MAT 242, Final Exam

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Name (printed): ____________________ Student ID: ____________

Honor Statement: By signing below, I confirm that I have neither given nor received any unauthorized assistance on this exam. This includes any use of a calculator beyond those uses specifically authorized by the School of Mathematical and Statistical Sciences and my instructor. Furthermore, I agree not to discuss this exam with anyone until the exam testing period is over.

Signature: ____________________________

DIRECTIONS:

- You have 1 hour and 50 minutes to complete the exam.
- The exam consists of 9 problems, with total score of 100 points.
- Read and follow the instructions for each problem. Be sure to understand each problem carefully before starting to work on it.
- **Box** your final answers and justify all your answers, especially when asked, and do it in the manner you are directed. Failing to do so may earn you 0 points for the problem.
- Write your solutions in the blank space left under each question. If you need more work space, extra paper will be provided on request.
- No books or notes are allowed on this test.
- No calculators with QWERTY keyboards or symbolic algebra capabilities are allowed.
- Your cell phone and other electronic devices should be turned off and put away for the entire testing period. If you use any electronic device, your score on the test will be 0.

(1) (**7 points**) Find a system of equation which, when solved, produces all solutions to the matrix equation given below. You do not need to simplify or solve these equations. (Your answer should be a list of equations, like $x^2 - 4x = 2$, $xy - 3x = 5 + 7y$, etc.)

$$\begin{bmatrix} x & y+2 \\ 5-y & 1 \end{bmatrix} \cdot \begin{bmatrix} y & 3 \\ 2x & x \end{bmatrix} = 2 \cdot \begin{bmatrix} 1 & 0 \\ 0 & x \end{bmatrix} - \begin{bmatrix} y & 1 \\ x & 0 \end{bmatrix}$$

(2) (**10 points**) Let W be the subspace of R^3 spanned by $\left\{ \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \right\}$. Apply the Gram-Schmidt procedure to this set to obtain an orthonormal basis for W.

(3) (**14 points**) Let $B = \{\vec{v_1}, \vec{v_2}, \vec{v_3}\}$ be the set of vectors, where

$$\vec{v_1} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \ \vec{v_2} = \begin{bmatrix} 0 \\ 0 \\ 3 \\ 5 \end{bmatrix}, \ \vec{v_3} = \begin{bmatrix} 4 \\ -2 \\ -5 \\ 3 \end{bmatrix}.$$

(a) Determine whether the set of vectors B is orthogonal. Show your work.

(b) Find the orthogonal projection of $\vec{u} = \begin{bmatrix} 1 \\ -2 \\ 3 \\ 5 \end{bmatrix}$, onto W=span$\{\vec{v_1}, \vec{v_2}, \vec{v_3}\}$, without inverting any matrices or solving any systems of linear equations.

(4) (**10 points**) Let W=span$\{\vec{v_1}, \vec{v_2}\}$ where $\vec{v_1} = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$, and $\vec{v_2} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$. Find the vector in W closest to the vector $\vec{u} = \begin{bmatrix} 3 \\ 0 \\ 8 \end{bmatrix}$.

(5) (**10 points**) Let W=span$\{\vec{v_1}, \vec{v_2}\}$ where $\vec{v_1} = \begin{bmatrix} 1 \\ -1 \\ 2 \\ 4 \end{bmatrix}$, and $\vec{v_2} = \begin{bmatrix} 1 \\ 4 \\ -9 \\ -8 \end{bmatrix}$. Find a basis for $W^{\perp}$, the orthogonal complement to W.

(6) (**15 points)** Find the best- fitting curve of each type below for the given data: $(0, 2)$, $(1, 4)$, $(2, 6)$, $(3, 9)$.

(a) Linear function $y = ax + b$.

(b) Quadratic function of the form $y = ax^2 + bx + c$.

(7) (**9 points**) Let $L : R^2 \to R^4$ be the linear transformation defined by $L(\begin{bmatrix} 2 \\ -1 \end{bmatrix}) = \begin{bmatrix} 1 \\ 1 \\ -3 \\ 4 \end{bmatrix}$ and $L(\begin{bmatrix} -1 \\ 0 \end{bmatrix}) = \begin{bmatrix} -5 \\ 6 \\ -3 \\ 7 \end{bmatrix}$. Find the matrix A that represents L, i.e. matrix A that satisfies the condition $L(\vec{x}) = A \cdot \vec{x}$, for all $\vec{x} \in R^2$.

(8) (**15 points**) Let $L : R^4 \to R^3$ be the linear transformation defined by $L(\vec{x}) = \begin{bmatrix} 1 & 2 & -3 & 12 \\ 2 & 4 & 2 & 0 \\ 3 & 6 & 1 & 6 \end{bmatrix} \cdot \vec{x}$.

(a) Find a basis for the kernel of L.

(b) Find a basis for the range (image) of L.

(9) **(10 points)** Find the least-squares solution to the system:
$x + y = 3$
$-2x + 3y = 1$
$2x - y = 2$