MAT 210 Brief Calculus
Sample Exam 1 (Practice Version A)
Solutions

Instructor	XXXXXXX
Full Name (Print clearly!)	
Student ID	

Important Disclaimer for Students:

- This is a **sample exam**, not the actual in-class exam. Problems are of similar difficulty but may differ in type, numbers, or context.
- Success in the course depends on practicing concepts beyond this sample: lecture notes, Edfinity, Group Work, quizzes, and homework.
- You should complete this exam as if you were in the real exam room: no notes, no internet, no calculators beyond what is allowed, and an approximately 60-minute time limit.
- After finishing, check your answers against the provided solutions and grade yourself.
- If you could not complete within the time limit, practice again and attempt another sample exam.
- Treat every problem in your coursework (notes, quizzes, assignments) as fair game for the actual exam.

Part I - Multiple Choice: Circle the response that best completes the statement or answers the question. Then write the letter of your answer in the table after Problem 7.

1. (5 points) The table below shows the median monthly electricity bill (in tens of dollars) for households in a city from January 2015 to January 2021, where t is years since the start of 2015. Compute the average rate of change of the bill from the start of 2017 to the start of 2021.

Year t (years after 2015)	0	1	2	3	4	5	6
Bill (tens of US \$)	12	13.2	14.5	15.5	16.4	17.5	18.1

A. From the start of 2017 to the start of 2021, the bill increased by 0.5 tens of dollars per year, on average.

B. From the start of 2017 to the start of 2021, the bill decreased by 0.6 tens of dollars per year, on average.

C. From the start of 2017 to the start of 2021, the bill increased by 0.75 tens of dollars per year, on average.

D. From the start of 2017 to the start of 2021, the bill increased by 0.9 tens of dollars per year, on average.

E. From the start of 2017 to the start of 2021, the bill decreased by 1.2 tens of dollars per year, on average.

Solution: "Start of 2017" corresponds to $t = 2$, and "start of 2021" corresponds to $t = 6$.

From the table:

$$\text{Bill at } t = 2 = 14.5 \quad \text{(tens of dollars)},$$

$$\text{Bill at } t = 6 = 18.1 \quad \text{(tens of dollars)}.$$

Average rate of change over $[2, 6]$:

$$\frac{18.1 - 14.5}{6 - 2} = \frac{3.6}{4} = 0.9 \text{ tens of dollars per year.}$$

Interpretation: the bill increased by $0.9 \times 10 = \$9.00$ per year on average.

Correct choice: D

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2. (5 points) A bacterial culture (in millions) is modeled by the function

$$B(t) = 12 + 4\ln(t),$$

where t is measured in hours after the culture is started. Find the instantaneous rate of change of the culture size at $t = 4$ hours. Round to the nearest whole number.

A. 4 million cells per hour

B. 1 million cells per hour

C. 12 million cells per hour

D. 16 million cells per hour

E. None of the above

Solution: The instantaneous rate of change is the derivative $B'(t)$.

Differentiate:

$$B'(t) = \frac{d}{dt}\big(12 + 4\ln t\big) = 4 \cdot \frac{1}{t} = \frac{4}{t}.$$

Evaluate at $t = 4$:

$$B'(4) = \frac{4}{4} = 1.$$

Units: millions of cells per hour.

Rounded to the nearest whole number: 1.

Correct choice: B

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3. (5 points) Determine the constant k such that $\lim_{x\to 2} g(x)$ exists, where

$$g(x) = \begin{cases} 3x + k, & x < 2, \\ 2x^2 - 1, & x > 2 \end{cases}$$

A. $k = \frac{7}{6}$

B. $k = 6$

C. $k = -1$

D. $k = 1$

E. None of the above

Solution: For $\lim_{x\to 2} g(x)$ to exist, the left-hand and right-hand limits must agree.

Left-hand limit (as $x \to 2^-$) using $3x + k$:

$$\lim_{x\to 2^-} 3x + k = 3(2) + k = 6 + k.$$

Right-hand limit (as $x \to 2^+$) using $2x^2 - 1$:

$$\lim_{x\to 2^+} 2x^2 - 1 = 2(2)^2 - 1 = 8 - 1 = 7.$$

Set equal:

$$6 + k = 7 \quad \Rightarrow \quad k = 1.$$

Correct choice: D

4. (5 points) At the age of t solaks, the height of a child is given by $M(t)$ (measured in molarks). Which of the following calculations gives the *average rate of change* of the child's height from age 3 solaks to 9 solaks?

A. $\dfrac{M(6)}{6}$

B. $\dfrac{M'(9) - M'(3)}{9-3}$

C. $\dfrac{M(9) - M(3)}{9-3}$

D. $\dfrac{M'(6)}{6}$

E. $\dfrac{M(9) - M(3)}{9}$

Solution: The average rate of change of a function M over $[a, b]$ is

$$\frac{M(b) - M(a)}{b-a}.$$

Here $a = 3$, $b = 9$, so

$$\frac{M(9) - M(3)}{9-3}.$$

This matches choice C.

Correct choice: C

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5. (5 points) Which of the following functions is *not continuous* at $x = 3$? There is only **one** correct answer.

A. $\dfrac{x-3}{x+3}$

B. $\dfrac{x+3}{x-3}$

C. $\ln(x+3)$

D. $(3x-9)^2$

E. None of the above

Solution: Check each at $x = 3$.

A: $\dfrac{x-3}{x+3}$. Denominator at 3 is $3+3 = 6 \neq 0$. So expression defined and continuous at 3.

B: $\dfrac{x+3}{x-3}$. Denominator at 3 is $3-3 = 0$ so function is not defined at $x = 3$ (vertical asymptote). Not continuous.

C: $\ln(x+3)$. At $x = 3$ the argument of ln is $3+3 = 6 > 0$, so defined and continuous at 3.

D: $(3x-9)^2$. This is a polynomial expression in x (after expansion) and is continuous everywhere, including at 3.

Therefore the only non-continuous one at $x = 3$ is B.

Correct choice: B

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6. (5 points) The number of bicycles in a city from 1990 to 2010 can be modeled by the function

$$B(t) = 80t^2 - 400t + 5000$$

where t is the number of years since 1990. Interpret the meaning of $B(15) = 17000$ and $B'(15) = 2000$.

A. In 2005, there were 17000 bicycles, and the number of bicycles was increasing at a rate of 2000 bicycles per year at that time.

B. In 2005, there were 17000 bicycles, and the total increase over the 15 years since 1990 was 2000 bicycles.

C. In 2005, there were 17000 bicycles, and the number of bicycles was increasing at an average of 2000 bicycles per year between 1990 and 2005.

D. In 2005, there were 17000 bicycles, and the number of bicycles was increasing at a rate of 2000 bicycles every 15 years after 1990.

E. None of the above

Solution: $t = 15$ corresponds to the year $1990 + 15 = 2005$.

$B(15) = 17000$ gives the number of bicycles at that time (17,000 bicycles).

$B'(t) = \dfrac{d}{dt}(80t^2 - 400t + 5000) = 160t - 400.$

Evaluate $B'(15) = 160(15) - 400 = 2400 - 400 = 2000$. Units: bicycles per year.

Interpretation: At the year 2005 the city had 17,000 bicycles and the number was increasing at 2,000 bicycles per year at that instant.

This matches choice A.

Correct choice: A

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7. (5 points) The graph of $f(x)$ is shown in Figure 1. Fill in the blanks: The function has a _____ discontinuity at $x =$ _____ and a _____ discontinuity at $x =$ _____.

A. removable, 1; infinite, -1

B. jump, -1; removable, 4

C. removable, -1; jump, 1

D. infinite 4; removable 1

E. None of the above

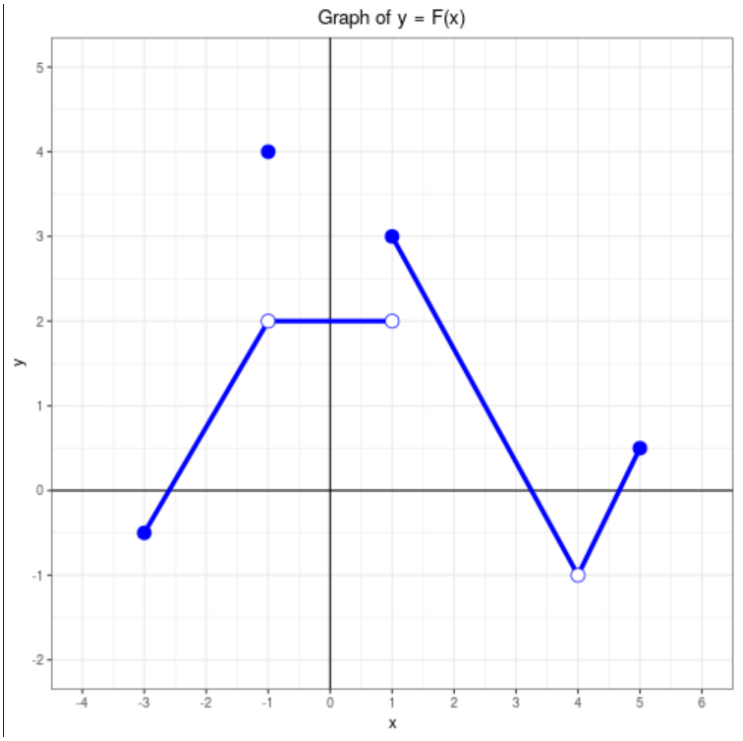


Figure 1: Sample discontinuities

Solution: To classify the discontinuities visually:

- A *removable* discontinuity (a hole) occurs where $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$ but $f(a)$ is either undefined or a different value (a hole with a missing dot).
- A *jump* discontinuity occurs when the left- and right-hand limits exist but are not equal.
- An *infinite* discontinuity occurs when one-sided limits blow up to $\pm\infty$.

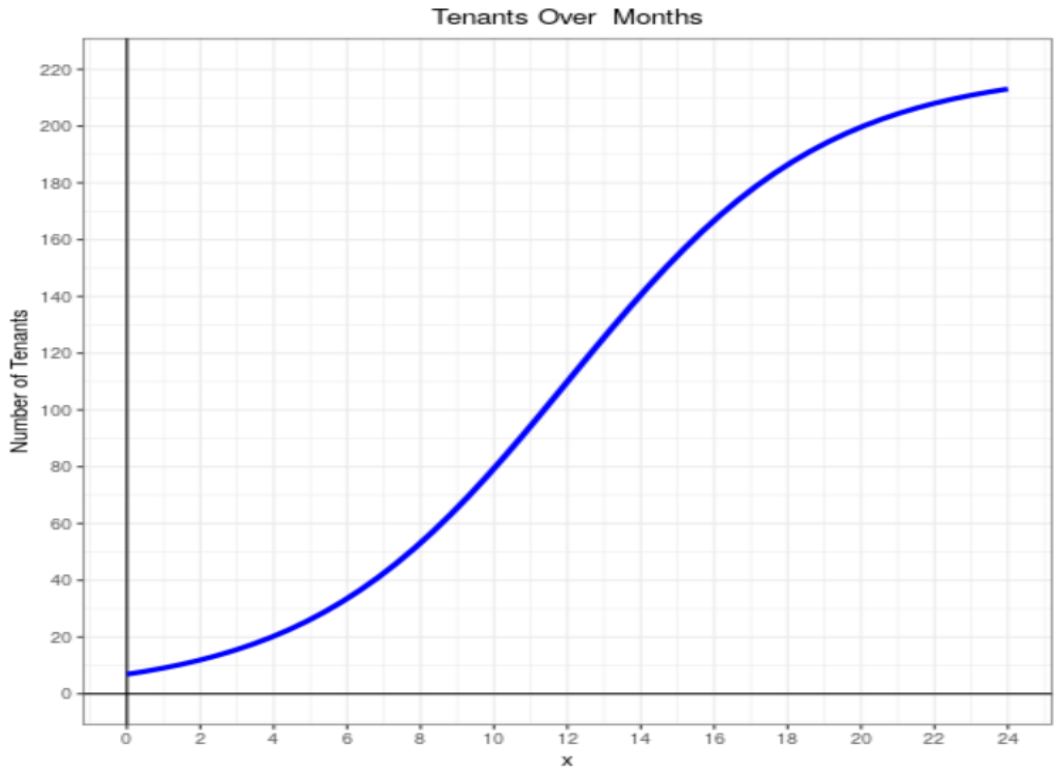
Figure shows jump at $x = 1$ and removable at $x = -1 \Rightarrow \mathbf{C}$

PART I Answers: Please write the letter of your selected answers to multiple choice questions 1 through 7 in the following table.

Problem	1	2	3	4	5	6	7
Answer	D	B	D	C	B	A	C

Part II - Fill in the blanks

8. (5 points) (1.25 points each) This graph is a model of the number of tenants in a particular apartment complex in its first 24 months. Answer the questions below.



Circle the correct answer to complete the following fill-in-the-blank questions:

A. From month 1 to month 11, the total population is (**increasing** / **decreasing**).

B. From month 1 to month 11, the average rate of change of the population is (**greater** / **less**) than the instantaneous rate of change in month 20. 0.25cm

C. The instantaneous rate of change in month 22 is (**greater** / **less**) than the average rate of change from month 1 to 24.

D. The instantaneous rate of change in month 4 is (**greater** / **less**) than the instantaneous rate of change in month 14.

Solution:

The steps below explain how to decide each answer from the graph:

A. Look at the plot between month 1 and month 11. The curve rises overall, hence the population is **increasing**.

B. The average rate of change from month 1 to 11 is the slope of the secant line between those two points. The instantaneous rate at month 20 is the slope of the tangent at $t = 20$. Compare the steepness: the tangent at $t = 20$ is shallower, avg ROC is **greater**.
Note: If the tangent at 20 was steeper (larger slope), then avg ROC would be **less** but that's not the case for the given graph.

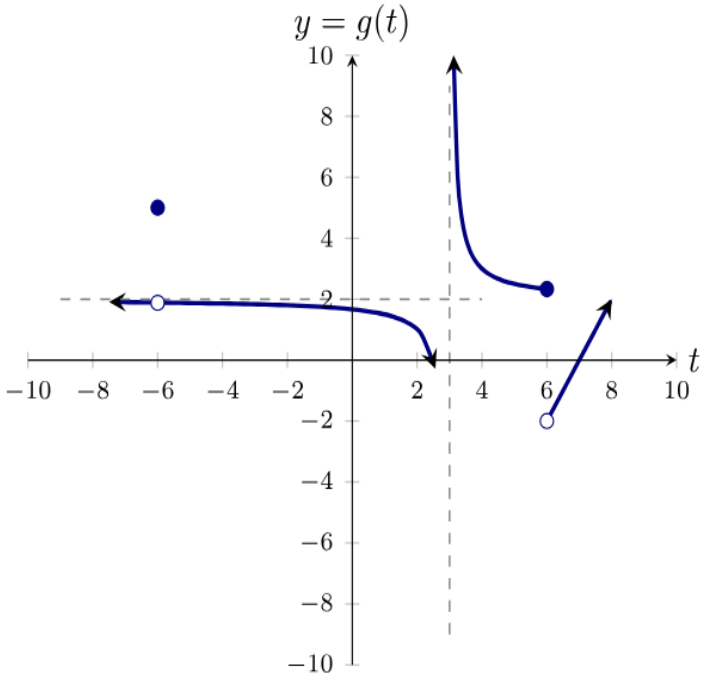
C. Compare instantaneous rate at 22 (tangent at 22) with the average rate over [1,24] (secant between months 1 and 24). The tangent at 22 is less steep than the secant.

D. Compare tangents at 4 and 14: visually see which is steeper. The slope at 14 is higher.

Part III - Free Response: You must show your work in the space provided to receive full credit. Correct final answers without sufficient supporting work may receive zero credit.

Please put a box around your final answers.

9. (5 points) (0.5 point each) Use the graph to calculate the following limits.



a) $\lim_{t\to -6^-} g(t) = 2$

b) $\lim_{t\to -6^+} g(t) = 2$

c) $\lim_{t\to -6} g(t) = 2$

d) $g(-6) = 5$

e) $\lim_{t\to 3^-} g(t) = -\infty$

f) $\lim_{t\to 3^+} g(t) = +\infty$

g) $\lim_{t\to 3} g(t) = DNE$

h) $g(3) = DNE$

i) $\lim_{t\to 6} g(t) = DNE$

j) $g(6) = 2$

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10. (10 points) Let $t \geq 1$ denote the number of weeks since a species of insects (*in hundreds*) were introduced in a greenhouse. The number of insects in the greenhouse at week t is modeled by

$$P(t) = 150 + 40 \ln t,$$

.

a) (4 points) Find $P'(t)$.

Solution: Differentiate $P(t)$ with respect to t.

$$P(t) = 150 + 40\ln t \quad \Rightarrow \quad P'(t) = 40 \cdot \frac{d}{dt}(\ln t) = \frac{40}{t}.$$

$\boxed{P'(t) = \dfrac{40}{t} \text{ (insects per week)}}$

b) (4 points) Evaluate $P'(8)$, round to 2 decimal places, include units.

Solution: Substitute $t = 8$ into $P'(t)$.

$$P'(8) = \frac{40}{8} = 5.$$

Units: hundred insects per week.

$\boxed{P'(8) = 500 \text{ insects/week}}$

c) (2 points) Interpret the meaning of your answer in part (b).

Interpretation: At week 8 the insect population is increasing at approximately 500 insects per week (i.e., the instantaneous growth rate at $t = 8$ is 500 insects/week).

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11. (10 points) Suppose an object is thrown upward with an initial velocity of 49 feet per second from a height of 135 feet. The height of the object t seconds after it is thrown is given by

$$h(t) = -2t^2 + 49t + 135.$$

a) (7 points) Find the average rate of change of the position of the object for the time period $t = 5$ to $t = 10$. Include units.

Solution: Average rate of change of h over $[5, 10]$ is

$$\frac{h(10) - h(5)}{10 - 5}.$$

Compute $h(5)$ and $h(10)$:

$$h(5) = -2(5)^2 + 49(5) + 135 = -2(25) + 245 + 135 = -50 + 245 + 135 = 330.$$

$$h(10) = -2(10)^2 + 49(10) + 135 = -2(100) + 490 + 135 = -200 + 490 + 135 = 425.$$

Now compute average rate:

$$\frac{425-330}{5} = \frac{95}{5} = 19.$$

Units: feet per second (ft/s), since h is feet and t is seconds.

$\boxed{\text{Average rate of change} = 19 \text{ ft/s}}$

b) (3 points) Interpret the meaning of your answer in part (a).

Interpretation: Between $t = 5$ s and $t = 10$ s the object's height is increasing at a rate of 19 feet per second. (This is the average vertical speed over that 5-second interval.)

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12. (15 points) (5 points each) Find the derivative of the following functions. Show your work. You **do NOT** need to simplify your results.

a) $p(y) = 14y^{\frac{9}{2}} - \dfrac{8}{y^3} - \dfrac{13}{9y^{-6}} + 121.7$

Solution: First rewrite the terms with powers:

$$p(y) = 14y^{9/2} - 8y^{-3} - \frac{13}{9}y^6 + 121.7$$

(note: $\frac{13}{9y^{-6}} = \frac{13}{9} \cdot y^6$ because dividing by y^{-6} is the same as multiplying by y^6.)

Differentiate term-by-term:

$$p'(y) = 14 \cdot \frac{9}{2}y^{7/2} - 8(-3)y^{-4} - \frac{13}{9} \cdot 6y^5 + 0.$$

Simplify coefficients:

$$p'(y) = 63y^{7/2} + 24y^{-4} - \frac{78}{9}y^5.$$

Reduce fraction $\frac{78}{9} = \frac{26}{3}$:

$$\boxed{p'(y) = 63y^{7/2} + 24y^{-4} - \tfrac{26}{3}y^5}$$

b) $m(z) = \sqrt[5]{z^{13}} - \sqrt[7]{z^{18}}$

Solution: Rewrite with rational exponents:

$$m(z) = z^{13/5} - z^{18/7}.$$

Differentiate:

$$m'(z) = \frac{13}{5}z^{13/5-1} - \frac{18}{7}z^{18/7-1} = \frac{13}{5}z^{8/5} - \frac{18}{7}z^{11/7}.$$

$$\boxed{m'(z) = \frac{13}{5}z^{8/5} - \frac{18}{7}z^{11/7}}$$

c) $r(s) = 6\ln(s) + 11e^{21s}$

Solution: Differentiate:

$$r'(s) = 6 \cdot \frac{1}{s} + 11 \cdot 21e^{21s} = \frac{6}{s} + 231e^{21s}.$$

$$\boxed{r'(s) = \frac{6}{s} + 231e^{21s}}$$

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13. (10 points) The number of bacteria in a lab culture from the start of the experiment can be modeled by

$$B(t) = 2e^{0.1t} + 4e^{0.3t} + 0.5t \quad \text{thousands of bacteria,}$$

where t is the number of hours since the start of the experiment ($t = 0$ represents the beginning of the experiment).

a) (4 points) Find $B'(t)$.

Solution: Differentiate each term (recall derivative of e^{at} is ae^{at}):

$$B'(t) = 2 \cdot 0.1e^{0.1t} + 4 \cdot 0.3e^{0.3t} + 0.5$$

So

$$\boxed{B'(t) = 0.2e^{0.1t} + 1.2e^{0.3t} + 0.5 \text{ (thousands of bacteria per hour)}}$$

b) (4 points) Calculate the instantaneous rate of change of total number of bacteria after **10 hours**. Include units. (Round your answer to the nearest whole number.)

Solution: Evaluate $B'(t)$ at $t = 10$:

$$B'(10) = 0.2e^{0.1\cdot 10} + 1.2e^{0.3\cdot 10} + 0.5 = 0.2e^1 + 1.2e^3 + 0.5.$$

Compute numerically (use $e^1 \approx 2.7182818,\ e^3 \approx 20.085537$):

$$0.2e^1 \approx 0.2(2.7182818) = 0.54365636,$$
$$1.2e^3 \approx 1.2(20.085537) = 24.1026444,$$

so

$$B'(10) \approx 0.5437 + 24.1026 + 0.5 = 25.1463 \text{ (thousands of bacteria per hour).}$$

Rounded to nearest whole number: $25,146$ bacteria per hour.

$\boxed{B'(10) \approx 25{,}146 \text{ bacteria per hour}}$

c) (2 points) Interpret your answer from part (b).

Interpretation: After 10 hours, the culture's population is increasing by 25,146 bacteria per hour (i.e., the instantaneous rate of change at $t = 10$ is ≈ 25 thousand bacteria per hour).

14. (10 points) Let

$$f(x) = 5x^2 - 6x.$$

Answer the following:

(a) (3 points) Find $f(x+h)$.

Solution:

$$f(x+h) = 5(x+h)^2 - 6(x+h) = 5(x^2 + 2xh + h^2) - 6x - 6h$$
$$= 5x^2 + 10xh + 5h^2 - 6x - 6h.$$

$\boxed{f(x+h) = 5x^2 + 10xh + 5h^2 - 6x - 6h}$

(b) (4 points) Find the difference quotient.

Solution: Difference quotient:

$$\frac{f(x+h) - f(x)}{h} = \frac{(5x^2 + 10xh + 5h^2 - 6x - 6h) - (5x^2 - 6x)}{h}$$

$$= \frac{10xh + 5h^2 - 6h}{h} = 10x + 5h - 6.$$

$\boxed{\dfrac{f(x+h) - f(x)}{h} = 10x + 5h - 6}$

(c) (3 points) Use the limit definition of the derivative to find $f'(x)$.

Solution: Take the limit as $h \to 0$ of the difference quotient:

$$f'(x) = \lim_{h \to 0} (10x + 5h - 6) = 10x - 6.$$

$\boxed{f'(x) = 10x - 6}$

(If asked for $f'(3)$: $f'(3) = 10(3) - 6 = 30 - 6 = 24$.)