MAT 210 Brief Calculus
Sample Exam 3 (Practice Version C)

Instructor	XXXXXXX
Full Name (Print clearly!)	
Student ID	

Important Disclaimer for Students:

- This is a **sample exam**, not the actual in-class exam. Questions are of similar difficulty but may differ in type, numbers, or context.
- Success in the course depends on practicing concepts beyond this sample: lecture notes, Edfinity assignments, group work, quizzes, and homework.
- Complete this exam under exam-like conditions: no notes, no internet, no communication, and about **60-75 minutes** to finish.
- After finishing, check your answers against the posted solutions and grade yourself honestly.
- If you cannot complete it within the time limit, review your weak areas and try another sample exam.
- Treat all concepts from class materials as fair game for the actual exam.

Part I - Multiple Choice: Circle the correct answer and record it in the answer grid.

1. (5 points) Suppose $f''(x) > 0$ for all x in an interval containing $x = 1$. What does this tell you about the graph of $f(x)$ on that interval?

   A. The graph is concave up

   B. The graph is concave down

   C. The function has a maximum at $x = 1$

   D. The function has a minimum at $x = 1$

   E. $f'(1) = 0$

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2. (5 points) The profit $P(x)$ (in dollars) from selling x units of a product has $P''(x) < 0$ for all relevant values of x. What does this indicate about the profit function?

   A. Profit increases at an increasing rate

   B. Profit increases at a decreasing rate

   C. Profit must be negative

   D. Profit has a minimum everywhere

   E. None of the above

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3. (5 points) Which statement correctly describes the indefinite integral of a derivative, $\int f'(x)dx$?

   A. It equals $f'(x) + C$

   B. It equals $\dfrac{df}{dx} + C$

   C. It equals $f(x) + C$

   D. It cannot be computed

   E. None of the above

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4. (5 points) The position of a particle moving along a line is given by $s(t)$. Suppose $s'(t) = 0$ and $s''(t) > 0$ at $t = 4$. What does this tell us about the particle at $t = 4$?

   A. The particle is moving with maximum speed

   B. The particle is momentarily at rest and at a relative minimum position

   C. The particle is moving backward

   D. The particle has constant velocity

   E. None of the above

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5. (5 points) The graph of a function represents the height of a ball thrown upward. If $h''(t) < 0$ for all t, what does this indicate about the motion?

   A. The ball is speeding up upward

   B. The ball experiences downward acceleration

   C. The ball moves with constant velocity

   D. The ball has no acceleration

   E. None of the above

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6. (5 points) If $f(x) = -\frac{1}{25}e^{-0.06x}$, what can we say about its concavity?

   A. Concave up for all real x

   B. Concave down for all real x

   C. Concave up for $x > 0$, concave down for $x < 0$

   D. Concave down for $x > 0$, concave up for $x < 0$

   E. None of the above

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7. (5 points) The velocity of a car is given by

$$v(t) = 6t - 2,$$

   where v is in m/s and t is in seconds.

   If we integrate $v(t)$ to find the position function $s(t)$, the constant of integration represents:

   A. The initial position $s(0)$

   B. The acceleration of the car

   C. The time when the car stops

   D. The velocity at $t = 0$

   E. None of the above

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8. (5 points) The rate at which water enters a tank is given by

$$\frac{dV}{dt} = 3t^2 - 4t,$$

   where V is the volume of water (in liters) and t is time (in minutes).

   The general solution for $V(t)$ must include which constant?

   A. The initial volume $V(0)$

   B. The flow rate 3

   C. The time when the tank becomes full

D. The coefficient of t^2

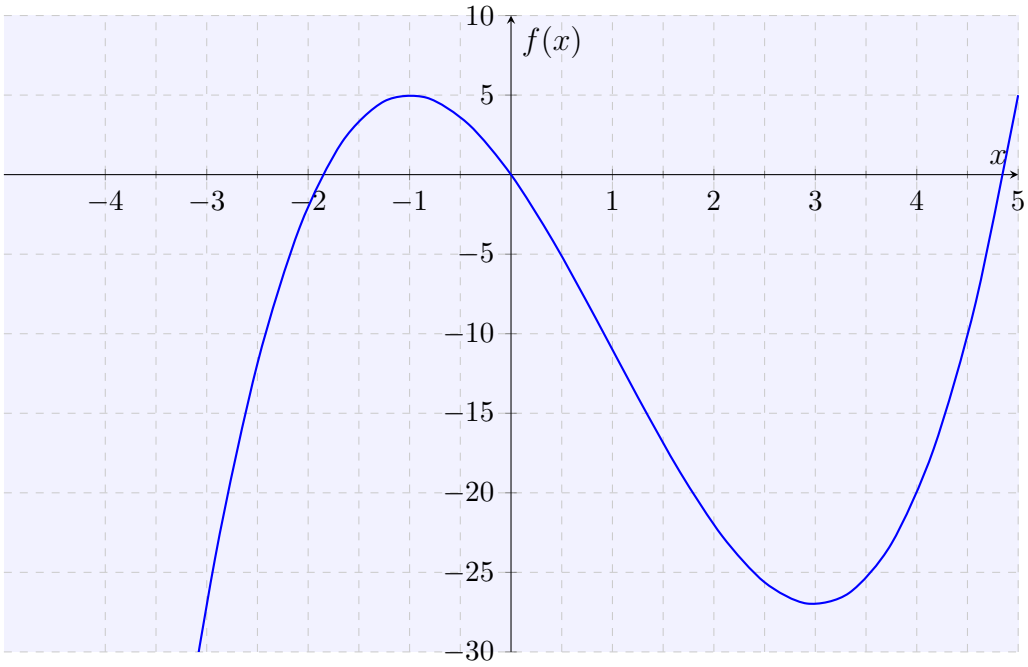
E. No constant is required

PART I Answers:

Problem	1	2	3	4	5	6	7	8
Answer								

Part II - Short Answer

9. (10 points) The graph of $f(x)$ is shown below for $x \in [-4, 5]$:



Answer the following questions about the graph of $f(x)$ on $[-4, 5]$. Write your answers using interval notation and separate them with commas.

a) **Increasing on:** _____
(Where the graph of $f(x)$ rises; i.e., $f'(x) > 0$.)

b) **Decreasing on:** _____
(Where the graph of $f(x)$ falls; i.e., $f'(x) < 0$.)

c) **Relative maximum at:** _____
(The x-value(s) where f changes from increasing to decreasing.)

d) **Relative minimum at:** _____
(The x-value(s) where f changes from decreasing to increasing.)

e) **Inflection point(s):** _____
(Where the concavity changes; i.e., $f''(x) = 0$ or undefined.)

f) **Concave up on:** _____
(Where $f''(x) > 0$ and the graph bends upward.)

g) **Concave down on:** _____
(Where $f''(x) < 0$ and the graph bends downward.)

h) **Critical values:** $x =$ _____
(List all x-values where $f'(x) = 0$.)

i) **Between $x = -4$ and $x = 5$:** _____
(Briefly describe the graph's shape using terms like increasing/decreasing, concave up/down, or relative max/min.)

Part III - Free Response
(Round your answers to 2 decimal places where needed)

10. (15 points) Compute the following integrals. Include the constant of integration, C.

a) $\int (2x^2 - 5x)(3x + 2)\, dx$

b) $\int \dfrac{3x^2 + 5\sqrt{x} - 4}{x^3}\, dx$

c) $\int x^{-7/2} + 6e^{-2x}\, dx$

d) $\int \dfrac{4x^2}{\sqrt{x^4}} - 2\sqrt{x} + 3x^3\, dx$

e) $\int \dfrac{7}{8x} - 5e^{-3x}\, dx$

11. (20 points) Solve the following initial value problems. For each part, find the function $f(x)$ that satisfies the initial condition.

a) $f'(x) = 3x^2 - 4x + 5, \qquad f(2) = 5.$

b) $f'(x) = 4e^{2x} + 3e^{-x}, \qquad f(0) = 0.$

c) $f'(x) = \frac{6}{x}, \qquad f(1) = 3 \qquad (x \geq 1).$

d) $f'(x) = \sqrt{x} - 2x^3, \qquad f(4) = 12, \qquad (x \geq 0).$

12. (10 points) A **hot chocolate dispenser** initially contains 800 milliliters of hot chocolate, so $H(0) = 800$. Hot chocolate is dispensed at a rate given by

$$H'(t) = -20e^{-0.05t}, \quad t \text{ in minutes.}$$

a) Find the **amount of hot chocolate** $H(t)$ remaining at any time $t \geq 0$. Include the constant of integration and determine its value using the initial condition.

b) How much hot chocolate remains after 10 minutes?

c) Describe whether the amount function is concave up or down over time, and explain what this means for the rate at which hot chocolate is dispensed.

d) Estimate the time when only 50 milliliters remain in the dispenser.

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13. (12 points) A **swimmer** moves along a straight lane in a pool, and her position (in meters) from the starting block is given by

$$y(t) = 5t^3 - 30t^2 + 45t - 4, \quad t \geq 0,$$

where t is time in seconds.

Answer the following (include appropriate units):

a) Find the **velocity** $v(t)$ and **acceleration** $a(t)$ as functions of t.

b) Find the **acceleration** at $t = 1.5$ seconds.

c) Find the **acceleration** at the instant when the velocity is zero.

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14. (10 points) A gardener tracks the height of a fast-growing plant (in centimeters) as a function of days $t > 0$ after germination. The growth model is

$$H(t) = t^3 - 9t^2 + 18t + 4.$$

Answer the following:

a) Find the first and second derivatives of $H(t)$.

$H'(t) =$ _____

$H''(t) =$ _____

b) Determine the intervals where $H(t)$ is concave up or concave down.

Concave up: _____

Concave down: _____

c) Find any inflection point(s) of $H(t)$. Give coordinates.

d) Interpret the concavity of $H(t)$: what does it tell you about the rate of plant growth over time?
