$

c) Shell Setup: $\boxed{V = \int_a^b 2\pi(r)(h)\, dx}$

$$V = \int_0^{1/2} 2\pi x(x - 2x^2)\, dx = 2\pi \int_0^{1/2} (x^2 - 2x^3)\, dx$$

d) Evaluation (Shell): $V = 2\pi \left[\frac{x^3}{3} - \frac{x^4}{2} \right]_0^{1/2} = 2\pi \left[\left(\frac{1}{24} - \frac{1}{32} \right) - 0 \right] = 2\pi \left(\frac{4-3}{96} \right) = \frac{\pi}{48}$

Question 12: Shell Method: $\boxed{V = \int_a^b 2\pi(r)(h)\,dx}$

a) Integral Setup:

$$V = \int_0^1 2\pi x e^{-x^2}\,dx$$

b) Evaluation via substitution:

Let $u = -x^2 \implies du = -2x\,dx$

$x = 0 \implies u = 0$

$x = 1 \implies u = -1$

$$V = -\pi \int_0^{-1} e^u\,du = \pi \int_{-1}^0 e^u\,du$$

$$= \pi\left[e^u\right]_{-1}^0 = \pi(e^0 - e^{-1}) = \boxed{\pi\left(1 - \frac{1}{e}\right)}$$

Question 13: Arc Length Formula: $\boxed{L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx}$

$$y = (7x+2)^{3/2}$$

$$\implies y' = \frac{3}{2}(7x+2)^{1/2}(7) = \frac{21}{2}(7x+2)^{1/2} \quad \textbf{(used chain rule)}$$

$$(y')^2 = \left(\frac{21}{2}(7x+2)^{1/2}\right)^2 = \frac{441}{4}(7x+2)$$

$$\boxed{L = \int_0^1 \sqrt{1 + \frac{441}{4}(7x+2)}\,dx}$$

Extra: If you want to integrate, use substitution:

Let $u = 1 + \frac{441}{4}(7x+2) \implies du = \frac{441}{4}(7)\,dx = \frac{3087}{4}\,dx$

Then

$$\int \sqrt{1 + \frac{441}{4}(7x+2)}\,dx = \frac{4}{3087}\int \sqrt{u}\,du$$

$$= \frac{4}{3087}\frac{2}{3}u^{\frac{3}{2}} + C = \frac{8}{9261}\left(1 + \frac{441}{4}(7x+2)\right)^{\frac{3}{2}} + C$$

Then

$$L = \int_0^1 \sqrt{1 + \frac{441}{4}(7x+2)}\,dx = \frac{8}{9261}\left[\left(1 + \frac{441}{4}(7(1)+2)\right)^{\frac{3}{2}} - \left(1 + \frac{441}{4}(0+2)\right)^{\frac{3}{2}}\right] = \boxed{2/3}$$