MAT 210 Brief Calculus
Sample Final Exam (Practice Version C)
Solutions

Instructor	XXXXXXX
Full Name (Print clearly!)	
Student ID	

Important Disclaimer for Students:

- This is a **sample Final exam**, not the actual in-class exam. Questions are of similar difficulty but may differ in type, numbers, or context.
- Success in the course depends on practicing concepts beyond this sample: lecture notes, Edfinity assignments, group work, quizzes, and homework.
- Complete this exam under exam-like conditions: no notes, no internet, no communication, and about **120-140 minutes** to finish.
- After finishing, check your answers against the posted solutions and grade yourself honestly.
- Repeat for all other versions posted on the MAT 210 website.
- If you cannot complete it within the time limit, review your weak areas and try another sample exam.
- **Round your answers to two decimal places where needed.**
- Treat all concepts from class materials as fair game for the actual exam.

Part I

Multiple Choice: Circle the response that best completes the statement or answers the question.

Then write the letter of your answer in the table after Problem 10.

1. (5 points) Identify which of the following definite integrals represents the area bounded by the curves $f(x)$ and $h(x)$ over the interval $[x_1, x_2]$ as shown below.

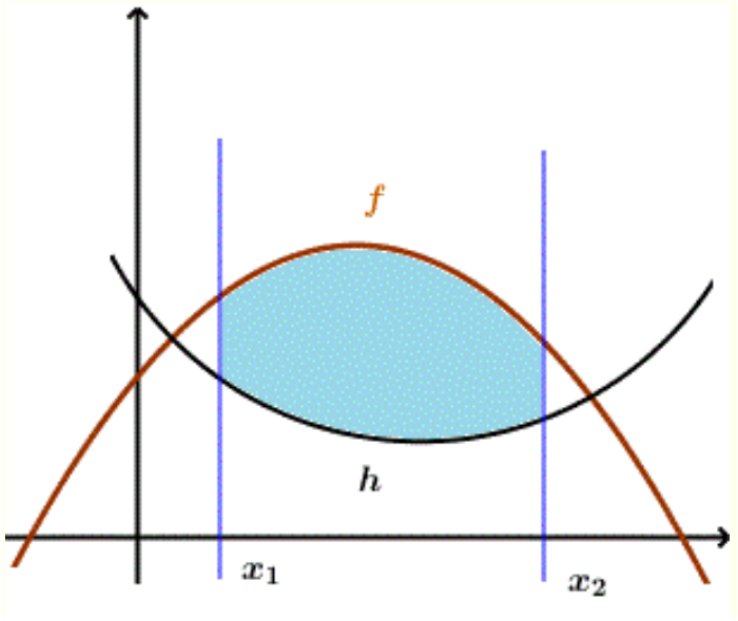

Figure 1

A. $\int_{x_1}^{x_2} \left[h(x) - f(x)\right] dx$

B. $\int_{x_1}^{x_2} \left[f(x) - h(x)\right] dx$

C. $\int_{x_2}^{x_1} \left[h(x) - f(x)\right] dx$

D. $\int_{x_2}^{x_1} \left[f(x) - h(x)\right] dx$

E. None of the above

Solution: The area between two curves is always given by the integral of the *top function minus the bottom function* over the interval. From the graph, $f(x)$ is above $h(x)$ on $[x_1, x_2]$, so the correct integral is:

$$\int_{x_1}^{x_2} \left[f(x) - h(x)\right] dx.$$

Correct Answer: B

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2. (5 points) A car moves forward along a straight road with velocity $v(t) \geq 0$ meters per second over the time interval $[0, 5]$. The definite integral $\int_0^5 v(t)\, dt$ represents:

A. The velocity at $t = 5$ seconds

B. The total distance traveled in the first 5 seconds

C. The average acceleration over 5 seconds

D. The change in velocity over 5 seconds

E. None of the above

Solution: The definite integral of a velocity function over a time interval gives the total displacement (or distance traveled if the velocity is always positive) during that time interval. Therefore:

$$\int_0^5 v(t)\,dt = \text{total distance traveled in the first 5 seconds.}$$

Correct Answer: B

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3. (5 points) Let $f(x)$ be continuous on $[0, 4]$. Which property allows us to write

$$\int_0^4 f(x)\,dx = \int_0^2 f(x)\,dx + \int_2^4 f(x)\,dx?$$

A. Linearity of the integral

B. Breakup of the definite integral over subintervals over a continuous interval

C. Reversing the limits of integration

D. Fundamental Theorem of Calculus

E. None of the above

Solution: This property is called the *additivity of definite integrals over adjacent subintervals*. It states that if f is continuous on $[a, b]$ and c is any point in $[a, b]$, then:

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx.$$

Here, $a = 0$, $b = 4$, and $c = 2$. Therefore, we can split the integral at $x = 2$ because f is continuous on $[0, 4]$.

Correct Answer: B

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4. (5 points) In general, which of the following is true about left Riemann sums for a continuous function $f(x)$ over $[a, b]$ as the number of subintervals n increases?

A. The approximation becomes less accurate.

B. The approximation becomes more accurate.

C. The approximation does not depend on the number of intervals, n.

D. The approximation always overestimates the integral.

E. None of the above.

vspace0.5cm **Solution:** As the number of subintervals n increases, each subinterval becomes smaller. For a continuous function, the left Riemann sum uses the left endpoint of each subinterval to approximate the area under the curve. Smaller subintervals mean the rectangles better conform to the shape of the function, so the approximation becomes closer to the true value of the integral.

Correct Answer: B — The approximation becomes more accurate as n increases.

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5. (5 points) Which statement is true about concavity?

   A. Concave up means $f'(x) < 0$

   B. Concave down means $f'(x)$ is increasing

   C. Concave up means $f'(x)$ is increasing

   D. Concave down means $f'(x) = 0$

   E. Concavity has no relation to $f'(x)$

Solution: Concavity describes the behavior of the derivative $f'(x)$:
- If $f''(x) > 0$ (concave up), then $f'(x)$ is increasing.
- If $f''(x) < 0$ (concave down), then $f'(x)$ is decreasing.

Therefore, the correct statement is that concave up means $f'(x)$ is increasing.

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6. (5 points) If $f(x) = e^{-0.06x}$, what can we say about its concavity?

   A. Concave up for all real x

   B. Concave down for all real x

   C. Concave up for $x > 0$, concave down for $x < 0$

   D. Concave down for $x > 0$, concave up for $x < 0$

   E. None of the above

Solution: To determine concavity, compute the second derivative:

$$f'(x) = -0.06e^{-0.06x}, \quad f''(x) = 0.0036e^{-0.06x}.$$

Since $e^{-0.06x} > 0$ for all real x, we have $f''(x) = 0.0036e^{-0.06x} > 0$, which means the graph is **concave up for all real** x

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7. (5 points) At the age of t blinks, the height of a child is given by $H(t)$ (measured in blinkets). Which of the following calculations gives the *instantaneous rate of change* of the child's height at the age of 2 blinks?

   A. $\dfrac{H(2)}{2}$

   B. $H'(2)$

   C. $\dfrac{H'(2) - H'(0)}{2 - 0}$

   D. $H(2)$

   E. $\dfrac{H'(2)}{2}$

Solution: The instantaneous rate of change of H at $t = 2$ is, by definition, the derivative evaluated at 2, i.e. $H'(2)$.

Option(B) is correct.

8. (5 points) The population of a certain bacterial culture (*in hundreds*) t hours after being placed in a nutrient solution is modeled by

$$P(t) = \sqrt{500t + 2800}.$$

Find the rate at which the population is growing *(in organisms per hour)* after 6 hours. (Round your answer to the nearest whole organism.)

A. After 6 hours the population is increasing by 250 organisms per hour

B. **After 6 hours the population is increasing by 328 organisms per hour**

C. After 6 hours the population is increasing by 500 organisms per hour

D. After 6 hours the population is decreasing by 250 organisms per hour

E. After 6 hours the population is decreasing by 500 organisms per hour

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Solution: We are given $P(t) = \sqrt{500t + 2800}$ (in hundreds).

$$P'(t) = \frac{1}{2}(500t + 2800)^{-1/2}(500) = \frac{250}{\sqrt{500t + 2800}}.$$

At $t = 6$:

$$P'(6) = \frac{250}{\sqrt{500(6) + 2800}} = \frac{250}{\sqrt{5800}} \approx 3.28.$$

Since $P(t)$ is in hundreds, the actual rate is

$$100 \times 3.28 = 328 \text{ organisms per hour.}$$

Therefore, the population is increasing at approximately 328 organisms per hour.

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9. (5 points) Find the derivative of $f(x) = x^2e^{3x^2} - (2x^2 - x + 1)^5$.

A. $2xe^{3x^2} + 6x^3e^{3x^2} - 5(4x + 1)(2x^2 - x + 1)^4$

B. $e^{3x^2} - 5(2x^2 - x + 1)^4$

C. $2xe^{3x^2} + 6x^3e^{3x^2} - 5(4x - 1)(2x^2 - x + 1)^4$

D. $2xe^{3x^2} + 6x^3e^{3x^2} - 5(2x^2 - x + 1)^4$

E. None of the above

Solution: Use the product rule for the first term and the chain rule for the second.

$$\frac{d}{dx}\left(x^2e^{3x^2}\right) = (2x)e^{3x^2} + x^2 \cdot e^{3x^2} \cdot (6x) = 2xe^{3x^2} + 6x^3e^{3x^2}.$$

$$\frac{d}{dx}\left((2x^2 - x + 1)^5\right) = 5(2x^2 - x + 1)^4 \cdot (4x - 1).$$

Putting the pieces together (and remembering the minus sign):

$$f'(x) = 2xe^{3x^2} + 6x^3e^{3x^2} - 5(4x - 1)(2x^2 - x + 1)^4,$$

so option **C** is correct.

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10. (5 points) Find the function $f(x)$ if $f'(x) = 2x^{-3} + \dfrac{1}{x}$ and $f(1) = 7$.

A. $\boxed{f(x) = -x^{-2} + \ln|x| + 8}$

B. $f(x) = -x^{-2} + \ln|x| + 7$

C. $f(x) = x^{-2} + \ln|x| + 8$

D. $f(x) = -\dfrac{1}{2}x^{-2} + \ln|x| + 8$

E. None of the above

Solution:

Integrate $f'(x)$:

$$\int f'(x)\,dx = \int 2x^{-3}\,dx + \int \frac{1}{x}\,dx = -x^{-2} + \ln|x| + C$$

Use $f(1) = 7$ to find C:

$$f(1) = -1 + 0 + C = 7 \implies C = 8$$

Hence,

$$f(x) = -x^{-2} + \ln|x| + 8$$

PART I Answers:

Problem	1	2	3	4	5	6	7	8	9	10
Answer										

Part II - Short Answer

True/False: Circle T for True or F for False next to each statement.

Fill in the blanks: Please read each question carefully.

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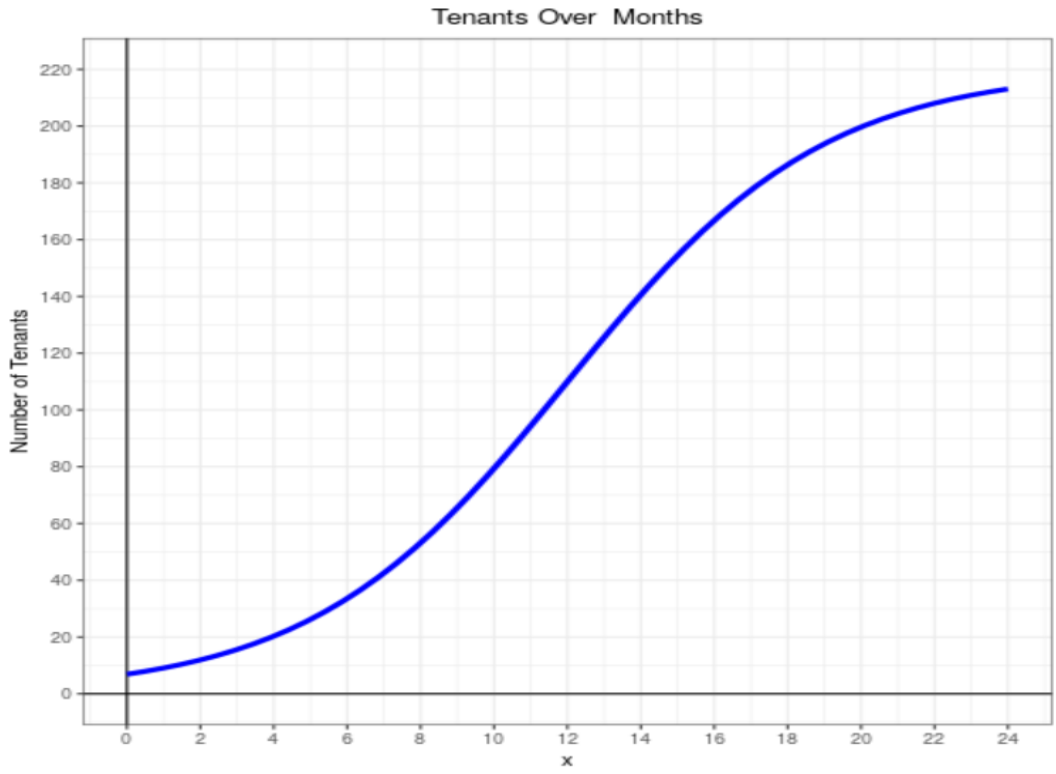
11. (5 points) (1.25 points each) This graph is a model of the number of tenants in a particular apartment complex in its first 24 months. Answer the questions below.



Circle the correct answer to complete the following fill-in-the-blank questions:

A. From month 1 to month 11, the total population is (**increasing** / **decreasing**).

B. From month 1 to month 11, the average rate of change of the population is (**greater** / **less**) than the instantaneous rate of change in month 20. 0.25cm

C. The instantaneous rate of change in month 22 is (**greater** / **less**) than the average rate of change from month 1 to 24.

D. The instantaneous rate of change in month 4 is (**greater** / **less**) than the instantaneous rate of change in month 14.

Solution:

The steps below explain how to decide each answer from the graph:

A. Look at the plot between month 1 and month 11. The curve rises overall, hence the population is **increasing**.

B. The average rate of change from month 1 to 11 is the slope of the secant line between those two points.

The instantaneous rate at month 20 is the slope of the tangent at $t = 20$. Compare the steepness: the tangent at $t = 20$ is shallower, avg ROC is **greater**.

Note: If the tangent at 20 was steeper (larger slope), then avg ROC would be **less** but that's not the case for the given graph.

C. Compare instantaneous rate at 22 (tangent at 22) with the average rate over [1,24] (secant between months 1 and 24). The tangent at 22 is less steep than the secant.

D. Compare tangents at 4 and 14: visually see which is steeper. The slope at 14 is higher.

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12. (5 points) (1 point each) Use the graph to calculate the following limits.

a) $\lim_{x \to -3^-} f(x) = -0.75$

b) $\lim_{x \to -3^+} f(x) = 0.75$

c) $\lim_{x \to -3} f(x) = DNE$

d) $\lim_{x \to 2} f(x) = -0.5$

e) $f(2) = -1$

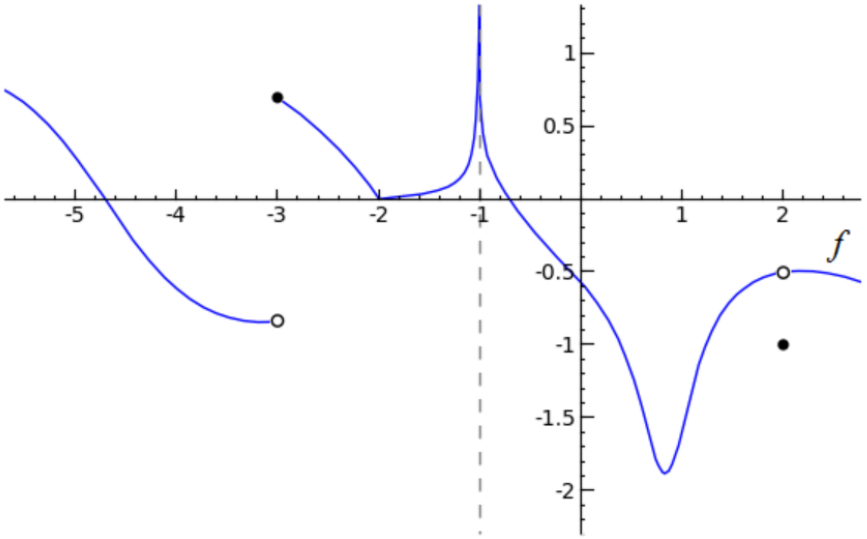

Figure 2

Solution (reading values from the graph):

a) $\lim_{x \to -3^-} f(x) = -0.75$. (Left-hand approach reaches $y = 2$.)

b) $\lim_{x \to -3^+} f(x) = 0.75$. (Right-hand approach reaches $y = 5$.)

c) $\lim_{x \to -3} f(x)$ does not exist because the left- and right-hand limits differ.

d) $\lim_{x \to 2} f(x) = -0.5$. (Both sides approach -0.5.)

e) $f(2) = -1$. (The filled dot at $x = 2$ is at $y = -1$, so the function value differs from the limit.)

Part III — Free Response

You must show your work in the space provided to receive full credit.
Correct final answers without sufficient supporting work may receive zero credit.

Please put a box around your final answers.

13. (15 points) Compute the following integrals. Include the constant of integration, C.

a) $\displaystyle\int (2x^3 - 5x^2)(x-1)\,dx$

$$\int (2x^4 - 2x^3 - 5x^3 + 5x^2)\,dx = \int (2x^4 - 7x^3 + 5x^2)\,dx = \frac{2}{5}x^5 - \frac{7}{4}x^4 + \frac{5}{3}x^3 + C$$

b) $\displaystyle\int \frac{3x^2 - 6\sqrt{x} + 4}{x^3}\,dx$

$$\int \left(\frac{3x^2}{x^3} - \frac{6\sqrt{x}}{x^3} + \frac{4}{x^3}\right)dx = \int \left(3x^{-1} - 6x^{-5/2} + 4x^{-3}\right)dx = 3\ln|x| + 4x^{-3/2} - 2x^{-2} + C$$

c) $\displaystyle\int x^{-3/2} + 6e^{-4x}\,dx$

$$\int x^{-3/2}\,dx + \int 6e^{-4x}\,dx = \frac{x^{-1/2}}{-1/2} - \frac{6}{4}e^{-4x} + C = -2x^{-1/2} - \frac{3}{2}e^{-4x} + C$$

d) $\displaystyle\int \frac{3x^2}{\sqrt{x^4}} - 6\sqrt{x} + 4x^3\,dx$

$$\int (3x^2x^{-2} - 6x^{1/2} + 4x^3)\,dx = \int (3 - 6x^{1/2} + 4x^3)\,dx = 3x - 4x^{3/2} + x^4 + C$$

e) $\displaystyle\int \frac{4}{7x} + 6e^{-4x}\,dx$

$$\int \frac{4}{7x}dx + \int 6e^{-4x}dx = \frac{4}{7}\ln|x| - \frac{3}{2}e^{-4x} + C$$

14. (10 points) The temperature $T(t)$ (in °C) of a cup of coffee after being poured into a mug is modeled by

$$T(t) = 80e^{-0.2t} + 20, \quad t \geq 0$$

where t is the time in minutes.

a) Find the first and second derivatives of $T(t)$.

$T'(t) = -16e^{-0.2t}$
$T''(t) = 3.2e^{-0.2t}$

b) Determine the intervals where $T(t)$ is concave up or concave down and identify any inflection point. Write your answers in the form $(x, f(x))$.
Inflection point: None. Since the second derivative is always positive.

Concave up: $(0, \infty)$

Concave down: None

c) Interpret what the concavity of $T(t)$ tells you about how the coffee is cooling over time.
Since $T''(t) > 0$, the cooling is slowing down over time; the rate at which temperature decreases is decreasing, so the coffee cools more gradually as time passes.

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15. (10 points) A drone ascends vertically. Its velocity (in meters per second) at time t seconds is modeled by

$$v(t) = \frac{8}{t} + t, \quad t \geq 1.$$

The drone is launched from ground level, so its height at $t = 1$ seconds is $s(1) = 0$ meters.

a) Find the height function $s(t)$ of the drone. Include the constant of integration and use the given initial condition to determine its value.
$s(t) = \int v(t)dt = \int \left(\frac{8}{t} + t\right) dt = 8 \ln t + \frac{t^2}{2} + C$
Use $s(1) = 0 : 8 \ln 1 + \frac{1^2}{2} + C = 0 \implies C = -0.5$
$s(t) = 8 \ln t + \frac{t^2}{2} - 0.5$ metres

b) Using your height function, calculate the **height of the drone at $t = 4$ seconds**.
$s(4) = 8 \ln 4 + \frac{16}{2} - 0.5 \approx 11.09 + 8 - 0.5 = 18.59$ m

c) Find the **velocity of the drone at $t = 2$ seconds**, $v(2)$.
$v(2) = \frac{8}{2} + 2 = 4 + 2 = 6$ m/s

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16. (10 points) A YouTuber tracks the growth of subscribers (in hundreds) as a function of weeks for $t > 0$ after their first video. The model is

$$S(t) = t^3 - 6t^2 + 9t + 2.$$

Answer the following:

a) Find the first and second derivatives of $S(t)$.

First derivative:

$$S'(t) = 3t^2 - 12t + 9$$

Second derivative:

$$S''(t) = 6t - 12$$

b) Determine the intervals where $S(t)$ is concave up or concave down.

Set $S''(t) = 0$ to find inflection point: $6t - 12 = 0 \implies t = 2$.
Concave up: $S''(t) > 0 \implies 6t - 12 > 0 \implies t > 2$ or $(2, \infty)$.
Concave down: $S''(t) < 0 \implies 6t - 12 < 0 \implies t < 2$ or $(0, 2)$.

c) Find any inflection point(s) of $S(t)$. Give coordinates.

Inflection point occurs at $t = 2$. Corresponding $S(2) = 2^3 - 6(2^2) + 9(2) + 2 = 4$.
Inflection point: $(2, 4)$.

d) Interpret the concavity of $S(t)$: what does it tell you about the rate of subscriber growth over time?

For $t < 2$ (concave down), the rate of subscriber growth is decreasing (slowing growth).
For $t > 2$ (concave up), the rate of subscriber growth is increasing (accelerating growth).
The inflection point at $t = 2$ marks the transition from slowing to accelerating subscriber growth.

17. (3 points) A water tank is being filled at a rate of

$$R(t) = 3t + 5 \quad \text{liters per minute}, \quad 0 \le t \le 10.$$

Compute the total volume of water added between $t = 2$ min and $t = 6$ min. Include units.

Solution: The total volume of water added is given by the definite integral of the rate function $R(t)$ over the time interval from 2 to 6 minutes:

$$\text{Volume} = \int_2^6 R(t)\,dt = \int_2^6 (3t + 5)\,dt.$$

Compute the integral:

$$\int_2^6 (3t + 5)\,dt = \int_2^6 3t\ + 5\,dt = \left[\frac{3t^2}{2} + 5t\right]_2^6 = 84 - 16 = 68.$$

Answer: 68 liters

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18. (6 points) A car accelerates along a straight road with velocity modeled by

$$v(t) = 3\sqrt{t} + 2 \quad \text{m/s}, \quad 0 \le t \le 9,$$

where t is measured in seconds from the start.

a) (3 points) Set up a definite integral for the total distance traveled from $t = 1$ s to $t = 4$ s.

Solution: The total distance traveled is given by the integral of the velocity over the given time interval. Therefore:

$$\text{Distance} = \int_1^4 v(t)\,dt = \int_1^4 (3\sqrt{t} + 2)\,dt.$$

b) (3 points) Evaluate the integral and provide the distance in meters.

Solution: Compute the integral from part (a):

$$\int_1^4 (3\sqrt{t} + 2)\,dt = 3 \cdot \frac{2}{3}t^{3/2} + 2t = \left[2t^{3/2} + 2t\right]_1^4.$$

Evaluate the limits:

$$\text{At } t = 4 : [2(4)^{3/2} + 2(4)] - [2(1)^{3/2} + 2(1)] = 24 - 4 = 20$$

$$\text{Distance} = 24 - 4 = 20 \text{ meters.}$$

Answer: 20 meters

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19. (10 points) Consider the two functions $f(x) = x^2$ and $g(x) = \sqrt{x}$.

a) (2 points) Solve algebraically for the intersection points.
Solution: Intersection points occur where $f(x) = g(x)$. So we set the two functions equal:

$$x^2 = \sqrt{x}.$$

Rewrite $\sqrt{x}$ as $x^{1/2}$ and move all terms to one side:

$$x^2 - x^{1/2} = 0.$$

Factor out $x^{1/2}$:

$$x^{1/2}(x^{3/2} - 1) = 0.$$

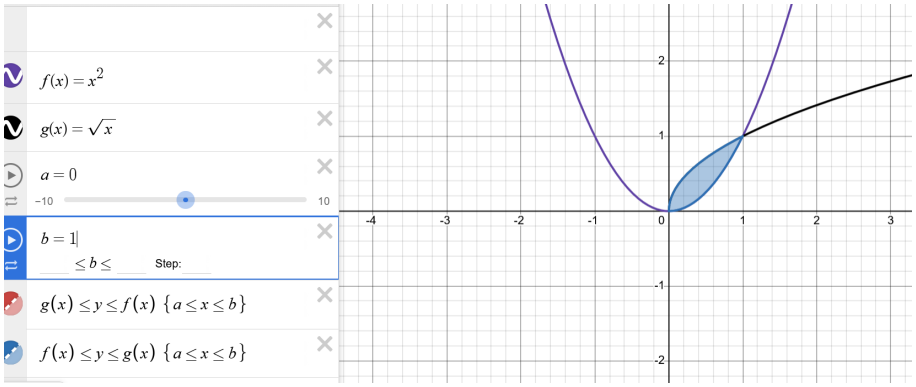
Set each factor equal to zero:

$$x^{1/2} = 0 \implies x = 0, \quad x^{3/2} - 1 = 0 \implies x^{3/2} = 1 \implies x = 1.$$

Therefore, the intersection points are:

$$\boxed{x = 0} \quad \text{and} \quad \boxed{x = 1}.$$

b) (3 points) Sketch the graphs of f and g and shade the region enclosed. **Solution:** You can use Desmos calculator to learn and practice for how to sketch functions.
Click to sketch this graph on Desmos.
Note: Desmos is NOT allowed on any in-person exams. The picture below is provided only to help you understand the setup and to practice.



c) (5 points) Write the definite integral corresponding to the area between f and g, then compute the area.
Solution: Observe the positions of functions in part (b). Since $g(x) > f(x)$ in this interval, the area is

$$\int_0^1 \left(g(x) - f(x)\right) dx = \int_0^1 \left[\sqrt{x} - x^2\right] dx = \left[\frac{2}{3}x^{3/2} - \frac{x^3}{3}\right].$$

Evaluate at the limits:

$$\left(\frac{2}{3}(1)^{3/2} - \frac{1^3}{3}\right) - \left(\frac{2}{3}(0)^{3/2} - \frac{0^3}{3}\right) = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}.$$

Answer: The area between the curves is $\boxed{\frac{1}{3}}$ square units.