MAT 266 Review Section 8.4

MAT 266 Review Section 8.4

1. Suppose a series $\sum_{n=1}^{\infty} a_n$ meets the conditions to use the ratio test. What, if any, is the most specific conclusion that may be drawn if:

   (a) $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \frac{5}{3}$

   (b) $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \frac{1}{2}$

   (c) $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 1$

   (d) $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 0$

2. Consider the series $\sum_{n=6}^{\infty} \frac{n+9}{n!}$. Find $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$.

3. Consider the series $\sum_{n=6}^{\infty} \frac{(5n+3)5^{n+2}}{8^n}$. Find $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$.

4. Determine whether the series $\sum_{n=0}^{\infty} \frac{(-5)^n}{(n+1)3^{2n+1}}$ is absolutely convergent, conditionally convergent, or divergent.

5. Determine whether the series $\sum_{n=0}^{\infty} \frac{n \cdot 5^n}{3^{2n+1}}$ is absolutely convergent, conditionally convergent, or divergent.

6. Determine whether the series $\sum_{n=1}^{\infty} \frac{e^{n-6}}{\sqrt{n+7}(n+8)!}$ is absolutely convergent, conditionally convergent, or divergent.

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ANSWERS

1. (a) Diverges
   (b) Converges absolutely
   (c) Cannot draw conclusion
   (d) Converges absolutely
2. 0
3. $\frac{5}{8}$
4. Converges absolutely by the Ratio Test
5. Converges absolutely by the Ratio Test
6. Converges absolutely by the Ratio Test

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MAT 266 Review Section 8.5

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1. Find the largest **open** interval of convergence and the radius of convergence for $\sum_{n=1}^{\infty} \frac{(3x)^n}{n!}$.
2. Find the largest **open** interval of convergence and the radius of convergence for $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n+7}$.
3. Find the largest **open** interval of convergence and the radius of convergence for $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n \cdot 2^n}$.
4. Find the largest **open** interval of convergence and the radius of convergence for $\sum_{n=1}^{\infty} \frac{(5x)^n}{n^4}$.
5. Find the largest **open** interval of convergence and the radius of convergence for $\sum_{n=1}^{\infty} \frac{(-1)^n x^{n+1}}{3^{n-1}}$.
6. Find the largest **open** interval of convergence and the radius of convergence for
$\sum_{n=1}^{\infty} n!(3x+1)^n$.

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ANSWERS

1. $(-\infty, \infty), \infty$
2. $(-1, 1), 1$
3. $(-1, 3), 2$
4. $\left(-\frac{1}{5}, \frac{1}{5}\right), \frac{1}{5}$
5. $(-3, 3), 3$
6. $\left\{-\frac{1}{3}\right\}, 0.$

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MAT 266 Review Section 8.6

MAT 266 Review Section 8.6

1. Find the power series representation of the function $f(x) = \dfrac{3}{4-x}$ for $|x| < 4$ by using the fact that $\dfrac{1}{1-x} = \displaystyle\sum_{n=0}^{\infty} x^n$ for $|x| < 1$.

2. Find the power series representation of the function $f(x) = \dfrac{x^2}{2+x}$ for $|x| < 2$ by using the fact that $\dfrac{1}{1-x} = \displaystyle\sum_{n=0}^{\infty} x^n$ for $|x| < 1$.

3. Find the power series representation of the function $f(x) = \dfrac{x^3}{4-x}$ for $|x| < 4$ by using the fact that $\dfrac{1}{1-x} = \displaystyle\sum_{n=0}^{\infty} x^n$ for $|x| < 1$.

4. Find the power series representation of the function $f(x) = \dfrac{x}{\frac{1}{2}-x}$ for $|x| < \dfrac{1}{2}$ by using the fact that $\dfrac{1}{1-x} = \displaystyle\sum_{n=0}^{\infty} x^n$ for $|x| < 1$.

5. Find a power series representation for $f(x) = \ln(1+x)$. *Hint:* $\ln(1+x) = \displaystyle\int \frac{1}{1+x}\,dx$.

6. Find a power series representation for $f(x) = \ln(1-x)$. *Hint:* $\ln(1-x) = \displaystyle\int \frac{-1}{1-x}\,dx$.

7. Find a power series representation for $f(x) = \arctan x$. *Hint:* $\arctan x = \displaystyle\int \frac{1}{1+x^2}\,dx$.

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ANSWERS

1. $\displaystyle\sum_{n=0}^{\infty} \frac{3x^n}{4^{n+1}}$
2. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+2}}{2^{n+1}}$
3. $\displaystyle\sum_{n=0}^{\infty} \frac{x^{n+3}}{4^{n+1}}$
4. $\displaystyle\sum_{n=0}^{\infty} (2x)^{n+1}$
5. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1}$ for $|x| < 1$
6. $\displaystyle\sum_{n=0}^{\infty} \frac{-x^{n+1}}{n+1}$ for $|x| < 1$
7. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$ for $|x| < 1$

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MAT 266 Review Section 8.7

MAT 266 Review Section 8.7

1. Find the first three non-zero terms of the Taylor series for $f(x) = x^{-3}$ about $a = 1$.

2. Find the first three non-zero terms of the Maclaurin series for $f(x) = (1 + x)^{-2}$.

3. Find the first three non-zero terms of the Maclaurin series for $f(x) = (1 - x)^{-2}$.

4. A function $f(x)$ has a Maclaurin Series on $(-1, 1)$. If possible, find the first three non-zero terms of the Maclaurin series if $f(0) = 5$; $f'(0) = -2$; $f''(0) = 2$.

5. A function $f(x)$ has a Maclaurin Series on $(-1, 1)$. If possible, find the first three non-zero terms of the Maclaurin series if $f(0) = 0$; $f'(0) = -1$; $f''(0) = 4$; $f'''(0) = 6$.

6. A function $f(x)$ has a Maclaurin Series on $(-1, 1)$. If possible, find the first three non-zero terms of the Maclaurin series if $f(0) = 2$; $f'(0) = 0$; $f''(0) = 1$.

7. Use the known Maclaurin series for $\sin x$ to find the first three **nonzero** terms of the Maclaurin series for $f(x) = \sin(x^2)$.

8. Use the known Maclaurin series for $\sin x$ to find the first three **nonzero** terms of the Maclaurin series for $f(x) = x \sin(3x)$.

9. Use the known Maclaurin series for $\cos x$ to find the Maclaurin series for $f(x) = \dfrac{x}{27} \cos(3x)$.

10. Use the known Maclaurin series for $\cos x$ to find the Maclaurin series for $f(x) = x \cos\left(\dfrac{x}{2}\right)$.

11. Use the known Maclaurin series for e^x to find the Maclaurin series for $f(x) = x^2 e^{3x}$.

12. Use the known Maclaurin series for e^x to find the Maclaurin series for $f(x) = e^{-x^2}$.

13. The resistivity ρ of a given metal depends on temperature and can be modeled by the equation

$$\rho(t) = \rho_{20} e^{\alpha(t-20)}$$

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where t is the temperature in $^\circ C$, and ρ_{20} and α are constants depending on the metal. At most temperatures, resistivity can be approximated by the second-degree (quadratic) Taylor polynomial centered at $t = 20$. Find an expression for the second-degree Taylor polynomial centered at $t = 20$.

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ANSWERS

1. $1 - 3(x-1) + 6(x-1)^2$
2. $1 - 2x + 3x^2$
3. $1 + 2x + 3x^2$
4. $5 - 2x + x^2$
5. $-x + 2x^2 + x^3$
6. Not possible with the given information
7. $x^2 - \dfrac{x^6}{6} + \dfrac{x^{10}}{120}$
8. $3x^2 - \dfrac{3^3 x^4}{3!} + \dfrac{3^5 x^6}{5!}$
9. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n} x^{2n+1}}{27(2n)!}$
10. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2^{2n}(2n)!}$
11. $\displaystyle\sum_{n=0}^{\infty} \frac{3^n x^{n+2}}{n!}$
12. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!}$
13. $\rho_{20} + \rho_{20}\alpha(t-20) + \dfrac{\rho_{20}\alpha^2}{2}(t-20)^2$

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MAT266 Section 9.1 Review

1. Eliminate the parameter to find the Cartesian equation for the curve represented by
$$x = \sqrt{t}, y = 4 - t.$$

2. Eliminate the parameter to find the Cartesian equation for the curve represented by
$$x = e^t - 8, y = e^{2t}.$$

3. Eliminate the parameter to find the Cartesian equation for the curve represented by
$$x = \sin\left(\frac{\theta}{2}\right), y = \cos\left(\frac{\theta}{2}\right).$$

4. Find the Cartesian equation of the curve $x = 5\cos t; y = 5\sin t, 0 \leq t \leq 2\pi$.

5. Find the Cartesian equation of the curve $x = 5\sin t; y = 3\cos t, 0 \leq t \leq 2\pi$.

6. Find the Cartesian equation of the curve $x = 2 + 3\cos t; y = 5 + 3\sin t, 0 \leq t \leq 2\pi$.

7. Describe the parametric curve for the set of parametric equations $x = 5\cos t; y = 2\sin t, 0 \leq t \leq 2\pi$. Include the starting point, the terminal point, and the direction.

8. Describe the parametric curve for the set of parametric equations $x = 2 + \cos t; y = 3 + \sin t, 0 \leq t \leq \pi$. Include the starting point, the terminal point, and the direction.

Solutions

1. $y = 4 - x^2$ for $x \geq 0$
2. $y = (x + 8)^2$ for $x \geq -8$
3. $x^2 + y^2 = 1$
4. $x^2 + y^2 = 25$
5. $\frac{x^2}{25} + \frac{y^2}{9} = 1$
6. $(x - 2)^2 + (y - 5)^2 = 9$
7. Ellipse centered at the origin with an x-radius of 5 and a y-radius of 2. It starts and stops at $(5, 0)$ and moves once counterclockwise.
8. Semicircle centered at $(2, 3)$ with a radius of 1. It starts at $(3, 3)$ and stops at $(1, 3)$, moving counterclockwise.

MAT266 Section 9.2 Review

1. For the parametric curve defined by $x = t^4 - t - 3$ and $y = 3\ln t - t^2 + 3$,
   (a) Algebraically find the equation of the line tangent to the curve at the point corresponding to $t = 1$.
   (b) Find all values of t where the tangent line to the graph of the curve is horizontal.
2. For the parametric curve defined by $x = 8t + \ln t$ and $y = 9t - \ln t$,
   (a) Algebraically find the equation of the line tangent to the curve at the point corresponding to $t = 1$.
   (b) Find all values of t where the tangent line to the graph of the curve is horizontal.
3. For the parametric curve defined by $x = t^3 - t$, $y = t^3 + 4t$, find the points, if any, where the graph has:
   (a) a horizontal tangent line
   (b) a vertical tangent line
4. For the parametric curve defined by $x = t - \frac{1}{t}$, $y = t + \frac{1}{t}$, find the points, if any, where the graph has:
   (a) a horizontal tangent line
   (b) a vertical tangent line
5. Find the equation of the line tangent to the graph of the parametric curve defined by $x = t\cos t$, $y = t\sin t$ for $t = \pi$.
6. If a projectile is fired with an initial velocity of v_0 at an angle of α above the horizontal and air resistance is assumed to be negligible, then its position after t seconds is given by the parametric equations
$$x = (v_0\cos\alpha)t \quad y = (v_0\sin\alpha)t - \frac{1}{2}gt^2$$
where g is the acceleration due to gravity (9.8 m/s^2), α is the angle of elevation, and v_0 is the initial velocity.
   (a) Find dy/dx.
   (b) Suppose a projectile is fired with $v_0 = 100$ m/s at an angle $\alpha = 60°$. At what time does it reach its maximum height? Round your answer to the nearest tenth of a second.
7. Find the exact length of the curve defined by the set of parametric equations $x = \sin(5t)$, $y = \cos(5t)$ for $0 \le t \le \frac{\pi}{4}$.
8. Find the exact length of the curve defined by the set of parametric equations $x = t^2 + 1$, $y = t^3$ for $0 \le t \le 1$.

Solutions

1. (a) $y = \frac{1}{3}x + 3$
   (b) $t = -\sqrt{\frac{3}{2}}; t = \sqrt{\frac{3}{2}}$
2. (a) $y = \frac{8}{9}x + \frac{17}{9}$
   (b) $t = 1/9$
3. (a) none
   (b) $\left(-\frac{2}{3\sqrt{3}}, \frac{13}{3\sqrt{3}}\right); \left(\frac{2}{3\sqrt{3}}, -\frac{13}{3\sqrt{3}}\right)$
4. (a) $(0, -2); (0, 2)$
   (b) none
5. $y = \pi x + \pi^2$
6. (a) $\frac{dy}{dx} = \frac{v_0 \sin\alpha - gt}{v_0 \cos\alpha}$
   (b) 8.8 seconds
7. $\frac{5\pi}{4}$
8. $\frac{1}{27}(13^{3/2} - 8)$