MAT 210 Brief Calculus
Sample Final Exam (Practice Version B)
Solutions

Instructor	XXXXXXX
Full Name (Print clearly!)	
Student ID	

Important Disclaimer for Students:

- This is a **sample Final exam**, not the actual in-class exam. Questions are of similar difficulty but may differ in type, numbers, or context.
- Success in the course depends on practicing concepts beyond this sample: lecture notes, Edfinity assignments, group work, quizzes, and homework.
- Complete this exam under exam-like conditions: no notes, no internet, no communication, and about **120-140 minutes** to finish.
- After finishing, check your answers against the posted solutions and grade yourself honestly.
- Repeat for all other versions posted on the MAT 210 website.
- If you cannot complete it within the time limit, review your weak areas and try another sample exam.
- **Round your answers to two decimal places where needed.**
- Treat all concepts from class materials as fair game for the actual exam.

Part I

Multiple Choice: Circle the response that best completes the statement or answers the question.

Then write the letter of your answer in the table after Problem 10.

1. (5 points) Identify which of the following definite integrals represents the area bounded by the curves $y = 4x + 16$ and $y = 2x^2 + 10$ over the interval $[0, 3]$.

A. $\displaystyle\int_{-1}^{3} (4x - 2x^2 + 6)\, dx$

B. $\displaystyle\int_{-1}^{3} \left[(2x^2 + 10) - (4x + 16)\right]\, dx$

C. $\displaystyle\int_{0}^{3} (2x^2 - 4x - 6)\, dx$

D. $\boxed{\displaystyle\int_{0}^{3} \left[(4x + 16) - (2x^2 + 10)\right]\, dx}$

E. None of the above

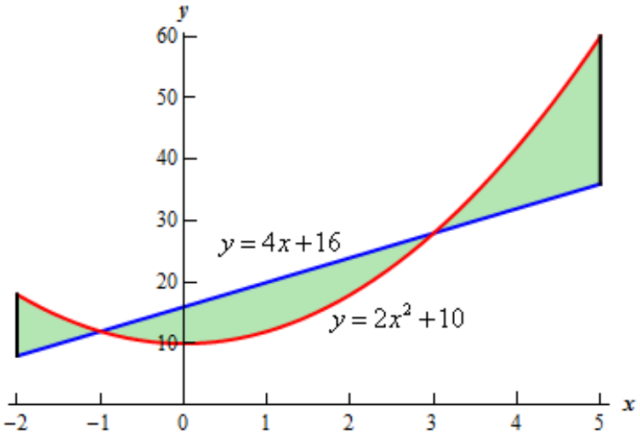

Figure 1

Solution:

The area between two curves is given by:

$$\text{Area} = \int_{x_1}^{x_2} \left[\text{top curve} - \text{bottom curve}\right] dx$$

- We want the area between the curves bounded by the lines $x = 0$ and $x = 3$.
- For $x \in [0, 3]$, the line $y = 4x + 16$ lies above the parabola $y = 2x^2 + 10$.
- Therefore, the integral for the area is:

$$\int_{0}^{3} \left[(4x + 16) - (2x^2 + 10)\right] dx = \int_{0}^{3} (-2x^2 + 4x + 6)\, dx$$

Correct Answer is D

2. (5 points) The concentration of a chemical in a tank changes according to $\dfrac{dC}{dt} = 5 - t$ (grams per minute). The integral $\displaystyle\int_{0}^{3} (5 - t)\, dt$ represents:

A. The rate of concentration at $t = 3$ minutes

B. The total increase in concentration over the first 3 minutes

C. The maximum concentration reached

D. The average concentration over 3 minutes

E. None of the above

Solution: Since $\frac{dC}{dt}$ is the *rate of change of concentration*, integrating it gives the total change in concentration over that time period.
Correct Answer: B - The total increase in concentration over the first 3 minutes

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3. (5 points) If $f(x)$ is continuous on $[1, 3]$, what does the integral

$$\int_3^1 f(x)\,dx$$

represent?

A. The negative of the integral, $\int_1^3 f(x)\,dx$

B. The same as $\int_1^3 f(x)\,dx$

C. Zero

D. The absolute value of $\int_1^3 f(x)\,dx$

E. None of the above

Correct Answer: A — Reversing the limits introduces a negative sign.

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4. (5 points) Let $f(x) = x^2$ on $[1, 5]$ with $\Delta x = 1$. Which of the following is the Right Riemann sum approximation of $\int_1^5 x^2 dx$?

A. $1(f(1) + f(2) + f(3) + f(4))$

B. $1(f(2) + f(3) + f(4) + f(5))$

C. $1(f(1) + f(2) + f(3) + f(4) + f(5))$

D. $4(f(2) + f(3) + f(4) + f(5))$

E. None of the above

Solution: For a Right Riemann sum on $[1, 5]$ with $\Delta x = 1$,
the right endpoints of the 4 subintervals are $2, 3, 4, 5$.
Thus the sum is $1\big(f(2) + f(3) + f(4) + f(5)\big)$.
Correct Answer: B

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5. (5 points) Determine the constant k such that $\lim_{x\to 2} g(x)$ exists, where

$$g(x) = \begin{cases} 3x + k, & x < 2, \\ 2x^2 - 1, & x > 2 \end{cases}$$

A. $k = \frac{7}{6}$

B. $k = 6$

C. $k = -1$

D. $\boxed{k = 1}$

E. None of the above

Solution: For $\lim_{x\to 2} g(x)$ to exist, the left-hand and right-hand limits must agree.

Left-hand limit (as $x \to 2^-$) using $3x + k$:

$$\lim_{x\to 2^-} 3x + k = 3(2) + k = 6 + k.$$

Right-hand limit (as $x \to 2^+$) using $2x^2 - 1$:

$$\lim_{x\to 2^+} 2x^2 - 1 = 2(2)^2 - 1 = 8 - 1 = 7.$$

Set equal:

$$6 + k = 7 \quad \Rightarrow \quad k = 1.$$

Correct choice: D

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6. (5 points) The number of trees in a park from 2000 to 2020 can be modeled by the function

$$T(t) = 50t^2 - 200t + 2500$$

where t is the number of years since 2000. Interpret the meaning of $T(10) = 5500$ and $T'(10) = 800$.

A. In 2010, there were 5,500 trees, and the number of trees was increasing at a rate of 800 trees per year at that time.

B. In 2010, there were 5,500 trees, and the total increase over the 10 years since 2000 was 800 trees.

C. In 2010, there were 5,500 trees, and the number of trees was increasing at an average of 800 trees per year between 2000 and 2010.

D. In 2010, there were 5,500 trees, and the number of trees was increasing at a rate of 800 trees every 10 years after 2000.

E. None of the above

Solution: $T(10) = 5500$ gives the number of trees when $t = 10$, i.e, the year 2010 and $T'(10) = 800$ is positive implies the number of trees are increasing at the rate of 800 trees per year in 2010. Option A matches the structure of the statement.

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7. (5 points) Let

$$h(x) = \frac{4x^3 + 2}{\ln(x+1)}, \qquad (x > -1).$$

Which of the following correctly represents $h'(x)$?

A. $h'(x) = \dfrac{12x^2}{\ln(x+1)} - \dfrac{4x^3+2}{x+1}$

B. $h'(x) = \dfrac{12x^2 \ln(x+1) + \frac{4x^3+2}{x+1}}{[\ln(x+1)]^2}$

C. $h'(x) = \dfrac{12x^2(\ln(x+1))^2 - (4x^3+2)}{[\ln(x+1)]^3}$

D. $h'(x) = \dfrac{12x^2 \ln(x+1) - \dfrac{4x^3+2}{x+1}}{[\ln(x+1)]^2}$

E. None of the above

Solution: Use the quotient rule. For $h(x) = \dfrac{u(x)}{v(x)}$ with $u(x) = 4x^3 + 2$ and $v(x) = \ln(x+1)$,

$$h'(x) = \frac{u'(x)v(x) - u(x)v'(x)}{[v(x)]^2}.$$

Here $u'(x) = 12x^2$ and $v'(x) = \dfrac{1}{x+1}$. Hence

$$h'(x) = \frac{12x^2 \ln(x+1) - \left(4x^3+2\right)\dfrac{1}{x+1}}{[\ln(x+1)]^2},$$

which matches option **D**.

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8. (5 points) Which of the following conditions must be true for $x = c$ to be a critical point of the function $g(x)$?

A. $g'(c)$ exists and equals 0

B. $g(c)$ is defined, and either $g'(c) = 0$ or $g'(c)$ does not exist.

C. $x = c$ must correspond to a local maximum or minimum of $g(x)$

D. $g'(c)$ is positive at $x = c$

E. None of the above

Solution: A critical point occurs where the derivative of the function is zero or does not exist, provided the function itself is defined at that point. Therefore, option B is correct.

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9. (5 points) If $f(x)$ is a polynomial with a single critical point at $x = 2$ and $f''(2) > 0$, which is true about $f(2)$?

   A. $f(2)$ is a relative minimum

   B. $f(2)$ is a relative maximum

   C. $f(2)$ is an inflection point

   D. There is not enough information

   E. None of the above

   Solution: The function $f(x)$ has a critical point at $x = 2$, so $f'(2) = 0$. By the second derivative test, since $f''(2) > 0$, the graph is concave up at $x = 2$, which means $f(2)$ is a relative minimum.

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10. (5 points) If $f''(x)$ changes from positive to negative at $x = 4$ and $f(x)$ is defined for all real numbers, what can we conclude?

    A. $f(x)$ has a relative maximum at $x = 4$

    B. $f(x)$ has a relative minimum at $x = 4$

    C. $f(x)$ has an inflection point at $x = 4$

    D. $f'(x)$ has a relative maximum at $x = 4$

    E. None of the above

    Solution: An **inflection point** occurs where the concavity of $f(x)$ changes, i.e., where $f''(x)$ changes sign. Here, $f''(x)$ changes from positive to negative at $x = 4$, so the graph changes from concave up to concave down. Therefore, $f(x)$ has an inflection point at $x = 4$.

PART I Answers:

Problem	1	2	3	4	5	6	7	8	9	10
Answer										

Part II - Short Answer

True/False: Circle T for True or F for False next to each statement.

Fill in the blanks: Please read each question carefully.

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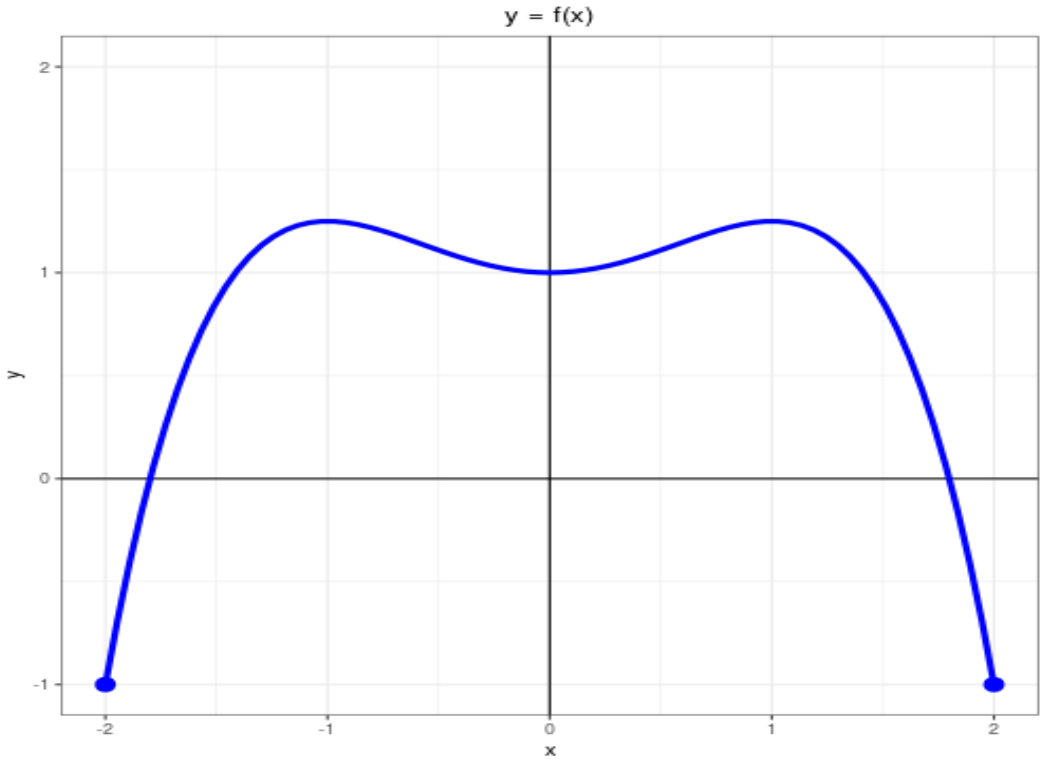
11. (12 points) The function $f(x)$ is graphed below. Use the graph to answer the following questions. Show all answers in the spaces provided.



Question	Answer
1. Find the value(s) of x where $f(x)$ has a relative minimum.	0
2. Find the value(s) of x where $f(x)$ has a relative maximum.	-1 and 1
3. Find the value(s) of x where $f(x)$ has an absolute minimum.	-2 and 2
4. Find the value(s) of x where $f'(x) = 0$.	-1, 0, and 1
5. Find the interval(s) where $f'(x) > 0$.	(-2,-1) and (0,1)
6. Find the interval(s) where $f'(x) < 0$.	(-1,0) and (1,2)

Solution: Observe the graph to determine the absolute and relative extrema based on where the function increases or decreases and where the tangent lines are horizontal.

Part III — Free Response

You must show your work in the space provided to receive full credit.
Correct final answers without sufficient supporting work may receive zero credit.

Please put a box around your final answers.

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12. (12 points) A coffee machine company determines that in order to sell x units of a new coffee maker, the price per unit (in dollars) is given by $u(x) = 210 - 2x$, and the cost of producing x units is given by $c(x) = 1000 + 10x$.

a) Write the profit function $P(x)$ in terms of x.

Solution: Revenue is $R(x) = u(x) \cdot x = (210 - 2x)x = 210x - 2x^2$.
Profit is revenue minus cost:

$$P(x) = R(x) - c(x) = (210x - 2x^2) - (1000 + 10x) = -2x^2 + 200x - 1000$$

b) Find $P'(x)$.

Solution:

$$P'(x) = \frac{d}{dx}(-2x^2 + 200x - 1000) = -4x + 200$$

c) Determine the price per unit that must be charged to maximize profit.

Solution: Set $P'(x) = 0$ to find the maximizing number of units:

$$-4x + 200 = 0 \quad \Rightarrow \quad x = 50$$

The price per unit is then:

$$u(50) = 210 - 2(50) = 210 - 100 = 110$$

$\Rightarrow$ To maximize profit, the company should charge \$110 per coffee machine.

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13. (10 points) A tank initially contains 1200 liters of water, so $V(0) = 1200$. Water drains from the tank at a rate given by

$$V'(t) = -30e^{-0.03t}, \quad t \text{ in hours.}$$

a) Find the **volume of water** $V(t)$ in the tank at any time $t \geq 0$. Include the constant of integration and use the initial condition to determine its value.

Integrate $V'(t)$:

$$V(t) = \int (-30e^{-0.03t})dt = -30 \cdot \frac{e^{-0.03t}}{-0.03} + C = 1000e^{-0.03t} + C$$

Use $V(0) = 1200$: $1000e^0 + C = 1200 \implies C = 200$.
Final answer: $V(t) = 1000e^{-0.03t} + 200$.

b) How much water is in the tank after 5 hours?

Use $V(t) = 1000e^{-0.03t} + 200$:

$$V(5) = 1000e^{-0.03(5)} + 200 = 1000e^{-0.15} + 200 \approx 1000(0.8607) + 200 \approx 1060.7 \text{ liters.}$$

c) Describe whether the volume function is concave up or down over time, and explain what this means for the rate of draining.

Compute second derivative: $V''(t) = (-30e^{-0.03t})' = 0.9e^{-0.03t} > 0$ for all $t \geq 0$.
So $V(t)$ is concave up.
Interpretation: The rate of draining is slowing down over time (the water leaves the tank more slowly as t increases).

d) Estimate the time when only 15 liters remain in the tank.

Set $V(t) = 15$:

$$1000e^{-0.03t} + 200 = 15 \implies 1000e^{-0.03t} = -185$$

This is impossible because $1000e^{-0.03t} > 0$ for all t.
Note: Only 200 liters is the limiting volume as $t \to \infty$, so the tank never reaches 15 liters. The minimum volume is 200 liters.

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14. (10 points) The monthly water usage for a household (in liters) can be modeled by

$$W(t) = 0.05t^3 - 0.4t^2 + 3t + 500,$$

where t measures time in months since the start of January 2020, and $W(t)$ is the water usage per month.

a) (4 points) Find $W'(t)$.

Solution: Differentiate term-by-term:

$$\boxed{W'(t) = 0.15t^2 - 0.8t + 3.}$$

b) (4 points) Evaluate $W'(12)$, round to 2 decimal places, include units.

Solution: Evaluate at $t = 12$:

$$W'(12) = 0.15(12)^2 - 0.8(12) + 3 = 0.15(144) - 9.6 + 3 = 21.6 - 9.6 + 3 = 15.0.$$

$\boxed{15.00}$ liters per month(i.e. increase in liters/month each month).

c) (2 points) Write a sentence to explain what the result from part (b) tells us about water usage.

Interpretation: At $t = 12$ months (one year after January 2020), the monthly water usage is increasing at about 15 liters per month. In other words, near that time the household's monthly usage is rising by about 15 liters each month.

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15. (10 points) The number of bacteria in a lab culture from the start of the experiment can be modeled by

$$B(t) = 2e^{0.1t} + 4e^{0.3t} + 0.5t \quad \text{thousands of bacteria,}$$

where t is the number of hours since the start of the experiment ($t = 0$ represents the beginning of the experiment).

a) (4 points) Find $B'(t)$.

Solution: Differentiate each term (recall derivative of e^{at} is ae^{at}):

$$B'(t) = 2 \cdot 0.1e^{0.1t} + 4 \cdot 0.3e^{0.3t} + 0.5$$

So

$$\boxed{B'(t) = 0.2e^{0.1t} + 1.2e^{0.3t} + 0.5 \text{ (thousands of bacteria per hour)}}$$

b) (4 points) Calculate the instantaneous rate of change of total number of bacteria after **10 hours**. Include units. (Round your answer to the nearest whole number.)

Solution: Evaluate $B'(t)$ at $t = 10$:

$$B'(10) = 0.2e^{0.1 \cdot 10} + 1.2e^{0.3 \cdot 10} + 0.5 = 0.2e^1 + 1.2e^3 + 0.5.$$

Compute numerically (use $e^1 \approx 2.7182818$, $e^3 \approx 20.085537$):

$$0.2e^1 \approx 0.2(2.7182818) = 0.54365636,$$
$$1.2e^3 \approx 1.2(20.085537) = 24.1026444,$$

so

$$B'(10) \approx 0.5437 + 24.1026 + 0.5 = 25.1463 \text{ (thousands of bacteria per hour).}$$

Rounded to nearest whole number: $25,146$ bacteria per hour.

$\boxed{B'(10) \approx 25{,}146 \text{ bacteria per hour}}$

c) (2 points) Interpret your answer from part (b).

Interpretation: After 10 hours, the culture's population is increasing by 25,146 bacteria per hour (i.e., the instantaneous rate of change at $t = 10$ is ≈ 25 thousand bacteria per hour).

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16. (3 points) A population grows at a rate $P'(t) = 2t + 1$ thousand people per year, for $0 \le t \le 6$. Write a definite integral that represents the total population increase over the first 4 years and evaluate the integral. Include units.

The total increase in population is given by the integral of the rate function over the time interval $0 \le t \le 4$. So we compute the definite integral $\int_0^4 (2t + 1)\, dt$.

Evaluate it and apply the limits: $(t^2 + t)\big|_0^4 = (16 + 4) - 0 = 20$.

Since the rate is measured in *thousands* of people per year, the final answer is $\boxed{20}$ thousand people of population increase over the first 4 years.

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17. (6 points) The rate at which electricity is consumed in a small town is modeled by

$$E'(t) = \frac{50}{t} \quad \text{megawatts per hour}, \quad 1 \le t \le 8,$$

where $t = 0$ corresponds to midnight.

a) (3 points) Set up a definite integral representing the total electricity consumed between 2 AM and 6 AM.
Solution: The total electricity consumed between 2 AM and 6 AM is given by the integral of the rate function $E'(t)$ from $t = 2$ to $t = 6$. So we write:

$$\int_2^6 \frac{50}{t}\, dt.$$

b) (3 points) Evaluate the integral and state the total electricity consumed in megawatts.

Solution: We evaluate the integral in part (a)

$$\int_2^6 \frac{50}{t}\, dt = 50 \int_2^6 \frac{1}{t}\, dt = 50\,[\ln|t|]_2^6 = 50(\ln 6 - \ln 2) \approx 54.93.$$

Therefore, the total electricity consumed between 2 AM and 6 AM is approximately $\boxed{54.93}$ megawatts.

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18. (10 points) Consider the two functions $f(x) = x^2 + 2x - 2$ and $g(x) = 3x + 4$.

a) (2 points) Solve algebraically for the intersection points. **Solution:** Intersection points occur where $f(x) = g(x)$. So we set the two functions equal:

$$x^2 + 2x - 2 = 3x + 4.$$

Bring all terms to one side to form a quadratic equation:

$$x^2 + 2x - 2 - 3x - 4 = 0 \implies x^2 - x - 6 = 0.$$

Factor the quadratic:

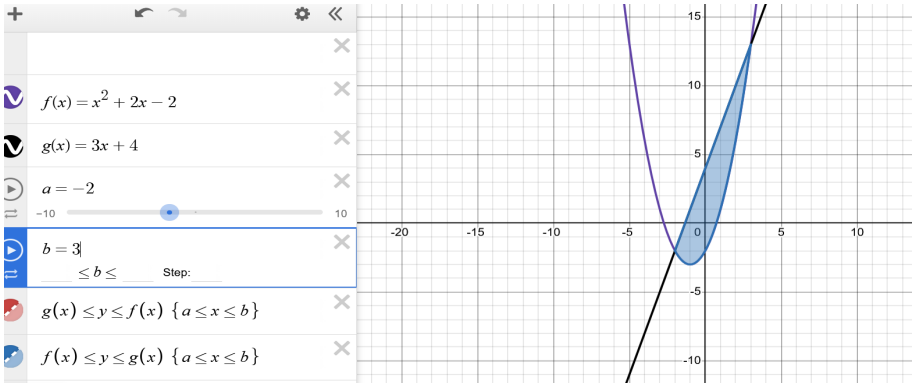
$$(x - 3)(x + 2) = 0.$$

Therefore, the intersection points are:

$$\boxed{x = 3} \quad \text{or} \quad \boxed{x = -2}.$$

b) (3 points) Sketch the graphs of f and g and shade the region enclosed.

Solution: You can use Desmos calculator to learn and practice for how to sketch functions.
Click to sketch this graph on Desmos.
Note: Desmos is NOT allowed on any in-person exams. The picture below is provided only to help you understand the setup and to practice.



c) (5 points) Write the definite integral corresponding to the area between f and g, then compute the area.
Solution: Observe the positions of functions in part (b). Since $g(x) > f(x)$ in this interval, the area is

$$\text{Area} = \int_{-2}^{3} (g(x) - f(x))\,dx = \int_{-2}^{3} (-x^2 + x + 6)\,dx = \left[-\frac{x^3}{3} + \frac{x^2}{2} + 6x\right]_{-2}^{3}.$$

Evaluate from -2 to 3:

$$\left[-\frac{27}{3} + \frac{9}{2} + 18\right] - \left[-\frac{-8}{3} + \frac{4}{2} - 12\right] = 13.5 - -\frac{22}{3},$$

Therefore, the area between the curves is approximately $\boxed{\mathbf{20.83}}$ square units.