MAT 210 Brief Calculus
Sample Exam 1 - (Practice Version B)
Solutions

Instructor	XXXXXXX
Full Name (Print clearly!)	
Student ID	

Important Disclaimer for Students:

- This is a **sample exam**, not the actual in-class exam. Problems are of similar difficulty but may differ in type, numbers, or context.
- Success in the course depends on practicing concepts beyond this sample: lecture notes, Edfinity, Group Work, quizzes, and homework.
- You should complete this exam as if you were in the real exam room: no notes, no internet, no calculators beyond what is allowed, and an approximately 60-minute time limit.
- After finishing, check your answers against the provided solutions and grade yourself.
- If you could not complete within the time limit, practice again and attempt another sample exam.
- Treat every problem in your coursework (notes, quizzes, assignments) as fair game for the actual exam.

Part I - Multiple Choice: Circle the response that best completes the statement or answers the question. Then write the letter of your answer in the table after Problem 7.

1. (5 points) The table below shows the median monthly water bill (in dollars) for households in a town from January 2018 to January 2024, where t is years since the start of 2018. Compute the average rate of change of the bill from the start of 2020 to the start of 2024.

Year t (years after 2018)	0	1	2	3	4	5	6
Bill ($)	45	47	50	52	55	57	60

A. From the start of 2020 to the start of 2024, the bill increased by \$2.5 per year, on average.
B. From the start of 2020 to the start of 2024, the bill increased by \$3 per year, on average.
C. From the start of 2020 to the start of 2024, the bill increased by \$2 per year, on average.
D. From the start of 2020 to the start of 2024, the bill increased by \$3.5 per year, on average.
E. From the start of 2020 to the start of 2024, the bill decreased by \$2 per year, on average.

Solution: Start of 2020 corresponds to $t = 2$ and start of 2024 to $t = 6$. The bill at $t = 2$ is \$50 and at $t = 6$ is \$60. The average rate of change is

$$\frac{60 - 50}{6 - 2} = \frac{10}{4} = 2.5 \text{ dollars per year.}$$

Option(A) is correct.

2. (5 points) A population of rabbits (*in hundreds*) in a farm is modeled by the function

$$R(t) = 50e^{0.02t},$$

where t is measured in months after the start of observation. Find the instantaneous rate of change of the rabbit population at $t = 5$ months. Round to the nearest whole number.

A. 1359 rabbits per month
B. 610 rabbits per month
C. 500 rabbits per month
D. 272 rabbits per month
E. None of the above

Solution: Differentiate: $R'(t) = 50 \cdot 0.02e^{0.02t} = 1 \cdot e^{0.02t}$. At $t = 5$:

$$R'(5) = 50 \cdot 0.02e^{0.02 \cdot 5} = e^{0.1} \approx 1.10517.$$

Rounded to the nearest whole number (units are in hundreds): 111 rabbits per month.

Option(E) is correct.

3. (5 points) Determine the constant c such that $\lim_{x\to 3} f(x)$ exists, where

$$f(x) = \begin{cases} 4x + c, & x < 3, \\ x^2 + 2, & x > 3 \end{cases}$$

A. $c = -4$

B. $c = -1$

C. $c = 2$

D. $c = 4$

E. None of the above

Solution: For the two one-sided limits to agree at $x = 3$,

$$\lim_{x\to 3^-} (4x + c) = 4(3) + c = 12 + c, \qquad \lim_{x\to 3^+} (x^2 + 2) = 3^2 + 2 = 11.$$

Set $12 + c = 11$ so $c = -1$.

Option(B) is correct.

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4. (5 points) At the age of t blinks, the height of a child is given by $H(t)$ (measured in blinkets). Which of the following calculations gives the *instantaneous rate of change* of the child's height at the age of 2 blinks?

A. $\dfrac{H(2)}{2}$

B. $H'(2)$

C. $\dfrac{H'(2) - H'(0)}{2 - 0}$

D. $H(2)$

E. $\dfrac{H'(2)}{2}$

Solution: The instantaneous rate of change of H at $t = 2$ is, by definition, the derivative evaluated at 2, i.e. $H'(2)$.

Option(B) is correct.

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5. (5 points) Which of the following functions is *not continuous* at $x = 5$? There is only **one** correct answer.

A. $\dfrac{x - 5}{x + 2}$

B. $\dfrac{x + 2}{x - 5}$

C. $\ln(x + 5)$

D. $(2x-10)^2$

E. None of the above

Solution: Check denominators and domains at $x=5$.

- A: $\dfrac{x-5}{x+2}$ at $x=5$ is $\frac{0}{7}=0$, is defined and continuous.
- B: $\dfrac{x+2}{x-5}$ has denominator $x-5$; at $x=5$ the denominator is 0, so the function is undefined there (vertical asymptote), hence is discontinuous.
- C: $\ln(x+5)$ the argument at $x=5$ is 10, $\ln 10$ defined and continuous.
- D: $(2x-10)^2$ is a polynomial expression (after simplification) and is continuous everywhere.

Therefore the non-continuous function at $x=5$ is option B.

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6. (5 points) The number of trees in a park from 2000 to 2020 can be modeled by the function

$$T(t) = 50t^2 - 200t + 2500$$

where t is the number of years since 2000. Interpret the meaning of $T(10)=5500$ and $T'(10)=800$.

A. In 2010, there were 5,500 trees, and the number of trees was increasing at a rate of 800 trees per year at that time.

B. In 2010, there were 5,500 trees, and the total increase over the 10 years since 2000 was 800 trees.

C. In 2010, there were 5,500 trees, and the number of trees was increasing at an average of 800 trees per year between 2000 and 2010.

D. In 2010, there were 5,500 trees, and the number of trees was increasing at a rate of 800 trees every 10 years after 2000.

E. None of the above

Solution: $T(10)=5500$ gives the number of trees when $t=10$, i.e, the year 2010 and $T'(10)=800$ is positive implies the number of trees are increasing at the rate of 800 trees per year in 2010. Option A matches the structure of the statement.

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7. (5 points) The graph of $f(x)$ is shown in Figure 1. Select all values of x where $f(x)$ is discontinuous.

A. $x = -3$

B. $x = -3,\ 1,\ 2$

C. $x = -3,\ 2$

D. All of the above

E. None of the above

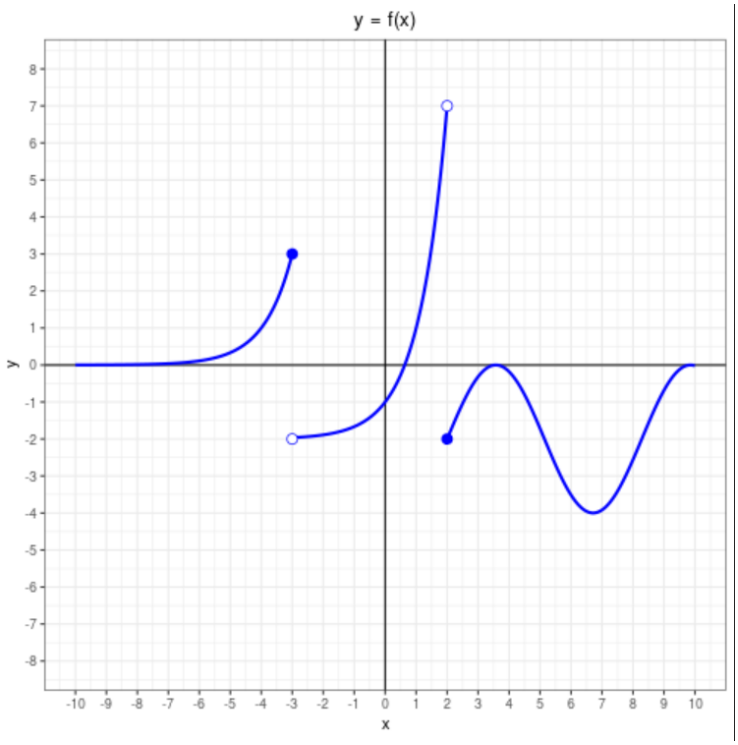

Figure 1

Solution: From the graph there is a jump at $x = -3$, and there is a jump at $x = 2$; the function is continuous at $x = 1$. Therefore, the discontinuities occur at $x = -3$ and $x = 2$.

Option(C) is correct.

PART I Answers: Please write the letter of your selected answers to multiple choice questions 1 through 7 in the following table.

Problem	1	2	3	4	5	6	7
Answer	A	E	B	B	B	A	C

Part II - Short Answer Type Problems

8. (5 points) The height (in meters) of a toy rocket above the ground after t seconds can be modeled by the quadratic function

$$h(t) = -5t^2 + 20t + 10,$$

where $t \geq 0$.

a) (2 points) Compute the average rate of change of the rocket's height over the interval $[3, 5]$ seconds. Include units.

Solution: Average rate of change on $[3, 5]$:

$$\frac{h(5) - h(3)}{5 - 3}.$$

Compute values: $h(5) = -5(25) + 20(5) + 10 = -125 + 100 + 10 = -15$. $h(3) = -5(9) + 20(3) + 10 = -45 + 60 + 10 = 25$. So average rate $= \dfrac{-15 - 25}{2} = \dfrac{-40}{2} = -20$ meters per second. (The rocket's height is decreasing on average by 20 m/s over that interval.)

b) (2 points) Find the instantaneous rate of change of the rocket's height at $t = 2$ seconds. Include units.

Solution: Instantaneous rate: $h'(t) = -10t + 20$. So $h'(2) = -10(2) + 20 = 0$ meters per second.

c) (1 point) Interpret the meaning of your answers from parts (a) and (b).

Interpretation: At $t = 2$ seconds the instantaneous velocity is 0 m/s which means that the rocket is momentarily stationary. In other words, the rocket's height is neither increasing nor decreasing at $t = 2$ seconds. The average rate on $[3, 5]$ being -20 m/s means that between 3 and 5 seconds the rocket's height decreased on average by 20 meters per second.

Part III - Free Response: You must show your work in the space provided to receive full credit. Correct final answers without sufficient supporting work may receive zero credit.

Please put a box around your final answers.

9. (5 points) (1 point each) Use the graph to calculate the following limits.

a) $\lim_{x \to -3^-} f(x) = -0.75$

b) $\lim_{x \to -3^+} f(x) = 0.75$

c) $\lim_{x \to -3} f(x) = DNE$

d) $\lim_{x \to 2} f(x) = -0.5$

e) $f(2) = -1$

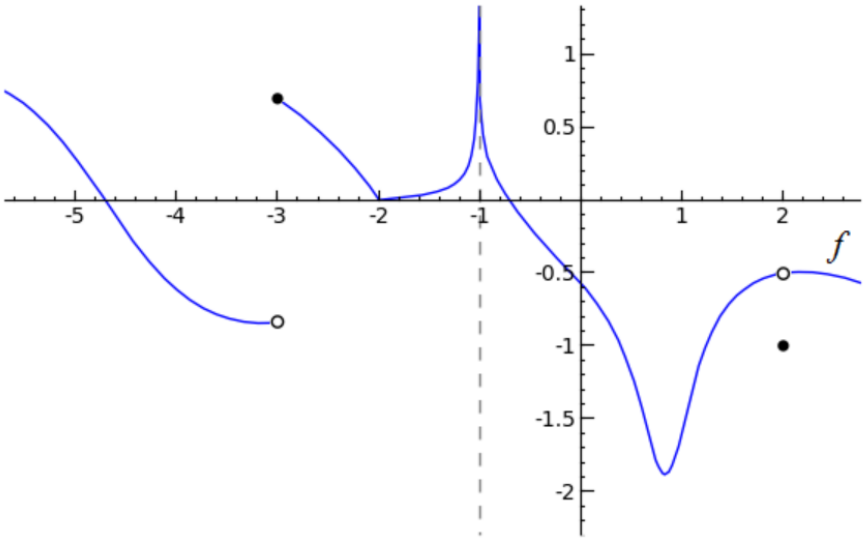

Figure 2

Solution (reading values from the graph):

a) $\lim_{x \to -3^-} f(x) = -0.75$. (Left-hand approach reaches $y = -0.75$)

b) $\lim_{x \to -3^+} f(x) = 0.75$. (Right-hand approach reaches $y = 0.75$)

c) $\lim_{x \to -3} f(x)$ does not exist because the left- and right-hand limits differ.

d) $\lim_{x \to 2} f(x) = -0.5$. (Both sides approach -0.5.)

e) $f(2) = -1$. (The filled dot at $x = 2$ is at $y = -1$, so the function value differs from the limit.)

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10. (10 points) The monthly water usage for a household (in liters) can be modeled by

$$W(t) = 0.05t^3 - 0.4t^2 + 3t + 500,$$

where t measures time in months since the start of January 2020, and $W(t)$ is the water usage per month.

a) (4 points) Find $W'(t)$.

Solution: Differentiate term-by-term:

$$\boxed{W'(t) = 0.15t^2 - 0.8t + 3.}$$

b) (4 points) Evaluate $W'(12)$, round to 2 decimal places, include units.

Solution: Evaluate at $t = 12$:

$$W'(12) = 0.15(12)^2 - 0.8(12) + 3 = 0.15(144) - 9.6 + 3 = 21.6 - 9.6 + 3 = 15.0.$$

$\boxed{15.00 \text{ liters per month}}$ (i.e. increase in liters/month each month).

c) (2 points) Write a sentence to explain what the result from part (b) tells us about water usage.

Interpretation: At $t = 12$ months (one year after January 2020), the monthly water usage is increasing at about 15 liters per month. In other words, near that time the household's monthly usage is rising by about 15 liters each month.

11. (10 points) Suppose a drone takes off with an initial velocity of 12 meters per second from a height of 50 meters. The height of the drone t seconds after taking off is given by

$$H(t) = -2t^2 + 12t + 50.$$

a) (7 points) Find the instantaneous rate of change of the drone's height at $t = 4$ seconds. Include units.

Solution: $H'(t) = -4t + 12$. Thus $H'(4) = -4(4) + 12 = -16 + 12 = \boxed{-4 \text{ meters per second.}}$

b) (3 points) Interpret the meaning of your answer in part (a).

Interpretation: At $t = 4$ seconds the drone's height is decreasing at 4 meters per second (i.e. the drone is moving downward at 4 m/s at that instant).

12. (15 points) (5 points each) Find the derivative of the following functions. Show your work. You **do NOT** need to simplify your results.

a) $f(x) = 9x^{\frac{7}{3}} - \dfrac{5}{x^4} + \dfrac{7}{2x^{-5}} + 88.2$

Solution: Rewrite terms: $9x^{7/3} - 5x^{-4} + \dfrac{7}{2}x^5 + 88.2$ (note: $\frac{7}{2x^{-5}} = \frac{7}{2} \cdot x^5$). Differentiate:

$$f'(x) = 9 \cdot \frac{7}{3}x^{4/3} - 5(-4)x^{-5} + \frac{7}{2} \cdot 5x^4 + 0.$$

So

$$\boxed{f'(x) = 21x^{4/3} + 20x^{-5} + \tfrac{35}{2}x^4.}$$

b) $g(t) = \sqrt[4]{t^{11}} - \sqrt[6]{t^{14}}$

Solution: Simplify exponents first: $\sqrt[4]{t^{11}} = t^{11/4}$, $\sqrt[6]{t^{14}} = t^{14/6} = t^{7/3}$. Then

$$\boxed{g'(t) = \frac{11}{4}t^{7/4} - \frac{7}{3}t^{4/3}}.$$

c) $h(u) = 4\ln(u) + 9e^{15u}$

Solution: $\boxed{h'(u) = 4 \cdot \dfrac{1}{u} + 9 \cdot 15e^{15u} = \dfrac{4}{u} + 135e^{15u}.}$

13. (10 points) The number of cells in a petri dish from the start of the observation can be modeled by

$$C(t) = 3e^{0.05t} + 2e^{0.2t} + 0.8t \quad \text{thousand cells,}$$

where t is the number of hours since the start of the observation ($t = 0$ represents the beginning of the observation).

a) (4 points) Find $C'(t)$.

Solution: Differentiate termwise:

$$C'(t) = 3(0.05)e^{0.05t} + 2(0.2)e^{0.2t} + 0.8 = \boxed{0.15e^{0.05t} + 0.4e^{0.2t} + 0.8 \quad \text{(thousand cells per hour).}}$$

b) (4 points) Estimate the instantaneous rate of change in total number of cells at **after 8 hours**. Include units. (Round your answer to the nearest whole number.)

Solution: Evaluate at $t = 8$:

$$C'(8) = 0.15e^{0.4} + 0.4e^{1.6} + 0.8.$$

Numerically: $e^{0.4} \approx 1.49182$, $e^{1.6} \approx 4.95303$. Thus

$$C'(8) \approx 0.15(1.49182) + 0.4(4.95303) + 0.8 \approx 0.22377 + 1.98121 + 0.8 \approx 3.00498$$

(thousand cells per hour). Rounded to nearest whole number:
$\boxed{(i.e.\ 3{,}005\ \textit{cells per hour})}$. Note units given in the model are thousand cells.

c) (2 points) Interpret your answer from part (b).

Interpretation: After 8 hours, the total number of cells is increasing at about 3.005 thousand cells per hour (roughly 3,005 thousand cells per hour), meaning the population is growing rapidly at that instant.

14. (10 points) Let $f(x) = 7x^2 - 11x$. Answer the following:

(a) (3 points) Find $f(x + h)$.

Solution: $f(x+h) = 7(x+h)^2 - 11(x+h) = 7(x^2+2xh+h^2) - 11x - 11h = 7x^2 + 14xh + 7h^2 - 11x - 11h.$

(b) (4 points) Find the difference quotient.

Solution: Difference quotient:

$$\frac{f(x+h) - f(x)}{h} = \frac{[7x^2 + 14xh + 7h^2 - 11x - 11h] - (7x^2 - 11x)}{h} = \frac{14xh + 7h^2 - 11h}{h} = \boxed{14x + 7h - 11}.$$

(c) (3 points) Use the limit definition of the derivative to find $f'(-2)$.

Solution: Derivative by limit: $f'(x) = \lim_{h \to 0}(14x + 7h - 11) = 14x - 11$. So $f'(-2) = 14(-2) - 11 = -28 - 11 = \boxed{-39}$.