MAT 210 Brief Calculus
Sample Final Exam (Practice Version C)

Instructor	XXXXXXX
Full Name (Print clearly!)	
Student ID	

Important Disclaimer for Students:

- This is a **sample Final exam**, not the actual in-class exam. Questions are of similar difficulty but may differ in type, numbers, or context.
- Success in the course depends on practicing concepts beyond this sample: lecture notes, Edfinity assignments, group work, quizzes, and homework.
- Complete this exam under exam-like conditions: no notes, no internet, no communication, and about **120-140 minutes** to finish.
- After finishing, check your answers against the posted solutions and grade yourself honestly.
- Repeat for all other versions posted on the MAT 210 website.
- If you cannot complete it within the time limit, review your weak areas and try another sample exam.
- **Round your answers to two decimal places where needed.**
- Treat all concepts from class materials as fair game for the actual exam.

Part I

Multiple Choice: Circle the response that best completes the statement or answers the question.

Then write the letter of your answer in the table after Problem 10.

1. (5 points) Identify which of the following definite integrals represents the area bounded by the curves $f(x)$ and $h(x)$ over the interval $[x_1, x_2]$ as shown below.

A. $\displaystyle\int_{x_1}^{x_2} \big[h(x) - f(x)\big]\, dx$

B. $\displaystyle\int_{x_1}^{x_2} \big[f(x) - h(x)\big]\, dx$

C. $\displaystyle\int_{x_2}^{x_1} \big[h(x) - f(x)\big]\, dx$

D. $\displaystyle\int_{x_2}^{x_1} \big[f(x) - h(x)\big]\, dx$

E. None of the above

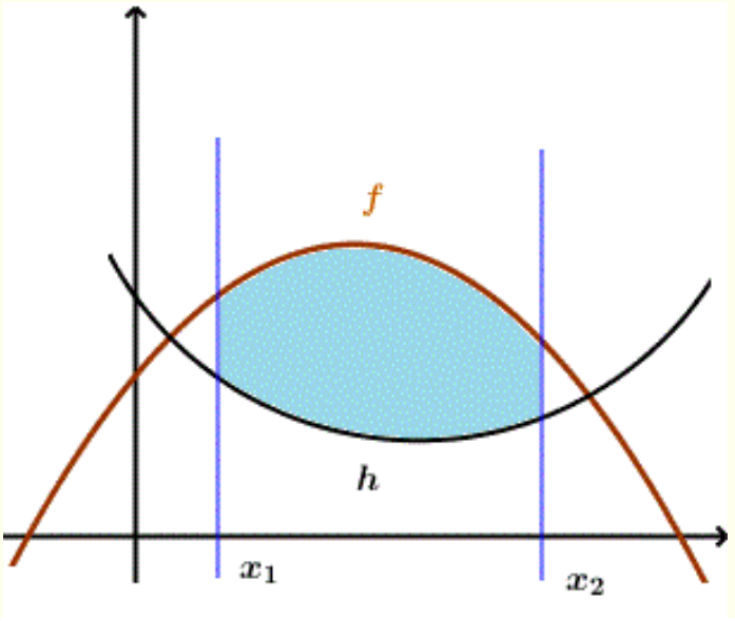

Figure 1

2. (5 points) A car moves forward along a straight road with velocity $v(t) \geq 0$ meters per second over the time interval $[0, 5]$. The definite integral $\displaystyle\int_0^5 v(t)\, dt$ represents:

A. The velocity at $t = 5$ seconds

B. The total distance traveled in the first 5 seconds

C. The average acceleration over 5 seconds

D. The change in velocity over 5 seconds

E. None of the above

3. (5 points) Let $f(x)$ be continuous on $[0, 4]$. Which property allows us to write

$$\int_0^4 f(x)\, dx = \int_0^2 f(x)\, dx + \int_2^4 f(x)\, dx?$$

A. Linearity of the integral

B. Breakup of a definite integral over subintervals over a continuous interval

C. Reversing the limits of integration

D. Fundamental Theorem of Calculus

E. None of the above

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4. (5 points) In general, which of the following is true about left Riemann sums for a continuous function $f(x)$ over $[a, b]$ as the number of subintervals n increases?

A. The approximation becomes less accurate.

B. The approximation becomes more accurate.

C. The approximation does not depend on the number of intervals, n.

D. The approximation always overestimates the integral.

E. None of the above.

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5. (5 points) Which statement is true about concavity?

A. Concave up means $f'(x) < 0$

B. Concave down means $f'(x)$ is increasing

C. Concave up means $f'(x)$ is increasing

D. Concave down means $f'(x) = 0$

E. Concavity has no relation to $f'(x)$

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6. (5 points) If $f(x) = e^{-0.06x}$, what can we say about its concavity?

A. Concave up for all real x

B. Concave down for all real x

C. Concave up for $x > 0$, concave down for $x < 0$

D. Concave down for $x > 0$, concave up for $x < 0$

E. None of the above

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7. (5 points) At the age of t blinks, the height of a child is given by $H(t)$ (measured in blinkets). Which of the following calculations gives the *instantaneous rate of change* of the child's height at the age of 2 blinks?

A. $\dfrac{H(2)}{2}$

B. $H'(2)$

C. $\dfrac{H'(2) - H'(0)}{2 - 0}$

D. $H(2)$

E. $\dfrac{H'(2)}{2}$

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8. (5 points) The population of a certain bacterial culture (*in hundreds*) t hours after being placed in a nutrient solution is modeled by

$$P(t) = \sqrt{500t + 2800}.$$

Find the rate at which the population is growing *(in organisms per hour)* after 6 hours. (Round your answer to the nearest whole organism.)

A. After 6 hours the population is increasing by 250 organisms per hour

B. After 6 hours the population is increasing by 328 organisms per hour

C. After 6 hours the population is increasing by 500 organisms per hour

D. After 6 hours the population is decreasing by 250 organisms per hour

E. After 6 hours the population is decreasing by 500 organisms per hour

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9. (5 points) Find the derivative of $f(x) = x^2 e^{3x^2} - (2x^2 - x + 1)^5$.

A. $2xe^{3x^2} + 6x^3 e^{3x^2} - 5(4x + 1)(2x^2 - x + 1)^4$

B. $e^{3x^2} - 5(2x^2 - x + 1)^4$

C. $2xe^{3x^2} + 6x^3 e^{3x^2} - 5(4x - 1)(2x^2 - x + 1)^4$

D. $2xe^{3x^2} + 6x^3 e^{3x^2} - 5(2x^2 - x + 1)^4$

E. None of the above

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10. (5 points) Find the function $f(x)$ if $f'(x) = 2x^{-3} + \dfrac{1}{x}$ and $f(1) = 7$.

A. $f(x) = -x^{-2} + \ln|x| + 8$

B. $f(x) = -x^{-2} + \ln|x| + 7$

C. $f(x) = x^{-2} + \ln|x| + 8$

D. $f(x) = -\dfrac{1}{2}x^{-2} + \ln|x| + 8$

E. None of the above

PART I Answers:

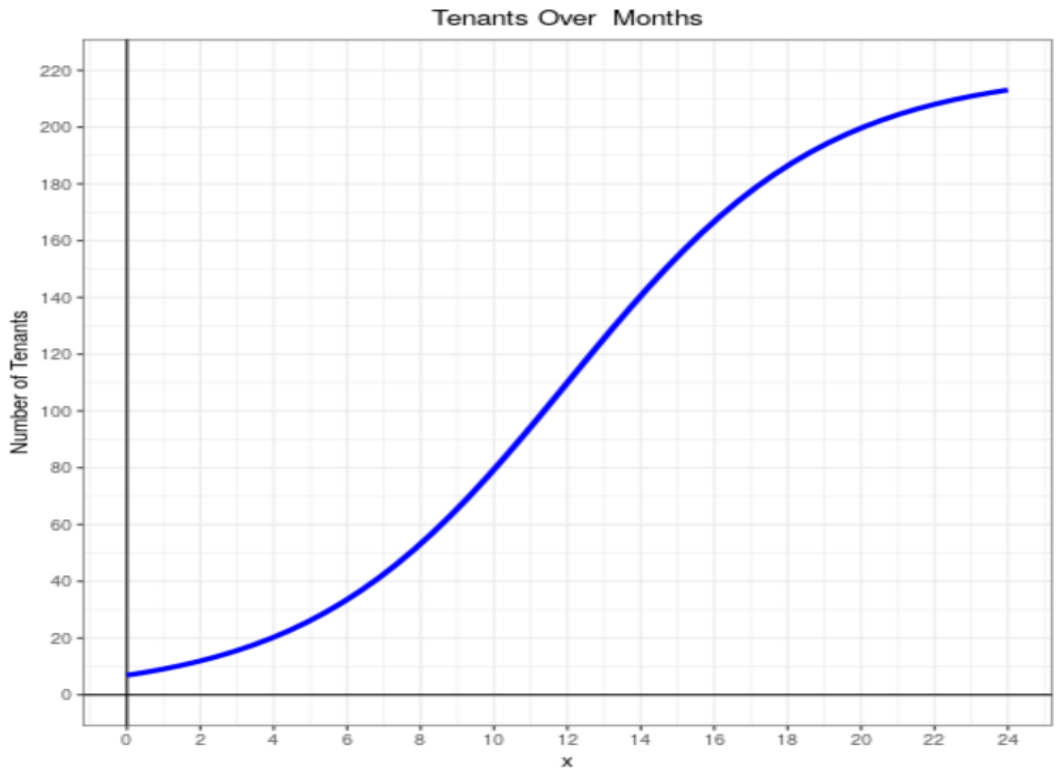
Problem	1	2	3	4	5	6	7	8	9	10
Answer										

Part II - Short Answer

True/False: Circle T for True or F for False next to each statement.

Fill in the blanks: Please read each question carefully.

11. (5 points) (1.25 points each) This graph is a model of the number of tenants in a particular apartment complex in its first 24 months. Answer the questions below.



Circle the correct answer to complete the following fill-in-the-blank questions:

A. From month 1 to month 11, the total population is **(increasing / decreasing)**.

B. From month 1 to month 11, the average rate of change of the population is **(greater / less)** than the instantaneous rate of change in month 20.

C. The instantaneous rate of change in month 22 is **(greater / less)** than the average rate of change from month 1 to 24.

D. The instantaneous rate of change in month 4 is **(greater / less)** than the instantaneous rate of change in month 14.

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12. (5 points) (1 point each) Use the graph to calculate the following limits.

a) $\lim_{x \to -3^-} f(x) =$

b) $\lim_{x \to -3^+} f(x) =$

c) $\lim_{x \to -3} f(x) =$

d) $\lim_{x \to 2} f(x) =$

e) $f(2) =$

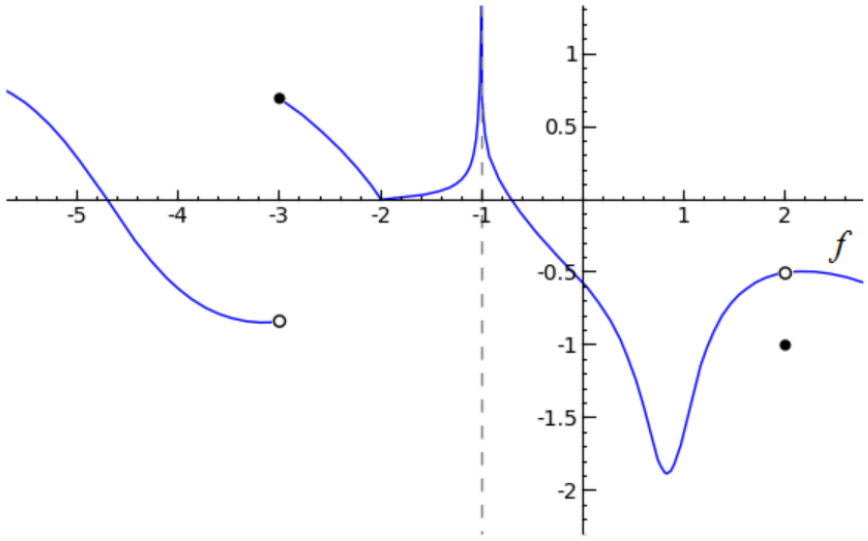

Figure 2

Part III — Free Response

You must show your work in the space provided to receive full credit.
Correct final answers without sufficient supporting work may receive zero credit.

Please put a box around your final answers.

13. (15 points) Compute the following integrals. Include the constant of integration, C.

a) $\int (2x^3 - 5x^2)(x - 1)\, dx$

b) $\int \dfrac{3x^2 - 6\sqrt{x} + 4}{x^3}\, dx$

c) $\int x^{-3/2} + 6e^{-4x}\, dx$

d) $\int \dfrac{3x^2}{\sqrt{x^4}} - 6\sqrt{x} + 4x^3\, dx$

e) $\int \frac{4}{7x} + 6e^{-4x}\,dx$

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14. (10 points) The temperature $T(t)$ (in °C) of a cup of coffee after being poured into a mug is modeled by

$$T(t) = 80e^{-0.2t} + 20, \quad t \geq 0$$

where t is the time in minutes.

a) Find the first and second derivatives of $T(t)$.

$T'(t) =$ _____
$T''(t) =$ _____

b) Determine the intervals where $T(t)$ is concave up or concave down and identify any inflection point. Write your answers in the form $(x, f(x))$.

Inflection point: _____

Concave up: _____

Concave down: _____

c) Interpret what the concavity of $T(t)$ tells you about how the coffee is cooling over time.

15. (10 points) A drone ascends vertically. Its velocity (in meters per second) at time t seconds is modeled by

$$v(t) = \frac{8}{t} + t, \quad t \geq 1.$$

The drone is launched from ground level, so its height at $t = 1$ seconds is $s(1) = 0$ meters.

Answer the following:

a) Find the height function $s(t)$ of the drone. Include the constant of integration and use the given initial condition to determine its value.

b) Using your height function, calculate the **height of the drone at $t = 4$ seconds**.

c) Find the **velocity of the drone at $t = 2$ seconds**, $v(2)$.

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16. (10 points) A YouTuber tracks the growth of subscribers (in hundreds) as a function of weeks for $t > 0$ after their first video. The model is

$$S(t) = t^3 - 6t^2 + 9t + 2.$$

Answer the following:

a) Find the first and second derivatives of $S(t)$.

$S'(t) =$ _____
$S''(t) =$ _____

b) Determine the intervals where $S(t)$ is concave up or concave down.

Concave up: _____
Concave down: _____

c) Find any inflection point(s) of $S(t)$. Give coordinates.

d) Interpret the concavity of $S(t)$: what does it tell you about the rate of subscriber growth over time?

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17. (3 points) A water tank is being filled at a rate of

$$R(t) = 3t + 5 \quad \text{liters per minute}, \quad 0 \le t \le 10.$$

Compute the total volume of water added between $t = 2$ min and $t = 6$ min. Include units.

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18. (6 points) A car accelerates along a straight road with velocity modeled by

$$v(t) = 3\sqrt{t} + 2 \quad \text{m/s}, \quad 0 \le t \le 9,$$

where t is measured in seconds from the start.

a) (3 points) Set up a definite integral for the total distance traveled from $t = 1$ s to $t = 4$ s.

b) (3 points) Evaluate the integral and provide the distance in meters.

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19. (10 points) Consider the two functions $f(x) = x^2$ and $g(x) = \sqrt{x}$.

a) (2 points) Solve algebraically for the intersection points.

b) (3 points) Sketch the graphs of f and g and shade the region enclosed.

c) (5 points) Write the definite integral corresponding to the area between f and g, then compute the area.