MAT 210 Brief Calculus
Sample Exam 3 (Practice Version A)

Instructor	XXXXXXX
Full Name (Print clearly!)	
Student ID	

Important Disclaimer for Students:

- This is a **sample exam**, not the actual in-class exam. Questions are of similar difficulty but may differ in type, numbers, or context.
- Success in the course depends on practicing concepts beyond this sample: lecture notes, Edfinity assignments, group work, quizzes, and homework.
- Complete this exam under exam-like conditions: no notes, no internet, no communication, and about **60-75 minutes** to finish.
- After finishing, check your answers against the posted solutions and grade yourself honestly.
- If you cannot complete it within the time limit, review your weak areas and try another sample exam.
- Treat all concepts from class materials as fair game for the actual exam.

Part I - Multiple Choice: Circle the correct answer and record it in the answer grid.

1. (5 points) If $f(x)$ is a polynomial with a single critical point at $x = 2$ and $f''(2) > 0$, which is true about $f(2)$?

   A. $f(2)$ is a relative minimum

   B. $f(2)$ is a relative maximum

   C. $f(2)$ is an inflection point

   D. There is not enough information

   E. None of the above

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2. (5 points) If $f''(x)$ changes from positive to negative at $x = 4$ and $f(x)$ is continuous for all real numbers, what can we conclude?

   A. $f(x)$ has a relative maximum at $x = 4$

   B. $f(x)$ has a relative minimum at $x = 4$

   C. $f(x)$ has an inflection point at $x = 4$

   D. $f'(x)$ has a relative maximum at $x = 4$

   E. None of the above

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3. (5 points) Which of the following statements about $\int f'(x)\,dx$ is true?

   A. It equals $f(x) + C$

   B. It equals $f'(x) + C$

   C. It equals $\dfrac{df}{dx} + C$

   D. It cannot be evaluated

   E. None of the above

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4. (5 points) Fill in the blank:
   If $f''(x) > 0$ on an interval, then the graph of f is _____ on that interval.

   A. Concave up

   B. Concave down

   C. Increasing

   D. Decreasing

   E. None of the above

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5. (5 points) Which statement is true about concavity?

A. Concave up means $f'(x) < 0$

B. Concave down means $f'(x)$ is increasing

C. Concave up means $f'(x)$ is increasing

D. Concave down means $f'(x) = 0$

E. Concavity has no relation to $f'(x)$

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6. (5 points) If $f(x) = e^{-0.06x}$, what can we say about its concavity?

A. Concave up for all real x

B. Concave down for all real x

C. Concave up for $x > 0$, concave down for $x < 0$

D. Concave down for $x > 0$, concave up for $x < 0$

E. None of the above

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7. (5 points) Evaluate:

$$\int \frac{x^3 - 4\sqrt{x} + 6}{x}\, dx$$

A. $\frac{1}{3}x^3 - 8x^{1/2} + 6\ln|x| + C$

B. $\frac{1}{4}x^4 - \frac{8}{3}x^{3/2} + 6\ln|x| + C$

C. $\frac{1}{3}x^3 - 4x^{1/2} + 6x + C$

D. $x^4 - 4x^{1/2} + 6\ln|x| + C$

E. None of the above

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8. (5 points) A drug concentration in the bloodstream changes at a rate given by

$$\frac{dC}{dt} = -0.4t + 5,$$

where C is in mg/L and t in hours. The general solution for $C(t)$ must include which constant of integration?

A. The initial concentration $C(0)$

B. The dose rate (mg/hr)

C. The elimination constant 0.4

D. No constant is needed for an indefinite integral

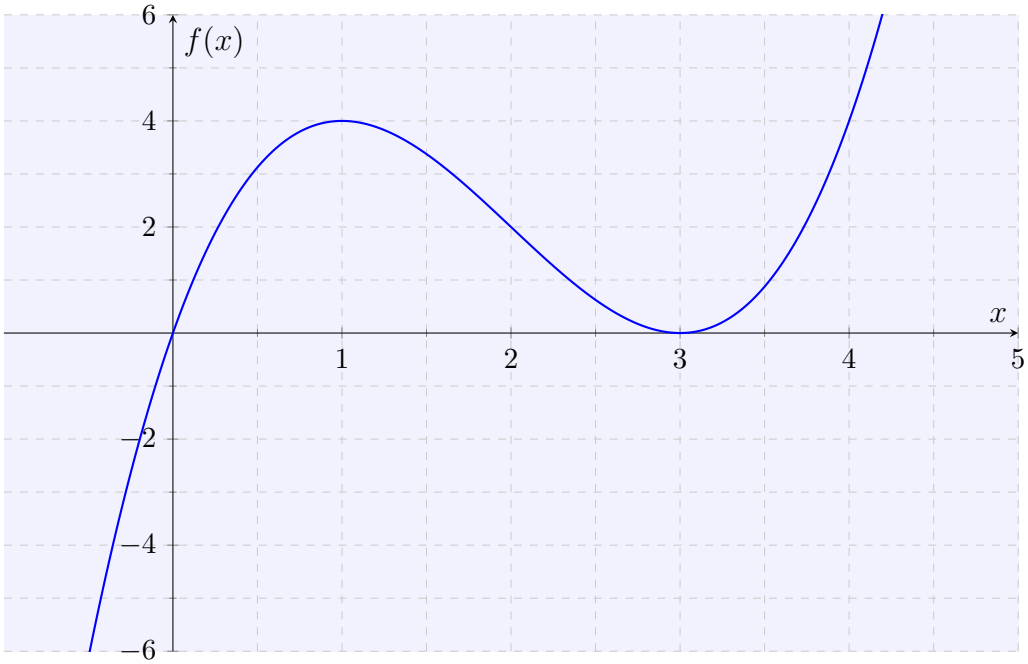
E. The time t at which the drug is measured

PART I Answers:

Problem	1	2	3	4	5	6	7	8
Answer								

Part II - Short Answer

9. (10 points) The graph of $f(x)$ is shown below for $x \in [0, 4]$:



Answer the following questions about the graph of $f(x)$ on $[0, 4]$. Write your answers using interval notation and separate them with commas.

a) **Increasing on:** _____
(Where the graph of $f(x)$ rises; i.e., $f'(x) > 0$.)

b) **Decreasing on:** _____
(Where the graph of $f(x)$ falls; i.e., $f'(x) < 0$.)

c) **Relative maximum at:** _____
(The x-value(s) where f changes from increasing to decreasing.)

d) **Relative minimum at:** _____
(The x-value(s) where f changes from decreasing to increasing.)

e) **Inflection point(s):** _____
(Where the concavity changes; i.e., $f''(x) = 0$ or undefined.)

f) **Concave up on:** _____
(Where $f''(x) > 0$ and the graph bends upward.)

g) **Concave down on:** _____
(Where $f''(x) < 0$ and the graph bends downward.)

h) **At $x = 1$:** slope = _____; the graph is _____.
(Use the graph to estimate the slope of $f(x)$ at $x = 1$ to determine the slope and whether f is increasing, decreasing, or flat.)

i) **Critical values:** $x =$ _____
(List all x-values where $f'(x) = 0$.)

j) **Between $x = 2$ and $x = 4$:** _____
(Briefly describe the graph's shape using terms like increasing/decreasing, concave up/down, or relative max/min.)

Part III - Free Response
(Round your answers to 2 decimal places where needed)

10. (15 points) Compute the following integrals. Include the constant of integration, C.

a) $\int (3x^2 - 4x)(2x + 1)\, dx$

b) $\int \dfrac{2x^2 + 7\sqrt{x} - 5}{x^3}\, dx$

c) $\int x^{-5/2} + 8e^{-3x}\, dx$

d) $\int \dfrac{5x^2}{\sqrt{x^4}} - 3\sqrt{x} + 2x^3\, dx$

e) $\int \dfrac{5}{9x} - 4e^{-2x}\, dx$

11. (20 points) Solve the following initial value problems. For each part, find the function $f(x)$ that satisfies the initial condition.

a) $f'(x) = 2x^2 - 5x + 6, \qquad f(0) = 4.$

b) $f'(x) = 5e^{3x} + 2e^{-x}, \qquad f(0) = 1.$

c) $f'(x) = \frac{4}{x}, \qquad f(1) = 2 \qquad (x \geq 1).$

d) $f'(x) = \sqrt{x} - x^3, \qquad f(1) = 0, \qquad (x \geq 0).$

12. (10 points) Let

$$f(x) = -3x^3 + 15x^2 - 9x + 7.$$

a) Find $f'(x)$ and $f''(x)$.

$f'(x) =$ _____

$f''(x) =$ _____

b) Find and classify critical points as relative maxima or minima. Write your answers in $(x, f(x))$ form. (**Hint:** Use the 2nd derivative test.)

c) Find the intervals of concavity.

Concave up on: _____

Concave down on: _____

d) Find inflection points. Write your answers in $(x, f(x))$ form.

e) Interpret what concavity reveals about the graph.

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13. (10 points) A tank initially contains 1200 liters of water, so $V(0) = 1200$. Water drains from the tank at a rate given by

$$V'(t) = -30e^{-0.03t}, \quad t \text{ in hours.}$$

a) Find the **volume of water** $V(t)$ in the tank at any time $t \geq 0$. Include the constant of integration and use the initial condition to determine its value.

b) How much water is in the tank after 5 hours?

c) Describe whether the volume function is concave up or down over time, and explain what this means for the rate of draining.

d) Estimate the time when only 15 liters remain in the tank.

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14. (12 points) A particle moves along a straight line, and its position (in meters) with respect to a reference point is given by

$$s(t) = 2t^3 - 12t^2 + 18t - 5, \quad t \geq 0,$$

where t is time in seconds.

Answer the following (include appropriate units):

a) Find the velocity $v(t)$ and acceleration $a(t)$ as functions of t.

b) Find the acceleration at $t = 2$ seconds.

c) Find the acceleration at the instant when the velocity is zero.

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15. (10 points) A YouTuber tracks the growth of subscribers (in hundreds) as a function of weeks for $t > 0$ after their first video. The model is

$$S(t) = t^3 - 6t^2 + 9t + 2.$$

Answer the following:

a) Find the first and second derivatives of $S(t)$.

$S'(t) =$ _____

$S''(t) =$ _____

b) Determine the intervals where $S(t)$ is concave up or concave down.

Concave up: _____

Concave down: _____

c) Find any inflection point(s) of $S(t)$. Give coordinates.

d) Interpret the concavity of $S(t)$: what does it tell you about the rate of subscriber growth over time?