MAT 210 Brief Calculus
Sample Exam 3 (Practice Version C)
Solutions

Instructor	XXXXXXX
Full Name (Print clearly!)	
Student ID	

Important Disclaimer for Students:

- This is a **sample exam**, not the actual in-class exam. Questions are of similar difficulty but may differ in type, numbers, or context.
- Success in the course depends on practicing concepts beyond this sample: lecture notes, Edfinity assignments, group work, quizzes, and homework.
- Complete this exam under exam-like conditions: no notes, no internet, no communication, and about **60-75 minutes** to finish.
- After finishing, check your answers against the posted solutions and grade yourself honestly.
- If you cannot complete it within the time limit, review your weak areas and try another sample exam.
- Treat all concepts from class materials as fair game for the actual exam.

Part I - Multiple Choice: Circle the correct answer and record it in the answer grid.

1. (5 points) Suppose $f''(x) > 0$ for all x in an interval containing $x = 1$. What does this tell you about the graph of $f(x)$ on that interval?

   A. The graph is concave up

   B. The graph is concave down

   C. The function has a maximum at $x = 1$

   D. The function has a minimum at $x = 1$

   E. $f'(1) = 0$

   Solution: Since $f''(x) > 0$ for all x in the interval, the graph of $f(x)$ is concave up throughout that interval. This means the function is "cupped upwards," but it does not necessarily tell us anything about maxima, minima, or the value of $f'(1)$.

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2. (5 points) The profit $P(x)$ (in dollars) from selling x units of a product has $P'(x) > 0$ and $P''(x) < 0$ for all relevant values of x. What does this indicate about the profit function?

   A. Profit increases at an increasing rate

   B. Profit increases at a decreasing rate

   C. Profit must be negative

   D. Profit has a minimum everywhere

   E. None of the above

   Solution: Since $P''(x) < 0$ for all relevant x, the graph of $P(x)$ is concave down. Also, $P'(x) > 0$ means that while profit is increasing, it is increasing at a decreasing rate—the slope of $P(x)$ is getting smaller as x increases. It does not mean profit is negative or that there is a minimum everywhere.

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3. (5 points) Which statement correctly describes the indefinite integral of a derivative, $\int f'(x)\,dx$?

   A. It equals $f'(x) + C$

   B. It equals $\dfrac{df}{dx} + C$

   C. It equals $f(x) + C$

   D. It cannot be computed

   E. None of the above

   Solution: We know that integrating a derivative gives back the original function. That is, $\int f'(x)\,dx = f(x) + C$, where C is the constant of integration.

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4. (5 points) The position of a particle moving along a line is given by $s(t)$. Suppose $s'(t) = 0$ and $s''(t) > 0$ at $t = 4$. What does this tell us about the particle at $t = 4$?

   A. The particle is moving with maximum speed

   B. The particle is momentarily at rest and at a relative minimum position

   C. The particle is moving backward

   D. The particle has constant velocity

   E. None of the above

Solution: We are given that $s'(4) = 0$, so the particle's velocity is zero at $t = 4$, that is, it is momentarily at rest. Since $s''(4) > 0$, the position function is concave up at $t = 4$, meaning the particle is at a relative minimum position.

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5. (5 points) The graph of a function represents the height of a ball thrown upward. If $h''(t) < 0$ for all t, what does this indicate about the motion?

   A. The ball is speeding up upward

   B. The ball experiences downward acceleration

   C. The ball moves with constant velocity

   D. The ball has no acceleration

   E. None of the above

Solution: Since $h''(t) < 0$ for all t, the height function is concave down. In terms of motion, $h''(t)$ represents the acceleration of the ball. A negative second derivative means the acceleration is downward (toward the ground).

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6. (5 points) If $f(x) = -\frac{1}{25}e^{-0.06x}$, what can we say about its concavity?

   A. Concave up for all real x

   B. Concave down for all real x

   C. Concave up for $x > 0$, concave down for $x < 0$

   D. Concave down for $x > 0$, concave up for $x < 0$

   E. None of the above

Solution: First, find the second derivative of $f(x)$:

$$f(x) = -\frac{1}{25}e^{-0.06x} \implies f'(x) = -\frac{1}{25}(-0.06)e^{-0.06x} = \frac{0.06}{25}e^{-0.06x}$$

$$f''(x) = \frac{0.06}{25}(-0.06)e^{-0.06x} = -\frac{0.0036}{25}e^{-0.06x} < 0 \text{ for all real } x$$

Since $f''(x) < 0$ for all x, the graph is *concave down* everywhere.

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7. (5 points) The velocity of a car is given by

$$v(t) = 6t - 2,$$

where v is in m/s and t is in seconds.

If we integrate $v(t)$ to find the position function $s(t)$, the constant of integration represents:

A. The initial position $s(0)$

B. The acceleration of the car

C. The time when the car stops

D. The velocity at $t = 0$

E. None of the above

Solution: The position function $s(t)$ is the integral of the velocity:

$$s(t) = \int v(t)\,dt = \int (6t - 2)\,dt = 3t^2 - 2t + C,$$

where C is the constant of integration. This constant represents the *initial position* of the car, $s(0)$.

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8. (5 points) The rate at which water enters a tank is given by

$$\frac{dV}{dt} = 3t^2 - 4t,$$

where V is the volume of water (in liters) and t is time (in minutes).

The general solution for $V(t)$ must include which constant?

A. The initial volume $V(0)$

B. The flow rate 3

C. The time when the tank becomes full

D. The coefficient of t^2

E. No constant is required

Solution: We are given the equation

$$\frac{dV}{dt} = 3t^2 - 4t.$$

To find $V(t)$, we integrate both sides:

$$V(t) = \int (3t^2 - 4t)\,dt = t^3 - 2t^2 + C,$$

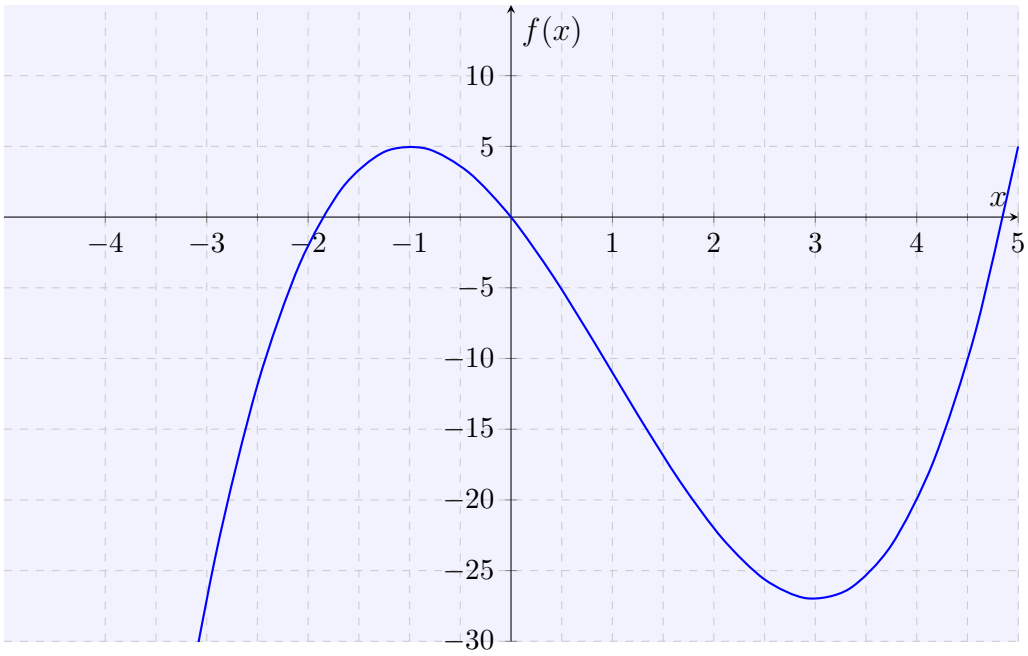
where C is the *constant of integration*. This constant represents the *initial volume of water*, $V(0)$.

PART I Answers:

Problem	1	2	3	4	5	6	7	8
Answer								

Part II - Short Answer

9. (10 points) The graph of $f(x)$ is shown below for $x \in [-3, 5]$:



Answer the following questions about the graph of $f(x)$ on $[-3, 5]$. Write your answers using interval notation and separate them with commas.

a) **Increasing on:** $(-3, -1), (3, 5)$
(Where the graph of $f(x)$ rises; i.e., $f'(x) > 0$.)

b) **Decreasing on:** $(-1, 3)$
(Where the graph of $f(x)$ falls; i.e., $f'(x) < 0$.)

c) **Relative maximum at:** $x = -1$
(The x-value(s) where f changes from increasing to decreasing.)

d) **Relative minimum at:** $x = 3$
(The x-value(s) where f changes from decreasing to increasing.)

e) **Inflection point(s):** $(1, -10)$
(Where the concavity changes; i.e., $f''(x) = 0$ or undefined.)

f) **Concave up on:** $(1, 5)$
(Where $f''(x) > 0$ and the graph bends upward.)

g) **Concave down on:** $(-3, 1)$
(Where $f''(x) < 0$ and the graph bends downward.)

h) **Critical values:** $x = -1,\ 3$
(List all x-values where $f'(x) = 0$.)

i) **Between $x = -3$ and $x = 5$:** Function changes concavity at $x = 1$ and reaches a relative maximum at $x = -1$, and a relative minimum at $x = 3$.
(Briefly describe the graph's shape using terms like increasing/decreasing, concave up/down, or relative max/min.)

Part III - Free Response
(Round your answers to 2 decimal places where needed)

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10. (15 points) Compute the following integrals. Include the constant of integration, C.

a) $\int (2x^2 - 5x)(3x + 2)\,dx$

First, expand: $(2x^2 - 5x)(3x + 2) = 6x^3 + 4x^2 - 15x^2 - 10x = 6x^3 - 11x^2 - 10x$.
Then integrate: $\int (6x^3 - 11x^2 - 10x)\,dx = \frac{6x^4}{4} - \frac{11x^3}{3} - \frac{10x^2}{2} + C = \frac{3}{2}x^4 - \frac{11}{3}x^3 - 5x^2 + C$

b) $\int \frac{3x^2 + 5\sqrt{x} - 4}{x^3}\,dx$

Rewrite each term: $\frac{3x^2}{x^3} = 3x^{-1} = \frac{3}{x}$, $\frac{5\sqrt{x}}{x^3} = 5x^{-5/2}$, $\frac{-4}{x^3} = -4x^{-3}$.
Integrate term by term: $\int 3x^{-1}dx + \int 5x^{-5/2}dx + \int (-4x^{-3})dx = 3\ln|x| - \frac{10}{3}x^{-3/2} + 2x^{-2} + C$

c) $\int x^{-7/2} + 6e^{-2x}\,dx$

Integrate each term: $\int x^{-7/2}dx = \frac{x^{-5/2}}{-5/2} = -\frac{2}{5}x^{-5/2}$, $\int 6e^{-2x}dx = 6 \cdot \frac{e^{-2x}}{-2} = -3e^{-2x}$.
Final answer: $-\frac{2}{5}x^{-5/2} - 3e^{-2x} + C$

d) $\int \frac{4x^2}{\sqrt{x^4}} - 2\sqrt{x} + 3x^3\,dx$

Simplify each term: $\frac{4x^2}{\sqrt{x^4}} = \frac{4x^2}{x^2} = 4$, $-2\sqrt{x} = -2x^{1/2}$, $3x^3$ stays the same.
Integrate term by term: $\int 4dx - \int 2x^{1/2}dx + \int 3x^3dx = 4x - \frac{4}{3}x^{3/2} + \frac{3}{4}x^4 + C$
Final answer: $4x - \frac{4}{3}x^{3/2} + \frac{3}{4}x^4 + C$

e) $\int \frac{7}{8x} - 5e^{-3x}\,dx$

Integrate each term: $\int \frac{7}{8x}dx = \frac{7}{8}\ln|x|$, $\int -5e^{-3x}dx = -5 \cdot \frac{e^{-3x}}{-3} = \frac{5}{3}e^{-3x}$.
Final answer: $\frac{7}{8}\ln|x| + \frac{5}{3}e^{-3x} + C$

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11. (20 points) Solve the following initial value problems. For each part, find the function $f(x)$ that satisfies the initial condition.

a) $f'(x) = 3x^2 - 4x + 5, \qquad f(2) = 5.$

Integrate: $f(x) = \int (3x^2 - 4x + 5)dx = \int 3x^2 dx - \int 4x dx + \int 5dx = x^3 - 2x^2 + 5x + C.$

Use the initial condition $f(2) = 5$: $2^3 - 2(2^2) + 5(2) + C = 8 - 8 + 10 + C = 10 + C = 5 \implies C = -5.$

Final answer: $f(x) = x^3 - 2x^2 + 5x - 5$

b) $f'(x) = 4e^{2x} + 3e^{-x}, \qquad f(0) = 0.$

Integrate: $f(x) = \int 4e^{2x}dx + \int 3e^{-x}dx = 4 \cdot \dfrac{e^{2x}}{2} + 3 \cdot \dfrac{e^{-x}}{-1} + C = 2e^{2x} - 3e^{-x} + C.$

Use the initial condition $f(0) = 0$: $2 - 3 + C = -1 + C = 0 \implies C = 1.$

Final answer: $f(x) = 2e^{2x} - 3e^{-x} + 1$

c) $f'(x) = \frac{6}{x}, \qquad f(1) = 3 \qquad (x \geq 1).$

Integrate: $f(x) = \int \dfrac{6}{x}dx = 6\ln|x| + C.$

Use the initial condition $f(1) = 3$: $6\ln 1 + C = 0 + C = 3 \implies C = 3.$

Final answer: $f(x) = 6\ln x + 3$

d) $f'(x) = \sqrt{x} - 2x^3, \qquad f(4) = 12, \qquad (x \geq 0).$

Rewrite: $\sqrt{x} = x^{1/2}$. Integrate term by term: $f(x) = \int x^{1/2}dx - \int 2x^3 dx = \dfrac{2}{3}x^{3/2} - \dfrac{2}{4}x^4 + C = \dfrac{2}{3}x^{3/2} - \dfrac{1}{2}x^4 + C.$

Use the initial condition $f(4) = 12$: $\frac{2}{3}(4)^{3/2} - \frac{1}{2}(4)^4 + C = \frac{2}{3} \cdot 8 - \frac{1}{2} \cdot 256 + C = 12 \implies C = \frac{404}{3}.$

Final answer: $f(x) = \frac{2}{3}x^{3/2} - \frac{1}{2}x^4 + \frac{404}{3}$

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12. (10 points) A **hot chocolate dispenser** initially contains 800 milliliters of hot chocolate, so $H(0) = 800$. Hot chocolate is dispensed at a rate given by

$$H'(t) = -20e^{-0.05t}, \quad t \text{ in minutes.}$$

a) Find the **amount of hot chocolate** $H(t)$ remaining at any time $t \geq 0$. Include the constant of integration and determine its value using the initial condition.

Integrate $H'(t)$:

$$H(t) = \int (-20e^{-0.05t})\,dt = -20 \cdot \frac{e^{-0.05t}}{-0.05} + C = 400e^{-0.05t} + C$$

Use the initial condition $H(0) = 800$:

$$400e^0 + C = 800 \implies 400 + C = 800 \implies C = 400$$

Final answer:

$$\boxed{H(t) = 400e^{-0.05t} + 400}$$

b) How much hot chocolate remains after 10 minutes?
Use $H(t) = 400e^{-0.05t} + 400$:

$$H(10) = 400e^{-0.05(10)} + 400 = 400e^{-0.5} + 400 \approx 400(0.6065) + 400 \approx 242.6 + 400 \approx 642.6 \text{ mL}$$

c) Describe whether the amount function is concave up or down over time, and explain what this means for the rate at which hot chocolate is dispensed.
Compute the second derivative:

$$H''(t) = \frac{d}{dt}(-20e^{-0.05t}) = 1.0e^{-0.05t} > 0 \text{ for all } t \geq 0$$

So $H(t)$ is concave up.
Interpretation: The rate at which hot chocolate is dispensed is slowing down over time; it dispenses quickly at first and then more slowly as t increases.

d) Estimate the time when only 50 milliliters remain in the dispenser.
Set $H(t) = 50$:

$$400e^{-0.05t} + 400 = 50 \implies 400e^{-0.05t} = -350$$

This is impossible because $400e^{-0.05t} > 0$ for all t.
Note: Only 400 mL is the limiting minimum as $t \to \infty$, so the dispenser never reaches 50 mL. The minimum amount is 400 mL.

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13. (12 points) A **swimmer** moves along a straight lane in a pool, and her position (in meters) from the starting block is given by

$$y(t) = 5t^3 - 30t^2 + 45t - 4, \quad t \geq 0,$$

where t is time in seconds.

Answer the following (include appropriate units):

a) Find the **velocity** $v(t)$ and **acceleration** $a(t)$ as functions of t.
Velocity is the derivative of position:

$$v(t) = y'(t) = 15t^2 - 60t + 45 \text{ m/s}.$$

Acceleration is the derivative of velocity:

$$a(t) = v'(t) = y''(t) = 30t - 60 \text{ m/s}^2.$$

b) Find the **acceleration** at $t = 1.5$ seconds.
Use $a(t) = 30t - 60$:

$$a(1.5) = 30(1.5) - 60 = 45 - 60 = -15 \text{ m/s}^2$$

c) Find the **acceleration** at the instant when the velocity is zero.
Set $v(t) = 0$:

$$15t^2 - 60t + 45 = 0 \implies t^2 - 4t + 3 = 0 \implies (t-1)(t-3) = 0$$

So $t = 1$ s or $t = 3$ s. Corresponding accelerations:

$$a(1) = 30(1) - 60 = -30 \text{ m/s}^2, \quad a(3) = 30(3) - 60 = 30 \text{ m/s}^2$$

Therefore, the swimmer's acceleration is -30 m/s^2 at $t = 1$ s and 30 m/s^2 at $t = 3$ s.

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14. (10 points) A gardener tracks the height of a fast-growing plant (in centimeters) as a function of days $t > 0$ after germination. The growth model is

$$H(t) = t^3 - 9t^2 + 18t + 4.$$

Answer the following:

a) Find the first and second derivatives of $H(t)$.
First derivative:

$$H'(t) = 3t^2 - 18t + 18$$

Second derivative:

$$H''(t) = 6t - 18$$

b) Determine the intervals where $H(t)$ is concave up or concave down.
Set $H''(t) = 0$ to find inflection point: $6t - 18 = 0 \implies t = 3$.
Concave up: $H''(t) > 0 \implies t > 3 \to (3, \infty)$
Concave down: $H''(t) < 0 \implies t < 3 \to (0, 3)$

c) Find any inflection point(s) of $H(t)$. Give coordinates.
Inflection point occurs at $t = 3$. Find $H(3)$:

$$H(3) = 3^3 - 9(3)^2 + 18(3) + 4 = 27 - 81 + 54 + 4 = 4$$

Inflection point: $(3, 4)$

d) Interpret the concavity of $H(t)$: what does it tell you about the rate of plant growth over time?
For $t < 3$ (concave down), the plant's growth rate is slowing.
For $t > 3$ (concave up), the plant's growth rate is increasing (accelerating growth).
The inflection point at $t = 3$ marks the transition from slowing to accelerating growth.