MAT 210 Brief Calculus
Sample Exam 3 (Practice Version B)
Solutions

Instructor	XXXXXXX
Full Name (Print clearly!)	
Student ID	

Important Disclaimer for Students:

- This is a **sample exam**, not the actual in-class exam. Questions are of similar difficulty but may differ in type, numbers, or context.
- Success in the course depends on practicing concepts beyond this sample: lecture notes, Edfinity assignments, group work, quizzes, and homework.
- Complete this exam under exam-like conditions: no notes, no internet, no communication, and about **60-75 minutes** to finish.
- After finishing, check your answers against the posted solutions and grade yourself honestly.
- If you cannot complete it within the time limit, review your weak areas and try another sample exam.
- Treat all concepts from class materials as fair game for the actual exam.

Part I - Multiple Choice: Circle the correct answer and record it in the answer grid.

1. (5 points) Evaluate:

$$\int \left(5x^{1/3} + \frac{3}{x^3}\right) dx$$

A. $\boxed{\frac{15}{4}x^{4/3} - \frac{3}{2x^2} + C}$

B. $\frac{5}{3}x^{4/3} - \frac{3}{2x^2} + C$

C. $\frac{15}{4}x^{4/3} + \frac{3}{2x^2} + C$

D. $5x^{4/3} + \frac{3}{x^2} + C$

E. None of the above

Solution: $\int 5x^{1/3}\,dx + \int 3x^{-3}\,dx = 5 \cdot \frac{x^{4/3}}{4/3} + 3 \cdot \frac{x^{-2}}{-2} + C = \frac{15}{4}x^{4/3} - \frac{3}{2x^2} + C$

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2. (5 points) Which of the following is an example of applying concavity in real-world contexts?

A. Determining when profit is maximized

B. Finding where cost stops decreasing

C. Computing total accumulated revenue

D. Plotting $f'(x)$

E. Predicting when growth rate of a population starts accelerating or slowing

Correct Answer: Predicting when growth rate of a population starts accelerating or slowing.

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3. (5 points) If $\frac{d}{dx}\big[F(x)\big] = f(x)$, then which of the following is always true?

A. $\int f(x)\,dx = F(x) + C$

B. $\int F(x)\,dx = f(x) + C$

C. $\int f(x)\,dx = F'(x) + C$

D. $\int f(x)\,dx = F(x)$

E. None of the above

Solution:
Since $\frac{d}{dx}[F(x)] = f(x)$, by the Fundamental Theorem of Calculus, the antiderivative of $f(x)$ is $F(x) + C$.
Hence, $\int f(x)\,dx = F(x) + C$.

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4. (5 points) Let $g(x)$ satisfy $g'(x) < 0$ and $g''(x) > 0$ for all x. Which statement is true?

   A. $g(x)$ is increasing and concave up

   B. $\boxed{g(x) \text{ is decreasing and concave up}}$

   C. $g(x)$ is increasing and concave down

   D. $g(x)$ is decreasing and concave down

   E. None of the above

Solution:
Since $g'(x) < 0$, the function is decreasing.
Since $g''(x) > 0$, the function is concave up.
Therefore, $g(x)$ is decreasing and concave up.

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5. (5 points) If $P'(x) = 5e^{2x} - 4e^{-x} + 6$, and $P(1) = 50$, find the profit function $P(x)$.

   A. $P(x) = \frac{5}{2}e^{2x} - 4e^{-x} + 6x + 24$

   B. $\boxed{P(x) = \frac{5}{2}e^{2x} + 4e^{-x} + 6x + 24}$

   C. $P(x) = 5e^{2x} - 4e^{-x} + 6x + 50$

   D. $P(x) = \frac{5}{2}e^{2x} + 4e^{-x} + 6x + 50$

   E. None of the above

Solution:
Integrate $P'(x)$:

$$\int P'(x)\,dx = \int (5e^{2x} - 4e^{-x} + 6)\,dx = \frac{5}{2}e^{2x} + 4e^{-x} + 6x + C$$

Use $P(1) = 50$ to find C:

$$\frac{5}{2}e^2 + 4e^{-1} + 6(1) + C = 50 \implies C = 50 - \frac{5}{2}e^2 - 4e^{-1} - 6 \approx 24$$

Hence,

$$P(x) = \frac{5}{2}e^{2x} - 4e^{-x} + 6x + 24.$$

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6. (5 points) Find the function $f(x)$ if $f'(x) = 2x^{-3} + \dfrac{1}{x}$ and $f(1) = 7$.

A. $\boxed{f(x) = -x^{-2} + \ln|x| + 8}$

B. $f(x) = -x^{-2} + \ln|x| + 7$

C. $f(x) = x^{-2} + \ln|x| + 8$

D. $f(x) = -\dfrac{1}{2}x^{-2} + \ln|x| + 8$

E. None of the above

Solution:

Integrate $f'(x)$:

$$\int f'(x)\,dx = \int 2x^{-3}\,dx + \int \frac{1}{x}\,dx = -x^{-2} + \ln|x| + C$$

Use $f(1) = 7$ to find C:

$$f(1) = -1 + 0 + C = 7 \implies C = 8$$

Hence,

$$f(x) = -x^{-2} + \ln|x| + 8$$

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7. (5 points) The velocity (in meters per second) of a particle moving along a line is given by $v(t) = 6t - 2t^2$. Find the general expression for the displacement $s(t)$.

A. $\boxed{s(t) = 3t^2 - \dfrac{2}{3}t^3 + C}$

B. $s(t) = 3t^2 - t^3 + C$

C. $s(t) = 6t^2 - 2t^3 + C$

D. $s(t) = 3t^3 - t^2 + C$

E. None of the above

Solution:

Displacement $s(t)$ (also known as the **position function**) is the antiderivative of velocity $v(t)$:

$$s(t) = \int v(t)\,dt = \int (6t - 2t^2)\,dt = \int 6t\,dt - \int 2t^2\,dt$$

$$\int 6t\,dt = 3t^2, \quad \int 2t^2\,dt = \frac{2}{3}t^3$$

So,

$$s(t) = 3t^2 - \frac{2}{3}t^3 + C$$

where C is the constant of integration.

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8. (5 points) If $C'(t) = \dfrac{20}{t}$, $t \geq 1$ and $C(1) = 100$, what best describes $C(t)$?

A. Increases at a constant rate

B. Increases quickly at first, then slows over time

C. Decreases exponentially

D. Oscillates around 100

E. None of the above

Solution:

We are given $C'(t) = \dfrac{20}{t}$.

Differentiating:

$$C''(t) = \frac{-20}{t^2}$$

Since time $t \geq 1$ the second derivative, $C''(t) < 0$.

The derivative $C'(t) > 0$ so the function is increasing but $C''(t) < 0$ so the function is conacve down, hence the function *increases quickly at first, then slows down.*

PART I Answers:

Problem	1	2	3	4	5	6	7	8
Answer	A	E	A	B	B	A	A	B

Part II - Short Answer

Round your answers to two decimal places where needed.

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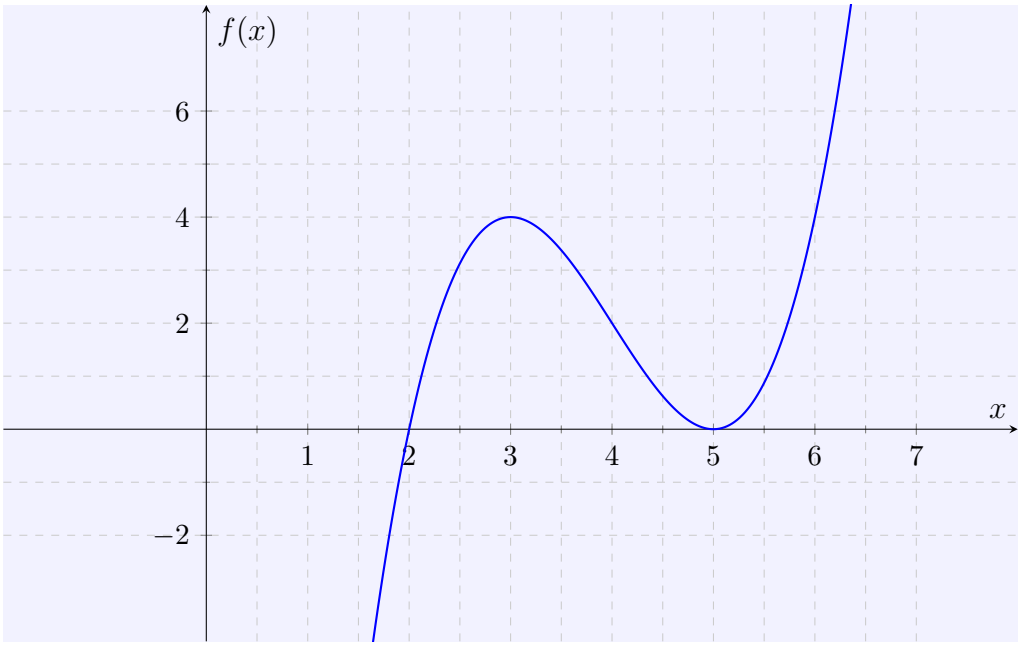
9. (10 points) The graph of $f(x)$ is shown below for $x \in [2, 6]$:



Answer the following briefly for $x \in [2, 6]$. Separate your answers with commas.

a) Increasing on: (2,3), (5,6)
(List the intervals where the function is rising; where $f'(x) > 0$.)

b) Decreasing on: (3,5)
(List the intervals where the function is falling; where $f'(x) < 0$.)

c) Relative maximum at: $x = 3$ or $(3, 4)$
(List x-values or $(x, f(x))$ points where the function changes from increasing to decreasing.)

d) Relative minimum at: $x = 5$ or $(5, 0)$
(List x-values or $(x, f(x))$ points where the function changes from decreasing to increasing.)

e) Inflection point(s): $x = 4$ or $(4, 2)$
(Points where the concavity changes; where $f''(x) = 0$ or undefined.)

f) $f''(x) > 0$ on: (4,6)
(Intervals where the function is concave up.)

g) $f''(x) < 0$ on: (2,4)
(Intervals where the function is concave down.)

h) At $x = 3$, $f'(3) =$ 0; the graph is flat.
(Use the graph to estimate the slope of $f(x)$ at $x = 3$ to determine the slope and whether f is increasing, decreasing, or flat.)

i) If $f'(x) = 0$, then $x = 3,\ 5$
(List all critical values (x-values) where the derivative is zero.)

j) General shape of $f(x)$ between $x = 3$ and $x = 5$: decreasing and changes concavity at $x = 4$
(Use terms like increasing/decreasing, concave up/down, relative maxima/minima.)

Part III - Free Response
Round your answers to two decimal places where needed.

10. (15 points) Compute the following integrals. Include the constant of integration, C.

a) $\int (2x^3 - 5x^2)(x-1)\,dx$

$\int (2x^4 - 2x^3 - 5x^3 + 5x^2)\,dx = \int (2x^4 - 7x^3 + 5x^2)\,dx = \frac{2}{5}x^5 - \frac{7}{4}x^4 + \frac{5}{3}x^3 + C$

b) $\int \dfrac{3x^2 - 6\sqrt{x} + 4}{x^3}\,dx$

$\int \left(\frac{3x^2}{x^3} - \frac{6\sqrt{x}}{x^3} + \frac{4}{x^3}\right)dx = \int \left(3x^{-1} - 6x^{-5/2} + 4x^{-3}\right)dx = 3\ln|x| + 4x^{-3/2} - 2x^{-2} + C$

c) $\int x^{-3/2} + 6e^{-4x}\,dx$

$\int x^{-3/2}\,dx + \int 6e^{-4x}\,dx = \frac{x^{-1/2}}{-1/2} - \frac{6}{4}e^{-4x} + C = -2x^{-1/2} - \frac{3}{2}e^{-4x} + C$

d) $\int \dfrac{3x^2}{\sqrt{x^4}} - 6\sqrt{x} + 4x^3\,dx$

$\int (3x^2x^{-2} - 6x^{1/2} + 4x^3)\,dx = \int (3 - 6x^{1/2} + 4x^3)\,dx = 3x - 4x^{3/2} + x^4 + C$

e) $\int \dfrac{4}{7x} + 6e^{-4x}\,dx$

$\int \frac{4}{7x}dx + \int 6e^{-4x}dx = \frac{4}{7}\ln|x| - \frac{3}{2}e^{-4x} + C$

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11. (20 points) Solve the following initial value problems. For each part, find the function $f(x)$ that satisfies the initial condition.

a) $f'(x) = 3x^2 - 6x + 4, \qquad f(0) = 5$

$f(x) = \int (3x^2 - 6x + 4)dx = x^3 - 3x^2 + 4x + C, \quad f(0) = 5 \implies C = 5 \Rightarrow f(x) = x^3 - 3x^2 + 4x + 5$

b) $f'(x) = 4e^{2x} - e^{-x}, \qquad f(0) = 3$

$f(x) = \int (4e^{2x} - e^{-x})dx = 2e^{2x} + e^{-x} + C, \quad f(0) = 3 \implies 2 + 1 + C = 3 \Rightarrow C = 0 \Rightarrow f(x) = 2e^{2x} + e^{-x}$

c) $f'(x) = \frac{5}{x}, \qquad f(1) = 0 \qquad (x \geq 1)$

$f(x) = \int \frac{5}{x}dx = 5\ln|x| + C, \quad f(1) = 0 \implies 5\ln 1 + C = 0 \Rightarrow C = 0 \Rightarrow f(x) = 5\ln x$

d) $f'(x) = \sqrt{x} + x^4, \qquad f(1) = 2, \qquad (x \geq 0)$

$$f(x) = \int (x^{1/2} + x^4)dx = \frac{2}{3}x^{3/2} + \frac{1}{5}x^5 + C, \quad f(1) = 2 \implies \frac{2}{3} + \frac{1}{5} + C = 2 \Rightarrow C = 2 - \frac{13}{15} = \frac{17}{15} \Rightarrow$$
$$f(x) = \frac{2}{3}x^{3/2} + \frac{1}{5}x^5 + \frac{17}{15}$$

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12. (10 points) Let
$$f(x) = -x^3 + 9x^2 - 18x + 12.$$

a) Find $f'(x)$ and $f''(x)$.

$f'(x) = -3x^2 + 18x - 18, \qquad f''(x) = -6x + 18$

b) Find and classify critical points as relative maxima or minima. Write your answers in the form $(x, f(x))$. (**Hint:** Use the 2nd derivative test.)

Solve $f'(x) = 0$ (using the quadratic formula):
$$-3x^2 + 18x - 18 = 0 \implies x^2 - 6x + 6 = 0 \implies x = 3 \pm \sqrt{3} \approx 1.27,\ 4.73$$

By using the Second derivative test:
- $f''(1.27) = -6(1.27) + 18 \approx 10.38 > 0 \implies$ relative minimum
- $f''(4.73) = -6(4.73) + 18 \approx -10.38 < 0 \implies$ relative maximum

Compute function values:
$$f(1.27) = -(1.27)^3 + 9(1.27)^2 - 18(1.27) + 12 = 1.61$$

Similarly:
$$f(4.73) = -(4.73)^3 + 9(4.73)^2 - 18(4.73) + 12 = 22.39$$

Relative minimum: $(1.27, 1.61)$, Relative maximum: $(4.73, 22.39)$

c) Find the intervals of concavity.

$f''(x) = -6(x - 3)$. You can use the number line and test points approach.
- $f''(x) > 0 \implies x < 3$ implies concave up
- $f''(x) < 0 \implies x > 3$ implies concave down

Concave up: $(-\infty, 3)$, Concave down: $(3, \infty)$

d) Find inflection points. Write your answers in the form $(x, f(x))$.

Solve $f''(x) = -6x + 18 = 0 \implies x = 3$
Function value: $f(3) = -27 + 81 - 54 + 12 = 12$
Inflection point: $(3, 12)$

e) Interpret what concavity reveals about the graph.

Concave up: tangent slopes are increasing;
Concave down: tangent slopes are decreasing;
inflection point at $(3, 12)$ is the point where graph changes concavity

13. (5 points) A particle moves so that its velocity is $v(t) = 2t^2 - 8t + 5$, with $v(t)$ in m/s and t in seconds. Its initial position is $s(2) = 0$.

a) Find $s(t)$.

$s(t) = \int v(t)\,dt = \int (2t^2 - 8t + 5)dt = \frac{2}{3}t^3 - 4t^2 + 5t + C$

Using $s(2) = 0$: $\frac{2}{3}(8) - 4(4) + 5(2) + C = \frac{16}{3} - 16 + 10 + C = \frac{16}{3} - 6 + C = -\frac{2}{3} + C = 0 \implies C = \frac{2}{3}$

$s(t) = \frac{2}{3}t^3 - 4t^2 + 5t + \frac{2}{3}$

b) Find its position at $t = 8$.

$s(8) = \frac{2}{3}(512) - 4(64) + 5(8) + \frac{2}{3} = \frac{1024}{3} - 256 + 40 + \frac{2}{3} = \frac{1026}{3} - 216 = 342 - 216 = 126$ m

c) When is the particle at rest?

Particle is at rest when $v(t) = 0$:

$$2t^2 - 8t + 5 = 0$$

Using the Quadratic formula: $t \approx 0.78,\ 3.22$

$t \approx 0.78$ seconds and 3.22 seconds

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14. (10 points) The temperature $T(t)$ (in °C) of a cup of coffee after being poured into a mug is modeled by

$$T(t) = 80e^{-0.2t} + 20, \quad t \geq 0$$

where t is the time in minutes.

a) Find the first and second derivatives of $T(t)$.

$T'(t) = -16e^{-0.2t}$

$T''(t) = 3.2e^{-0.2t}$

b) Determine the intervals where $T(t)$ is concave up or concave down and identify any inflection point. Write your answers in the form $(x, f(x))$.

Inflection point: None. Since the second derivative is always positive.

Concave up: $(0, \infty)$

Concave down: None

c) Interpret what the concavity of $T(t)$ tells you about how the coffee is cooling over time.

Since $T''(t) > 0$, the cooling is slowing down over time; the rate at which temperature decreases is decreasing, so the coffee cools more gradually as time passes.

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15. (10 points) A drone ascends vertically. Its velocity (in meters per second) at time t seconds is modeled by

$$v(t) = \frac{8}{t} + t, \quad t \geq 1.$$

The drone is launched from ground level, so its height at $t = 1$ seconds is $s(1) = 0$ meters.

a) Find the height function $s(t)$ of the drone. Include the constant of integration and use the given initial condition to determine its value.

$s(t) = \int v(t)dt = \int \left(\frac{8}{t} + t\right) dt = 8\ln t + \frac{t^2}{2} + C$

Use $s(1) = 0 : 8\ln 1 + \frac{1^2}{2} + C = 0 \implies C = -0.5$

$s(t) = 8\ln t + \frac{t^2}{2} - 0.5$ metres

b) Using your height function, calculate the **height of the drone at $t = 4$ seconds**.

$s(4) = 8\ln 4 + \frac{16}{2} - 0.5 \approx 11.09 + 8 - 0.5 = 18.59$ m

c) Find the **velocity of the drone at $t = 2$ seconds**, $v(2)$.

$v(2) = \frac{8}{2} + 2 = 4 + 2 = 6$ m/s