MAT 210 Brief Calculus
Sample Exam 1 (Practice Version C)

Instructor	XXXXXXX
Full Name (Print clearly!)	
Student ID	

Important Disclaimer for Students:

- This is a **sample exam**, not the actual in-class exam. Problems are of similar difficulty but may differ in type, numbers, or context.
- Success in the course depends on practicing concepts beyond this sample: lecture notes, Edfinity, Group Work, quizzes, and homework.
- You should complete this exam as if you were in the real exam room: no notes, no internet, no calculators beyond what is allowed, and an approximately 60-minute time limit.
- After finishing, check your answers against the provided solutions and grade yourself.
- If you could not complete within the time limit, practice again and attempt another sample exam.
- Treat every problem in your coursework (notes, quizzes, assignments) as fair game for the actual exam.

Part I - Multiple Choice: Circle the response that best completes the statement or answers the question. Then write the letter of your answer in the table after Problem 7.

1. (5 points) The height of a launched drone (in meters) above the ground is modeled by the quadratic function

$$h(t) = -2t^2 + 12t + 5,$$

where t is the time in seconds after launch.

Compute the average rate of change of the drone's height from $t = 3$ to $t = 5$.

A. The drone's height is decreasing at a rate of 6 meters per second.

B. The drone's height is decreasing at a rate of 4 meters per second.

C. The drone's height is decreasing at a rate of 2 meters per second.

D. The drone's height is increasing at a rate of 2 meters per second.

E. The drone's height is increasing at a rate of 4 meters per second.

---

2. (5 points) The distance traveled by an autonomous delivery robot (in kilometers) is modeled by the function

$$D(t) = 3e^{0.5t},$$

where t is measured in hours after the robot begins moving.

Find the instantaneous rate of change of the distance traveled at $t = 2$ hours. Round your answer to the nearest whole number.

A. 3 kilometers per hour

B. 4 kilometers per hour

C. 5 kilometers per hour

D. 7 kilometers per hour

E. 9 kilometers per hour

---

3. (5 points) Determine the constant k such that $\lim_{x \to 1} g(x)$ exists, where

$$g(x) = \begin{cases} 4x + k, & x < 1, \\ x^2 + 3, & x > 1 \end{cases}$$

A. $k = 0$

B. $k = 2$

C. $k = 4$

D. $k = 6$

E. None of the above

4. (5 points) The height of a plant is given by $H(t)$, where t is measured in weeks and height is measured in centimeters.

   Which statement best describes the *average rate of change* of the plant's height from week 2 to week 8?

   A. The height of the plant at week 5.

   B. The total change in height of the plant over the interval.

   C. The change in height divided by the length of the time interval.

   D. The instantaneous growth rate of the plant at week 5.

   E. The derivative of the height function evaluated at week 8.

---

5. (5 points) Which of the following functions is *not continuous* at $x = 2$? There is only **one** correct answer.

   A. $\dfrac{x-2}{x+2}$

   B. $\dfrac{x+2}{x-2}$

   C. $\ln(x+2)$

   D. $(2x-4)^2$

   E. None of the above

---

6. (5 points) The population of a town (in thousands) is modeled by the function

$$P(t) = 2t^2 - 12t + 50,$$

   where t is the number of years since 2000.

   Which statement correctly compares the *average rate of change* of the population from $t = 4$ to $t = 10$ with the *instantaneous rate of change* at $t = 7$?

   A. The average rate of change equals the instantaneous rate of change at $t = 7$.

   B. The average rate of change represents how fast the population is changing at exactly $t = 7$.

   C. The average rate of change describes the overall change in population per year between $t = 4$ and $t = 10$, while the instantaneous rate of change gives the rate of change at a specific moment.

   D. The instantaneous rate of change measures the total change in population over the interval from $t = 4$ to $t = 10$.

   E. None of the above

---

7. (5 points) The graph of $g(x)$ is shown in Figure 1. Fill in the blanks: The function has a _____ discontinuity at $x =$ _____ and a _____ discontinuity at $x =$ _____.

A. removable, 1; removable, 3

B. jump, 1; jump, 3

C. infinite, 0; removable, 3

D. infinite 5; removable 1

E. None of the above

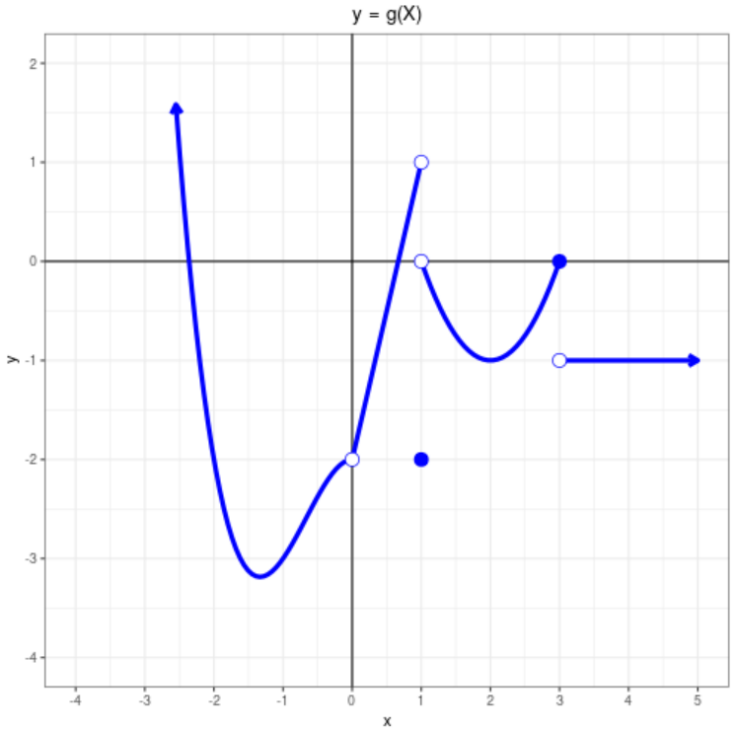


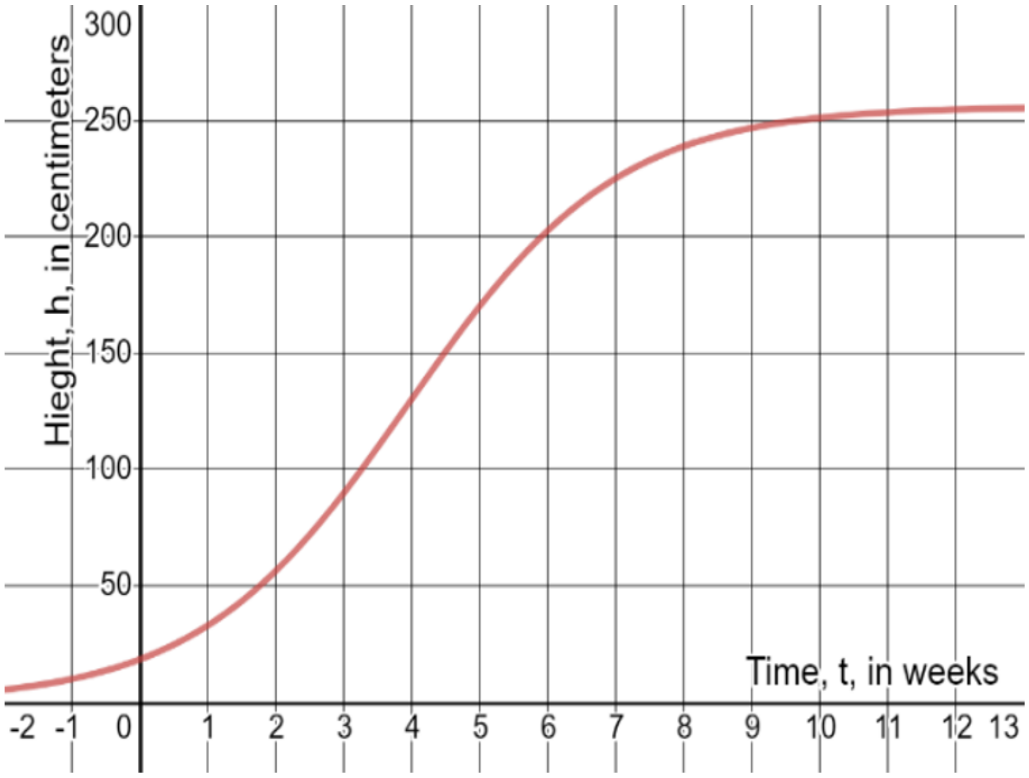
Figure 1: Sample discontinuities

PART I Answers: Please write the letter of your selected answers to multiple choice questions 1 through 7 in the following table.

Problem	1	2	3	4	5	6	7
Answer							

Part II - Fill in the blanks

8. (5 points) (1.25 points each) A sunflower seedling is planted, and its height h *(in centimeters)* over time t *(in weeks)* is graphed below. Determine whether each statement is True or False.



Circle **T** for True or **F** for False for each statement below.

T F a) The average rate of change of h over the interval $[0, 12]$ is greater than the instantaneous rate of change of h at $t = 4$.

T F b) After 4 weeks, the instantaneous rate of change of the height decreases over time, reaching zero when the graph flattens.

T F c) The average rate of change of h over the interval $[1, 5]$ is less than the instantaneous rate of change of the height at $t = 10$.

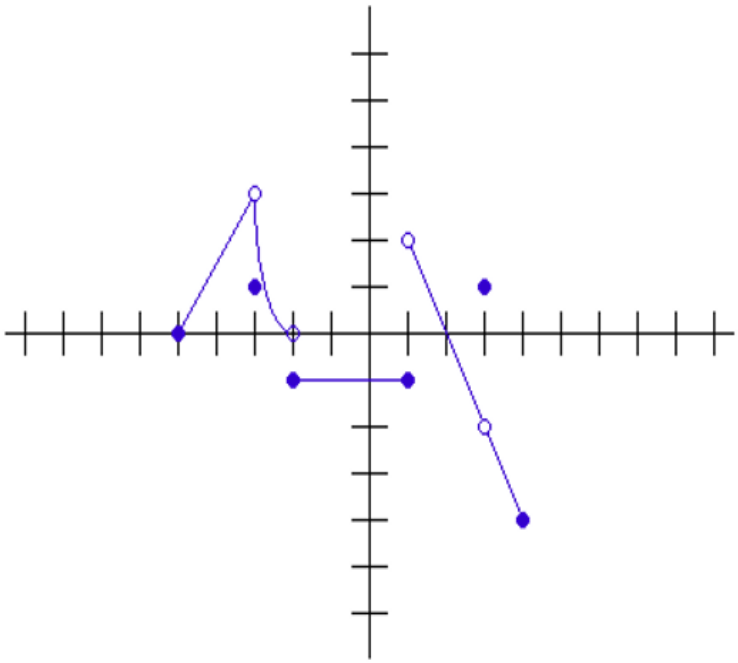
T F d) The sunflower seedling grows faster during week $t = 4$ than during week $t = 8$.

Part III - Free Response: You must show your work in the space provided to receive full credit. Correct final answers without sufficient supporting work may receive zero credit.

Please put a box around your final answers.

9. (5 points) (0.5 point each) Use the graph to calculate the following limits.

Graph of $y = f(x)$. Each tick mark is 1 unit.



a) $\lim_{x \to -3^-} f(x) =$

b) $\lim_{x \to -3^+} f(x) =$

c) $\lim_{x \to -3} f(x) =$

d) $\lim_{x \to 1} =$

e) $\lim_{x \to 1^-} f(x) =$

f) $\lim_{x \to 1^+} f(x) =$

g) $\lim_{x \to 3} f(x) =$

h) $f(1) =$

i) $\lim_{x \to -2} f(x) =$

j) $f(3) =$

---

10. (10 points) Let $t \geq 1$ denote the number of months since a new public library opened. The number of active library memberships (in hundreds) after t months is modeled by

$$M(t) = 220 + 35 \ln(t).$$

a) (4 points) Find $M'(t)$.

b) (4 points) Evaluate $M'(8)$, round to 2 decimal places, include units.

c) (2 points) Interpret the meaning of your answer in part (b).

---

11. (10 points) The table below shows the median monthly water bill (in tens of dollars) for households in a city from January 2010 to January 2016, where t is years since the start of 2010.

Year t (years after 2010)	0	1	2	3	4	5	6
Bill (tens of US $)	12	13.2	14.5	15.5	16.4	17.5	18.1

a) Compute the average rate of change of the bill from the start of 2012 to the start of 2015. Include units.

b) Interpret your answer from part (a).

---

12. (15 points) (5 points each) Find the derivative of the following functions. Show your work. You **do NOT** need to simplify your results.

a) $f(x) = 9x^{\frac{7}{2}} - \dfrac{5}{x^4} - \dfrac{10}{7x^{-5}} + 84.3$

b) $g(t) = \sqrt[4]{t^{11}} - \sqrt[6]{t^{15}}$

c) $h(u) = 8\ln(u) + 7e^{12u}$

---

13. (10 points) The total energy stored in a hybrid power system is modeled by

$$E(t) = 5e^{0.06t} + 3e^{0.15t} + 0.8t \quad \text{(megajoules)},$$

where t is the number of minutes since the system was turned on ($t = 0$ represents the start).

a) (4 points) Find $E'(t)$.

b) (4 points) Calculate the instantaneous rate of change of the total energy after **12 minutes**. Include units. (Round your answer to the nearest whole number.)

c) (2 points) Interpret your answer from part (b).

---

14. (10 points) Let $f(x) = 5x + 3$. Answer the following:

(a) (3 points) Find $f(x + h)$.

(b) (4 points) Find the difference quotient.

(c) (3 points) Use the limit definition of the derivative to find $f'(a)$.