MAT 267: Calculus III For Engineers
Test 3 Review

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1. Evaluate the triple integral $\iiint_E \frac{e^{-2x}\sin^2(z)}{\sqrt{y}}\, dV$ where $E = [0,1] \times [1,2] \times [0,\pi]$.

2. Evaluate $\iiint_E 6xy\, dV$ where E lies below the plane $z = 1 + x + y$ and above the region in the xy plane bounded by the curves $y = \sqrt{x}$, $y = 0$ and $x = 1$.

3. Use a triple integral to find the volume of the region bounded by the cylinder $x^2 + z^2 = 4$ and the planes $y = -1$ and $y + z = 4$.

4. Convert the triple integral to cylindrical coordinates and evaluate it:
$$\int_{-2}^{2} \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_{\sqrt{x^2+y^2}}^{2} z\, dz\, dx\, dy.$$

5. Use cylindrical coordinates to evaluate $\iiint_E z\, dV$ where E is the solid region bounded by the paraboloid $z = 3(x^2 + y^2)$ and the plane $z = 1$.

6. Use cylindrical coordinates to evaluate $\iiint (x + y + z)\, dV$ where E is the region above the xy plane and below the paraboloid $z = 4 - x^2 - y^2$.

7. Use cylindrical coordinates to find the volume of the region that is inside the cylinder $x^2+y^2 = 16$, above the xy plane, and below the cone $z = 3\sqrt{x^2 + y^2}$.

8. Convert the triple integral to spherical coordinates and evaluate it.
$$\int_{-4}^{4} \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \int_{0}^{\sqrt{16-x^2-y^2}} (x^2 + y^2 + z^2)^{-1/2}\, dz\, dy\, dx.$$

9. Evaluate $\iiint_E xe^{(x^2+y^2+z^2)^2}\, dV$ where E is the portion of the unit ball in the first octant.

10. Use spherical coordinates to the find the volume of the region that lies above the cone $z = \sqrt{3(x^2 + y^2)}$ and below the hemisphere $z = \sqrt{9 - x^2 - y^2}$.

11. Evaluate $\iiint_E x^2 + y^2\, dV$ where E is the region between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 4$.

12. Consider the region inside the cone $z = \sqrt{x^2 + y^2}$ and under the plane $z = 4$. Set up the integral for the volume of this region in Cartesian, cylindrical and spherical coordinates. Which is the best choice?

13. Evaluate the line integral $\int_C (x + y)\, ds$ where C is the line segment from $(1, 3)$ to $(4, 6)$.

14. Evaluate the line integral $\int xy\, ds$ where C is the portion of the circle $x^2 + y^2 = 9$ in the first quadrant.

15. Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x, y) = \langle -y, x \rangle$ and C is the top half of the unit circle.

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16. Find the work (ignoring units) done by the force field $\mathbf{F}(x, y, z) = \langle -y, xz, xy \rangle$ on a particle that moves from the origin to $(1, 1, 1)$ along the curve $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$.

17. Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x, y, z) = \langle x, y, xy \rangle$ and C is the helix $\mathbf{r}(t) = \langle \cos(t), \sin(t), t \rangle$ with $0 \leq t \leq \pi/4$.

18. Let $f(x, y) = e^{-3x} \sin(x + 2y)$. Without doing any integration, find the value of $\int_C \nabla f \cdot d\mathbf{r}$ where C is the arc of the parabola $y = x^2$ starting at $(-1, 1)$ and ending at $(1, 1)$.

19. Verify that the vector field $\mathbf{F}(x, y) = \langle 2xy^2 + 4x, 2x^2y + 8y \rangle$ is conservative and find a potential function f such that $\mathbf{F} = \nabla f$.

20. Verify that the vector field $\mathbf{F}(x, y) = \langle 6x + y, 2y + x \rangle$ is conservative and find a potential function f such that $\mathbf{F} = \nabla f$.

21. The vector field $\mathbf{F}(x, y, z) = \langle 2x + yz, 2y + xz, xy \rangle$ is conservative.
(a) Find a potential function $f(x, y, z)$ such that $\mathbf{F} = \nabla f$.
(b) Use the answer to (a) to evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the helix parameterized as $\mathbf{r}(t) = \langle \sin t, \cos t, t \rangle,\ 0 \leq t \leq \pi/4$.

22. Use Green's Theorem to evaluate the line integral $\oint_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \langle xy + \sin(x), xy^2 + e^y \rangle$ and C is the positively oriented triangle with vertices $(0, 0)$, $(1, 2)$ and $(0, 2)$.

23. Use Green's Theorem to evaluate the line integral $\oint_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \langle 7x + 6y, 8x - 3y \rangle$ and C is the positively-oriented boundary curve of the region bounded by $y = x^2$ and $y = 4$.

24. Use Green's Theorem to evaluate the line integral $\oint_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \langle 4, x^3 \rangle$ and C is the positively-oriented unit circle.

Answers

1. $\frac{\pi}{2}(1 - e^{-2})(\sqrt{2} - 1)$
2. $65/28$
3. 20π
4. 4π
5. $\pi/9$
6. $32\pi/3$
7. 128π
8. 16π
9. $\pi(e - 1)/16$
10. $18\pi(1 - \sqrt{3}/2)$
11. $248\pi/15$
12. Cartesian: $\int_{-4}^{4} \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \int_{\sqrt{x^2+y^2}}^{4} dz\ dy\ dx$

    Cylindrical: $\int_0^{2\pi} \int_0^4 \int_r^4 r\ dz\ dr\ d\theta$

    Spherical: $\int_0^{2\pi} \int_0^{\pi/4} \int_0^{4\sec\phi} \rho^2 \sin\phi\ d\rho\ d\phi\ d\theta$

    Cylindrical coordinates are the best choice.
13. $21\sqrt{2}$
14. $27/2$
15. π
16. $1/2$
17. $1/4$
18. $e^{-3}\sin 3 - e^3 \sin 1$
19. $Q_x = P_y = 4xy \Rightarrow$ conservative, $f(x, y) = x^2y^2 + 2x^2 + 4y^2 + C$
20. $Q_x = P_y = 1 \Rightarrow$ conservative, $f(x, y) = 3x^2 + xy + y^2$
21. a) $f(x, y, z) = xyz + x^2 + y^2 + C$ b) $\pi/8$
22. $5/3$
23. $64/3$
24. $3\pi/4$