MAT 210 Brief Calculus
Sample Exam 3 (Practice Version B)

Instructor	XXXXXXX
Full Name (Print clearly!)	
Student ID	

Important Disclaimer for Students:

- This is a **sample exam**, not the actual in-class exam. Questions are of similar difficulty but may differ in type, numbers, or context.
- Success in the course depends on practicing concepts beyond this sample: lecture notes, Edfinity assignments, group work, quizzes, and homework.
- Complete this exam under exam-like conditions: no notes, no internet, no communication, and about **60-75 minutes** to finish.
- After finishing, check your answers against the posted solutions and grade yourself honestly.
- If you cannot complete it within the time limit, review your weak areas and try another sample exam.
- Treat all concepts from class materials as fair game for the actual exam.

Part I - Multiple Choice: Circle the correct answer and record it in the answer grid.

1. (5 points) Evaluate:

$$\int \left(5x^{1/3} + \frac{3}{x^3}\right) dx$$

A. $\frac{15}{4}x^{4/3} - \frac{3}{2x^2} + C$

B. $\frac{5}{3}x^{4/3} - \frac{3}{2x^2} + C$

C. $\frac{15}{4}x^{4/3} + \frac{3}{2x^2} + C$

D. $5x^{4/3} + \frac{3}{x^2} + C$

E. None of these

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2. (5 points) Which of the following is an example of applying concavity in real-world contexts?

A. Determining when profit is maximized

B. Finding where cost stops decreasing

C. Computing total accumulated revenue

D. Plotting $f'(x)$

E. Predicting when growth rate of a population starts accelerating or slowing

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3. (5 points) If $\frac{d}{dx}\big[F(x)\big] = f(x)$, then which of the following is always true?

A. $\int f(x)\,dx = F(x) + C$

B. $\int F(x)\,dx = f(x) + C$

C. $\int f(x)\,dx = F'(x) + C$

D. $\int f(x)\,dx = F(x)$

E. None of the above

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4. (5 points) Let $g(x)$ satisfy $g'(x) < 0$ and $g''(x) > 0$ for all x. Which statement is true?

A. $g(x)$ is increasing and concave up

B. $g(x)$ is decreasing and concave up

C. $g(x)$ is increasing and concave down

D. $g(x)$ is decreasing and concave down

E. None of the above

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5. (5 points) If $P'(x) = 5e^{2x} - 4e^{-x} + 6$, and $P(1) = 50$, find the profit function $P(x)$.

   A. $P(x) = \frac{5}{2}e^{2x} - 4e^{-x} + 6x + 24$

   B. $P(x) = \frac{5}{2}e^{2x} + 4e^{-x} + 6x + 24$

   C. $P(x) = 5e^{2x} - 4e^{-x} + 6x + 50$

   D. $P(x) = \frac{5}{2}e^{2x} + 4e^{-x} + 6x + 50$

   E. None of the above

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6. (5 points) Find the function $f(x)$ if $f'(x) = 2x^{-3} + \frac{1}{x}$ and $f(1) = 7$.

   A. $f(x) = -x^{-2} + \ln|x| + 8$

   B. $f(x) = -x^{-2} + \ln|x| + 7$

   C. $f(x) = x^{-2} + \ln|x| + 8$

   D. $f(x) = -\frac{1}{2}x^{-2} + \ln|x| + 8$

   E. None of the above

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7. (5 points) The velocity (in meters per second) of a particle moving along a line is given by $v(t) = 6t - 2t^2$. Find the general expression for the displacement $s(t)$.

   A. $s(t) = 3t^2 - \frac{2}{3}t^3 + C$

   B. $s(t) = 3t^2 - t^3 + C$

   C. $s(t) = 6t^2 - 2t^3 + C$

   D. $s(t) = 3t^3 - t^2 + C$

   E. None of the above

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8. (5 points) If $C'(t) = \frac{20}{t}$ and $C(1) = 100$, what best describes $C(t)$?

   A. Increases at a constant rate

   B. Increases quickly at first, then slows over time

   C. Decreases exponentially

   D. Oscillates around 100

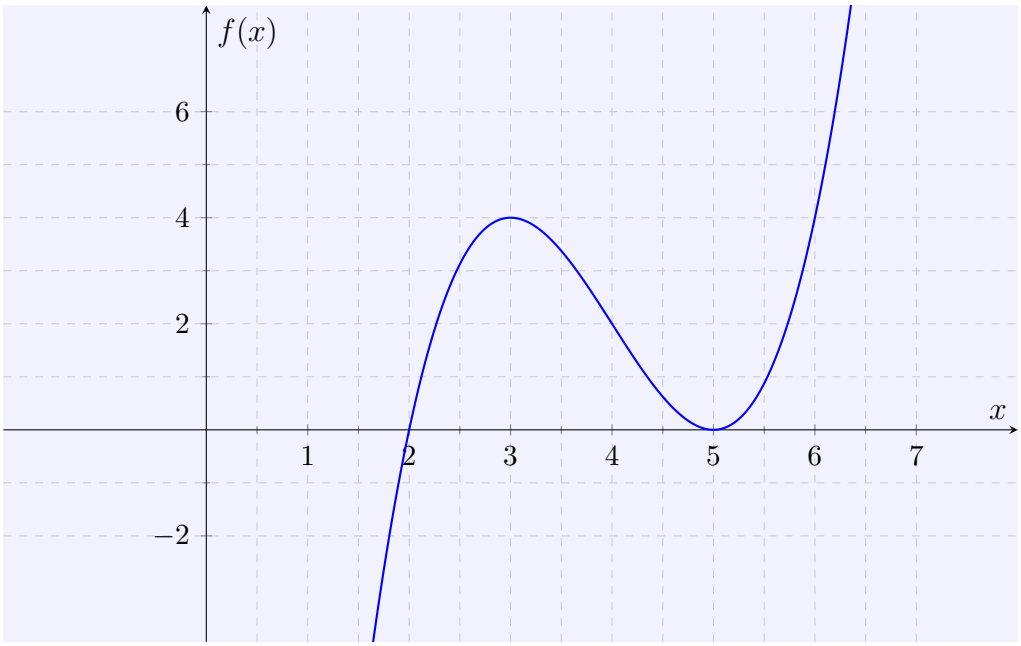
   E. None of the above

PART I Answers:

Problem	1	2	3	4	5	6	7	8
Answer								

Part II - Short Answer

9. (10 points) The graph of $f(x)$ is shown below for $x \in [2, 6]$:



Answer the following briefly for $x \in [2, 6]$. Separate your answers with commas.

a) Increasing on: _____
(List the intervals where the function is rising; where $f'(x) > 0$.)

b) Decreasing on: _____
(List the intervals where the function is falling; where $f'(x) < 0$.)

c) Relative maximum at: _____
(List x-values or $(x, f(x))$ points where the function changes from increasing to decreasing.)

d) Relative minimum at: _____
(List x-values or $(x, f(x))$ points where the function changes from decreasing to increasing.)

e) Inflection point(s): _____
(Points where the concavity changes; where $f''(x) = 0$ or undefined.)

f) $f''(x) > 0$ on: _____
(Intervals where the function is concave up.)

g) $f''(x) < 0$ on: _____
(Intervals where the function is concave down.)

h) At $x = 3$, $f'(3) =$ _____; the graph is _____.
(Use the graph to estimate the slope of $f(x)$ at $x = 3$ to determine the slope and whether f is increasing, decreasing, or flat.)

i) If $f'(x) = 0$, then _____
(List all critical values (x- values) where the derivative is zero.)

j) General shape of $f(x)$ between $x = 3$ and $x = 5$: _____
(Use terms like increasing/decreasing, concave up/down, relative maxima/minima.)

Part III - Free Response
Round your answers to two decimal places where needed.

10. (15 points) Compute the following integrals. Include the constant of integration, C.

a) $\int (2x^3 - 5x^2)(x - 1)\, dx$

b) $\int \dfrac{3x^2 - 6\sqrt{x} + 4}{x^3}\, dx$

c) $\int x^{-3/2} + 6e^{-4x}\, dx$

d) $\int \dfrac{3x^2}{\sqrt{x^4}} - 6\sqrt{x} + 4x^3\, dx$

e) $\int \dfrac{4}{7x} + 6e^{-4x}\, dx$

11. (20 points) Solve the following initial value problems. For each part, find the function $f(x)$ that satisfies the initial condition.

a) $f'(x) = 3x^2 - 6x + 4, \qquad f(0) = 5.$

b) $f'(x) = 4e^{2x} - e^{-x}, \qquad f(0) = 3.$

c) $f'(x) = \frac{5}{x}, \qquad f(1) = 0 \qquad (x \geq 1).$

d) $f'(x) = \sqrt{x} + x^4, \qquad f(1) = 2, \qquad (x \geq 0).$

12. (10 points) Let

$$f(x) = -x^3 + 9x^2 - 18x + 12.$$

a) Find $f'(x)$ and $f''(x)$.

b) Find and classify critical points as relative maxima or minima. Write your answers in the form $(x, f(x))$. (**Hint:** Use the 2nd derivative test.)

c) Find the intervals of concavity.

d) Find inflection points. Write your answers in the form $(x, f(x))$.

e) Interpret what concavity reveals about the graph.

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13. (5 points) A particle moves so that its velocity is $v(t) = 2t^2 - 8t + 5$, with $v(t)$ in m/s and t in seconds. Its initial position is $s(2) = 0$.

a) Find $s(t)$.

b) Find its position at $t = 8$.

c) When is the particle at rest?

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14. (10 points) The temperature $T(t)$ (in °C) of a cup of coffee after being poured into a mug is modeled by

$$T(t) = 80e^{-0.2t} + 20, \quad t \geq 0$$

where t is the time in minutes.

a) Find the first and second derivatives of $T(t)$.

$T'(t) =$ _____

$T''(t) =$ _____

b) Determine the intervals where $T(t)$ is concave up or concave down and identify any inflection point. Write your answers in the form $(x, f(x))$.

Inflection point: _____

Concave up: _____

Concave down: _____

c) Interpret what the concavity of $T(t)$ tells you about how the coffee is cooling over time.

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15. (10 points) A drone ascends vertically. Its velocity (in meters per second) at time t seconds is modeled by

$$v(t) = \frac{8}{t} + t, \quad t \geq 1.$$

The drone is launched from ground level, so its height at $t = 1$ seconds is $s(1) = 0$ meters.

Answer the following:

a) Find the height function $s(t)$ of the drone. Include the constant of integration and use the given initial condition to determine its value.

b) Using your height function, calculate the **height of the drone at $t = 4$ seconds**.

c) Find the **velocity of the drone at $t = 2$ seconds**, $v(2)$.