MAT 210 Brief Calculus
Sample Final Exam (Practice Version B)

Instructor	XXXXXXX
Full Name (Print clearly!)	
Student ID	

Important Disclaimer for Students:

- This is a **sample Final exam**, not the actual in-class exam. Questions are of similar difficulty but may differ in type, numbers, or context.
- Success in the course depends on practicing concepts beyond this sample: lecture notes, Edfinity assignments, group work, quizzes, and homework.
- Complete this exam under exam-like conditions: no notes, no internet, no communication, and about **120-140 minutes** to finish.
- After finishing, check your answers against the posted solutions and grade yourself honestly.
- Repeat for all other versions posted on the MAT 210 website.
- If you cannot complete it within the time limit, review your weak areas and try another sample exam.
- **Round your answers to two decimal places where needed.**
- Treat all concepts from class materials as fair game for the actual exam.

Part I

Multiple Choice: Circle the response that best completes the statement or answers the question.

Then write the letter of your answer in the table after Problem 10.

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1. (5 points) Identify which of the following definite integrals represents the area bounded by the curves $y = 4x + 16$ and $y = 2x^2 + 10$ over the interval $[0, 3]$.

   A. $\displaystyle\int_{-1}^{3} (4x - 2x^2 + 6)\, dx$

   B. $\displaystyle\int_{-1}^{3} \left[(2x^2 + 10) - (4x + 16)\right]\, dx$

   C. $\displaystyle\int_{0}^{3} (2x^2 - 4x - 6)\, dx$

   D. $\displaystyle\int_{0}^{3} \left[(4x + 16) - (2x^2 + 10)\right]\, dx$

   E. None of the above

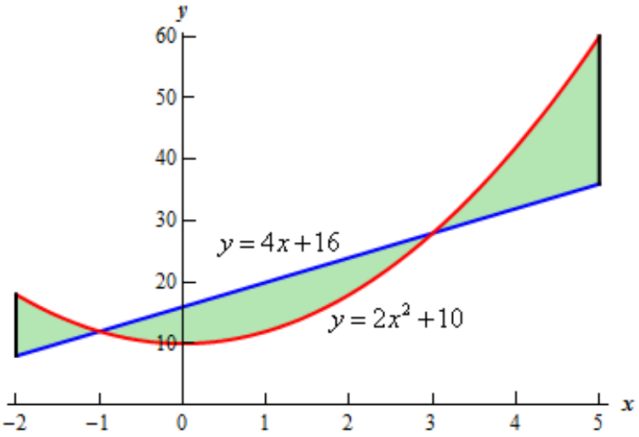

Figure 1

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2. (5 points) The concentration of a chemical in a tank changes according to $\dfrac{dC}{dt} = 5 - t$ (grams per minute). The integral $\displaystyle\int_{0}^{3} (5 - t)\, dt$ represents:

   A. The rate of concentration at $t = 3$ minutes

   B. The total increase in concentration over the first 3 minutes

   C. The maximum concentration reached

   D. The average concentration over 3 minutes

   E. None of the above

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3. (5 points) If $f(x)$ is continuous on $[1, 3]$, what does the integral

$$\int_{3}^{1} f(x)\, dx$$

represent?

A. The negative of the integral, $\int_1^3 f(x)\,dx$

B. The same as $\int_1^3 f(x)\,dx$

C. Zero

D. The absolute value of $\int_1^3 f(x)\,dx$

E. None of the above

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4. (5 points) Let $f(x) = x^2$ on $[1, 5]$ with $\Delta x = 1$. Which of the following is the Right Riemann sum approximation of $\int_1^5 x^2 dx$?

A. $1(f(1) + f(2) + f(3) + f(4))$

B. $1(f(2) + f(3) + f(4) + f(5))$

C. $1(f(1) + f(2) + f(3) + f(4) + f(5))$

D. $4(f(2) + f(3) + f(4) + f(5))$

E. None of the above

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5. (5 points) Determine the constant k such that $\lim_{x\to 2} g(x)$ exists, where

$$g(x) = \begin{cases} 3x + k, & x < 2, \\ 2x^2 - 1, & x > 2 \end{cases}$$

A. $k = \frac{7}{6}$

B. $k = 6$

C. $k = -1$

D. $k = 1$

E. None of the above

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6. (5 points) The number of trees (in hundreds) in a park from 2000 to 2020 can be modeled by the function

$$T(t) = 50t^2 - 200t + 2500$$

where t is the number of years since 2000. Interpret the meaning of $T(10) = 5500$ and $T'(10) = 800$.

A. In 2010, there were 5,500 trees, and the number of trees was increasing at a rate of 800 trees per year at that time.

B. In 2010, there were 5,500 trees, and the total increase over the 10 years since 2000 was 800 trees.

C. In 2010, there were 5,500 trees, and the number of trees was increasing at an average of 800 trees per year between 2000 and 2010.

D. In 2010, there were 5,500 trees, and the number of trees was increasing at a rate of 800 trees every 10 years after 2000.

E. None of the above

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7. (5 points) Let

$$h(x) = \frac{4x^3 + 2}{\ln(x+1)}, \qquad (x > -1).$$

Which of the following correctly represents $h'(x)$?

A. $h'(x) = \dfrac{12x^2}{\ln(x+1)} - \dfrac{4x^3+2}{x+1}$

B. $h'(x) = \dfrac{12x^2 \ln(x+1) + \frac{4x^3+2}{x+1}}{[\ln(x+1)]^2}$

C. $h'(x) = \dfrac{12x^2(\ln(x+1))^2 - (4x^3+2)}{[\ln(x+1)]^3}$

D. $h'(x) = \dfrac{12x^2 \ln(x+1) - \frac{4x^3+2}{x+1}}{[\ln(x+1)]^2}$

E. None of the above

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8. (5 points) Which of the following conditions must be true for $x = c$ to be a critical point of the function $g(x)$?

A. $g'(c)$ exists and equals 0.

B. $g(c)$ is defined, and either $g'(c) = 0$ or $g'(c)$ does not exist.

C. $x = c$ must correspond to a local maximum or minimum of $g(x)$.

D. $g'(c)$ is positive at $x = c$.

E. None of the above.

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9. (5 points) If $f(x)$ is a polynomial with a single critical point at $x = 2$ and $f''(2) > 0$, which is true about $f(2)$?

A. $f(2)$ is a relative minimum

B. $f(2)$ is a relative maximum

C. $f(2)$ is an inflection point

D. There is not enough information

E. None of the above

10. (5 points) If $f''(x)$ changes from positive to negative at $x = 4$ and $f(x)$ is defined for all real numbers, what can we conclude?

A. $f(x)$ has a relative maximum at $x = 4$

B. $f(x)$ has a relative minimum at $x = 4$

C. $f(x)$ has an inflection point at $x = 4$

D. $f'(x)$ has a relative maximum at $x = 4$

E. None of the above

PART I Answers:

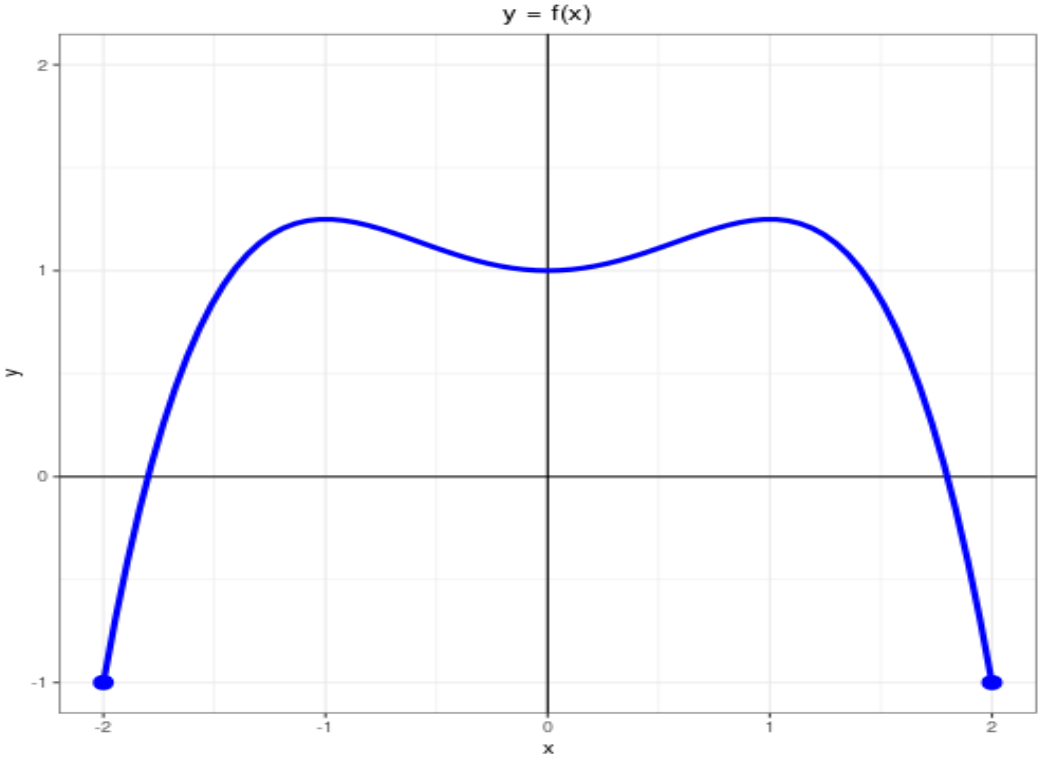
Problem	1	2	3	4	5	6	7	8	9	10
Answer										

Part II - Short Answer

True/False: Circle T for True or F for False next to each statement.

Fill in the blanks: Please read each question carefully.

11. (12 points) The function $f(x)$ is graphed below. Use the graph to answer the following questions. Show all answers in the spaces provided.



Question	**Answer**
1. Find the value(s) of x where $f(x)$ has a relative minimum.	
2. Find the value(s) of x where $f(x)$ has a relative maximum.	
3. Find the value(s) of x where $f(x)$ has an absolute minimum.	
4. Find the value(s) of x where $f'(x) = 0$.	
5. Find the interval(s) where $f'(x) > 0$.	
6. Find the interval(s) where $f'(x) < 0$.	

Part III — Free Response

You must show your work in the space provided to receive full credit.
Correct final answers without sufficient supporting work may receive zero credit.

Please put a box around your final answers.

12. (12 points) A coffee machine company determines that in order to sell x units of a new coffee maker, the price per unit (in dollars) is given by $u(x) = 210 - 2x$, and the cost of producing x units is given by $c(x) = 1000 + 10x$.

    a) (5 points) Write the profit function $P(x)$ in terms of x.

    b) (2 points) Find $P'(x)$.

    c) (5 points) Determine the price per unit that must be charged to maximize profit.

13. (10 points) A tank initially contains 1200 liters of water, so $V(0) = 1200$. Water drains from the tank at a rate given by

$$V'(t) = -30e^{-0.03t}, \quad t \text{ in hours.}$$

    a) Find the **volume of water** $V(t)$ in the tank at any time $t \geq 0$. Include the constant of integration and use the initial condition to determine its value.

b) How much water is in the tank after 5 hours?

c) Describe whether the volume function is concave up or down over time, and explain what this means for the rate of draining.

d) Estimate the time when only 15 liters remain in the tank.

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14. (10 points) The monthly water usage for a household (in liters) can be modeled by

$$W(t) = 0.05t^3 - 0.4t^2 + 3t + 500,$$

where t measures time in months since the start of January 2020, and $W(t)$ is the water usage per month.

a) (4 points) Find $W'(t)$.

b) (4 points) Evaluate $W'(12)$, round to 2 decimal places, include units.

c) (2 points) Write a sentence to explain what the result from part (b) tells us about water usage.

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15. (10 points) The number of bacteria in a lab culture from the start of the experiment can be modeled by

$$B(t) = 2e^{0.1t} + 4e^{0.3t} + 0.5t \quad \text{thousands of bacteria,}$$

where t is the number of hours since the start of the experiment ($t = 0$ represents the beginning of the experiment).

a) (4 points) Find $B'(t)$.

b) (4 points) Calculate the instantaneous rate of change of total number of bacteria after **10 hours**. Include units. (Round your answer to the nearest whole number.)

c) (2 points) Interpret your answer from part (b).

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16. (3 points) A population grows at a rate $P'(t) = 2t + 1$ thousand people per year, for $0 \leq t \leq 6$. Write a definite integral that represents the total population increase over the first 4 years and evaluate the integral. Include units.

17. (6 points) The rate at which electricity is consumed in a small town is modeled by

$$E(t) = \frac{50}{t} \quad \text{megawatts per hour}, \quad 1 \leq t \leq 8,$$

where $t = 0$ corresponds to midnight.

a) (3 points) Set up a definite integral representing the total electricity consumed between 2 AM and 6 AM.

b) (3 points) Evaluate the integral and state the total electricity consumed in megawatts.

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18. (10 points) Consider the two functions $f(x) = x^2 + 2x - 2$ and $g(x) = 3x + 4$.

a) (2 points) Solve algebraically for the intersection points.

b) (3 points) Sketch the graphs of f and g and shade the region enclosed.

c) (5 points) Write the definite integral corresponding to the area between f and g, then compute the area.