MAT 210 Brief Calculus
Sample Final Exam (Practice Version A)
Solutions

Instructor	XXXXXXX
Full Name (Print clearly!)	
Student ID	

Important Disclaimer for Students:

- This is a **sample Final exam**, not the actual in-class exam. Questions are of similar difficulty but may differ in type, numbers, or context.
- Success in the course depends on practicing concepts beyond this sample: lecture notes, Edfinity assignments, group work, quizzes, and homework.
- Complete this exam under exam-like conditions: no notes, no internet, no communication, and about **120-140 minutes** to finish.
- After finishing, check your answers against the posted solutions and grade yourself honestly.
- Repeat for all other versions posted on the MAT 210 website.
- If you cannot complete it within the time limit, review your weak areas and try another sample exam.
- **Round your answers to two decimal places where needed.**
- Treat all concepts from class materials as fair game for the actual exam.

Part I

Multiple Choice: Circle the response that best completes the statement or answers the question.

Then write the letter of your answer in the table after Problem 10.

1. (5 points) Identify which of the following definite integrals corresponds to the area bounded by the curves $y = 4x + 16$ and $y = 2x^2 + 10$ shown below.

A. $\int_{-1}^{3} (4x - 2x^2 + 6)\, dx$

B. $\int_{-1}^{3} \left[(2x^2 + 10) - (4x + 16)\right]\, dx$

C. $\int_{0}^{4} (4x - 2x^2 + 6)\, dx$

D. $\int_{-1}^{3} \left[(4x + 16) + (2x^2 + 10)\right]\, dx$

E. None of the above

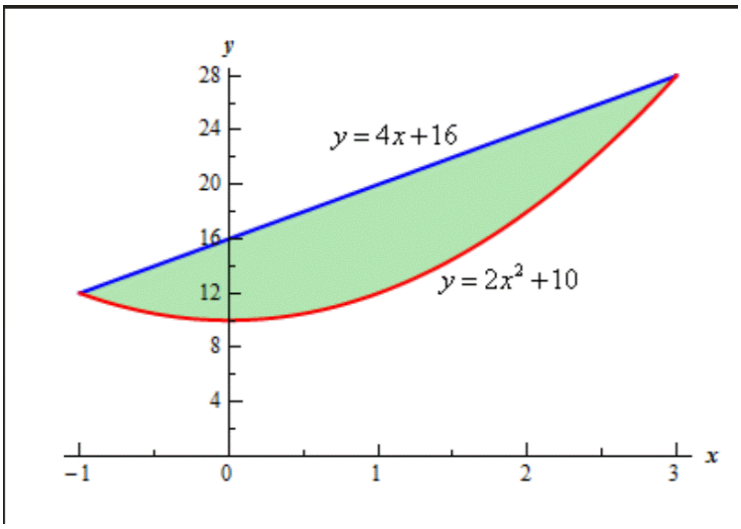

Figure 1

Solution:

The area between two curves is given by:

$$\text{Area} = \int_{x_1}^{x_2} \left[\text{top curve} - \text{bottom curve}\right] dx$$

- The curves intersect at $x = -1$ and $x = 3$ (solve $4x + 16 = 2x^2 + 10$).
- For $x \in [-1, 3]$, the line $y = 4x + 16$ lies above the parabola $y = 2x^2 + 10$.
- Therefore, the integral for the area is:

$$\int_{-1}^{3} \left[(4x + 16) - (2x^2 + 10)\right] dx = \int_{-1}^{3} (-2x^2 + 4x + 6)\, dx$$

Correct answer: A

2. (5 points) Evaluate

$$\int_1^4 (3x^2 - 2x)\,dx.$$

A. 33

B. 48

C. 30

D. 24

E. None of the above

Solution:

Step 1: Find the antiderivative of the integrand:

$$\int (3x^2 - 2x)\,dx = x^3 - x^2 + C$$

Step 2: Evaluate the definite integral from 1 to 4:

$$\int_1^4 (3x^2 - 2x)\,dx = \left[x^3 - x^2\right]_1^4 = (4^3 - 4^2) - (1^3 - 1^2)$$

Step 3: Simplify:

$$(64 - 16) - (1 - 1) = 48 - 0 = 48$$

Correct Answer: B

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3. (5 points) Which of the following is true for any continuous function $f(x)$ on $[a, b]$?

A. $\int_a^b f(x)\,dx + \int_b^a f(x)\,dx = 0$

B. $\int_a^b f(x)\,dx = 0$

C. $\int_a^b f(x)\,dx = \int_a^b |f(x)|\,dx$

D. $\int_a^b f(x)\,dx = f(b) - f(a)$

E. None of the above

Solution:
Property: Reversing the limits of integration changes the sign of the integral:

$$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx.$$

Therefore,

$$\int_a^b f(x)\,dx + \int_b^a f(x)\,dx = \int_a^b f(x)\,dx - \int_a^b f(x)\,dx = 0.$$

Option A is always true for any continuous function.
Correct Answer: A

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4. (5 points) For $f(x) = x^3$ on $[0, 2]$ with $\Delta x = 0.5$, the Left Riemann sum is given by:

   A. $\boxed{0.5(f(0) + f(0.5) + f(1) + f(1.5))}$
   B. $0.5(f(0.5) + f(1) + f(1.5) + f(2))$
   C. $0.5(f(0) + f(0.5) + f(1) + f(1.5) + f(2))$
   D. $2(f(0) + f(0.5) + f(1) + f(1.5))$
   E. None of the above

Solution:
A Left Riemann sum on the interval $[0, 2]$ with $\Delta x = 0.5$ uses the left endpoints:

$$0,\ 0.5,\ 1,\ 1.5$$

So the correct expression is:

$$0.5(f(0) + f(0.5) + f(1) + f(1.5)).$$

Correct Answer: A.

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5. (5 points) If $h(2)$ exists, what can we conclude if $h'(2)$ does not exist?

   A. $x = 2$ cannot be a critical point since the derivative does not exist.
   B. $x = 2$ must be a relative maximum or a relative minimum.
   C. $\boxed{x = 2 \text{ is a critical point.}}$
   D. $x = 2$ cannot be a relative extrema point.
   E. Nothing can be concluded without more information about the function.

Solution: A critical point occurs where the derivative is zero or does not exist, provided the function is defined at that point. Since $h(2)$ exists and $h'(2)$ does not exist, $x = 2$ is a critical point.

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6. (5 points) A small business models revenue with a nonlinear demand function:

$$q(p) = 750 - 10p^2, \quad R(p) = p \cdot q(p)$$

where p is the price per unit (in dollars) and $q(p)$ is the number of units sold.

Determine the price that maximizes revenue.

A. $p = 4$

B. $\boxed{p = 5}$

C. $p = 6$

D. $p = 7$

E. None of the above

Solution: To maximize revenue, find $R'(p)$ and set it to zero:

$$R(p) = p(750 - 10p^2) = 750p - 10p^3, \quad R'(p) = 750 - 30p^2.$$

Set $R'(p) = 0$: $750 - 30p^2 = 0 \implies p^2 = 25 \implies p = 5$ (positive price).
Thus, the revenue is maximized at \$5 per unit.

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7. (5 points) Let $g(x)$ satisfy $g'(x) < 0$ and $g''(x) > 0$ for all x. Which statement is true?

A. $g(x)$ is increasing and concave up

B. $\boxed{g(x) \text{ is decreasing and concave up}}$

C. $g(x)$ is increasing and concave down

D. $g(x)$ is decreasing and concave down

E. None of the above

Solution:
Since $g'(x) < 0$, the function is decreasing.
Since $g''(x) > 0$, the function is concave up.
Therefore, $g(x)$ is decreasing and concave up.

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8. (5 points) If $P'(x) = 5e^{2x} - 4e^{-x} + 6$, and $P(1) = 50$, find the profit function $P(x)$.

A. $P(x) = \frac{5}{2}e^{2x} - 4e^{-x} + 6x + 24$

B. $\boxed{P(x) = \frac{5}{2}e^{2x} + 4e^{-x} + 6x + 24}$

C. $P(x) = 5e^{2x} - 4e^{-x} + 6x + 50$

D. $P(x) = \frac{5}{2}e^{2x} + 4e^{-x} + 6x + 50$

E. None of the above

Solution:

Integrate $P'(x)$:

$$\int P'(x)\,dx = \int (5e^{2x} - 4e^{-x} + 6)\,dx = \frac{5}{2}e^{2x} + 4e^{-x} + 6x + C$$

Use $P(1) = 50$ to find C:

$$\frac{5}{2}e^2 + 4e^{-1} + 6(1) + C = 50 \implies C = 50 - \frac{5}{2}e^2 - 4e^{-1} - 6 \approx 24$$

Hence,

$$P(x) = \frac{5}{2}e^{2x} - 4e^{-x} + 6x + 24.$$

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9. (5 points) A bacterial culture (in millions) is modeled by the function

$$B(t) = 12 + 4\ln(t),$$

where t is measured in hours after the culture is started. Find the instantaneous rate of change of the culture size at $t = 4$ hours. Round to the nearest whole number.

A. 4 million cells per hour

B. $\boxed{\text{1 million cells per hour}}$

C. 12 million cells per hour

D. 16 million cells per hour

E. None of the above

Solution: The instantaneous rate of change is the derivative $B'(t)$.

Differentiate:

$$B'(t) = \frac{d}{dt}\left(12 + 4\ln t\right) = 4 \cdot \frac{1}{t} = \frac{4}{t}.$$

Evaluate at $t = 4$:

$$B'(4) = \frac{4}{4} = 1.$$

Units: millions of cells per hour.

Rounded to the nearest whole number: 1.

$\boxed{\text{Correct choice: B}}$

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10. (5 points) The table below shows the median monthly water bill (in dollars) for households in a town from January 2018 to January 2024, where t is years since the start of 2018. Compute the average rate of change of the bill from the start of 2020 to the start of 2024.

Year t (years after 2018)	0	1	2	3	4	5	6
Bill ($)	45	47	50	52	55	57	60

A. From the start of 2020 to the start of 2024, the bill increased by $2.5 per year, on average.

B. From the start of 2020 to the start of 2024, the bill increased by $3 per year, on average.

C. From the start of 2020 to the start of 2024, the bill increased by $2 per year, on average.

D. From the start of 2020 to the start of 2024, the bill increased by $3.5 per year, on average.

E. From the start of 2020 to the start of 2024, the bill decreased by $2 per year, on average.

Solution: Start of 2020 corresponds to $t = 2$ and start of 2024 to $t = 6$. The bill at $t = 2$ is \$50 and at $t = 6$ is \$60. The average rate of change is

$$\frac{60 - 50}{6 - 2} = \frac{10}{4} = 2.5 \text{ dollars per year.}$$

Option(A) is correct.

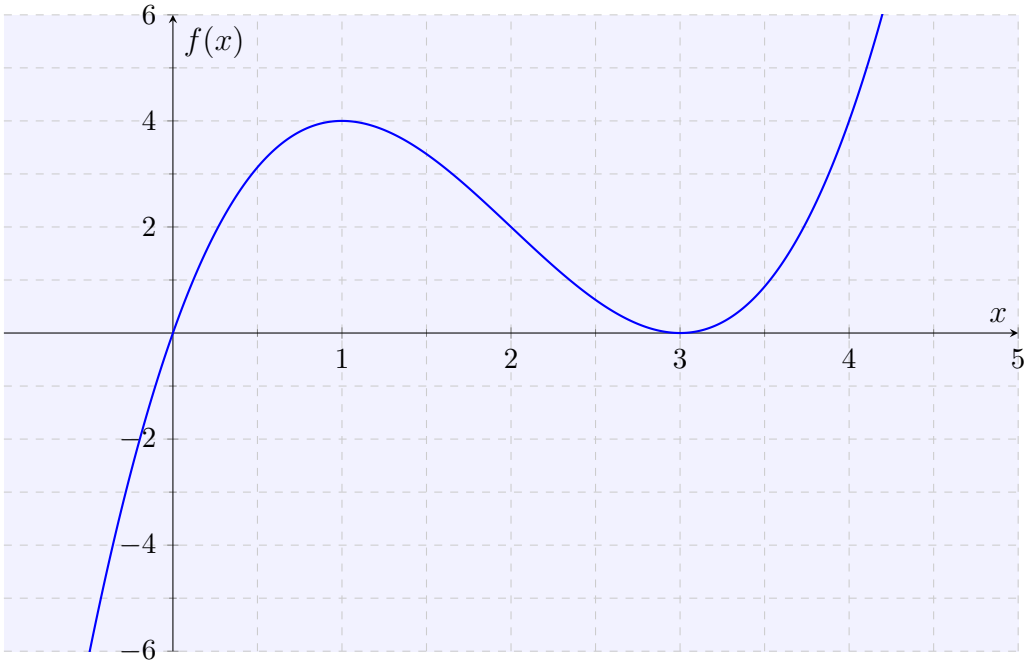
PART I Answers:

Problem	1	2	3	4	5	6	7	8	9	10
Answer										

Part II - Short Answer

True/False: Circle T for True or F for False next to each statement.
Fill in the blanks: Please read each question carefully.

11. (10 points) The graph of $f(x)$ is shown below for $x \in [0, 4]$:



Answer the following questions about the graph of $f(x)$ on $[0, 4]$. Write your answers using interval notation and separate them with commas.

a) **Increasing on:** $(0, 1)$, $(3, 4)$
(Where the graph of $f(x)$ rises; i.e., $f'(x) > 0$.)

b) **Decreasing on:** $(1, 3)$
(Where the graph of $f(x)$ falls; i.e., $f'(x) < 0$.)

c) **Relative maximum at:** $x = 1$
(The x-value(s) where f changes from increasing to decreasing.)

d) **Relative minimum at:** $x = 3$
(The x-value(s) where f changes from decreasing to increasing.)

e) **Inflection point(s):** $x = 2$
(Where the concavity changes; i.e., $f''(x) = 0$ or undefined.)

f) **Concave up on:** $(2, 4)$
(Where $f''(x) > 0$ and the graph bends upward.)

g) **Concave down on:** $(0, 2)$
(Where $f''(x) < 0$ and the graph bends downward.)

h) **At $x = 1$:** slope $= 0$; the graph is flat.
(Use the graph to estimate the slope of $f(x)$ at $x = 1$ to determine the slope and whether f is increasing, decreasing, or flat.)

i) **Critical values:** $x = 1,\ 3$
(List all x-values where $f'(x) = 0$.)

j) **Between $x = 2$ and $x = 4$:** concave up and reaches a relative minimum at $x = 3$
(Briefly describe the graph's shape using terms like concave up/down, or relative max/min.)

Part III — Free Response

You must show your work in the space provided to receive full credit.
Correct final answers without sufficient supporting work may receive zero credit.

Please put a box around your final answers.

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12. (20 points) Solve the following initial value problems. For each part, find the function $f(x)$ that satisfies the initial condition.

a) $f'(x) = 2x^2 - 5x + 6, \qquad f(0) = 4.$

Integrate: $f(x) = \int (2x^2 - 5x + 6)\,dx = \frac{2x^3}{3} - \frac{5x^2}{2} + 6x + C.$
Use $f(0) = 4$: $0 - 0 + 0 + C = 4 \implies C = 4.$
Final answer: $f(x) = \frac{2}{3}x^3 - \frac{5}{2}x^2 + 6x + 4$

b) $f'(x) = 5e^{3x} + 2e^{-x}, \qquad f(0) = 1.$

Integrate: $\int 5e^{3x}dx + \int 2e^{-x}dx = \frac{5}{3}e^{3x} - 2e^{-x} + C.$
Use $f(0) = 1$: $\frac{5}{3} - 2 + C = 1 \implies C = \frac{4}{3}.$
Final answer: $f(x) = \frac{5}{3}e^{3x} - 2e^{-x} + \frac{4}{3}$

c) $f'(x) = \frac{4}{x}, \qquad f(1) = 2 \qquad (x \geq 1).$

Integrate: $f(x) = \int \frac{4}{x}dx = 4\ln|x| + C.$
Use $f(1) = 2$: $4\ln 1 + C = 2 \implies C = 2.$
Final answer: $f(x) = 4\ln x + 2$

d) $f'(x) = \sqrt{x} - x^3, \qquad f(1) = 0, \qquad (x \geq 0).$

Rewrite: $\sqrt{x} = x^{1/2}$. Integrate: $\int x^{1/2}dx - \int x^3 dx = \frac{2}{3}x^{3/2} - \frac{1}{4}x^4 + C.$

Use $f(1) = 0$: $\frac{2}{3} - \frac{1}{4} + C = 0 \implies C = -\frac{5}{12}$.

Final answer: $f(x) = \frac{2}{3}x^{3/2} - \frac{1}{4}x^4 - \frac{5}{12}$

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13. (10 points) Let

$$f(x) = -3x^3 + 15x^2 - 9x + 7.$$

a) Find $f'(x)$ and $f''(x)$.

$f'(x) = -9x^2 + 30x - 9$

$f''(x) = -18x + 30$

b) Find and classify critical points as relative maxima or minima. Write your answers in $(x, f(x))$ form. (**Hint:** Use the 2nd derivative test.)

Critical points: Solve $f'(x) = -9x^2 + 30x - 9 = 0 \implies 9x^2 - 30x + 9 = 0.$
Use the Quadratic formula: $x = 3$ or $x = 1/3$.
Use 2nd derivative test: $f''(1/3) = -18(1/3) + 30 = -6 + 30 = 24 > 0 \implies$ local minimum.
$f''(3) = -18(3) + 30 = -54 + 30 = -24 < 0 \implies$ local maximum.
Corresponding $f(x)$ values: $f(1/3) = -3(1/27) + 15(1/9) - 9(1/3) + 7 = 50/9 \approx 5.556$.
$f(3) = -3(27) + 15(9) - 27 + 7 = 34$.
Final answers: Local minimum: $\left(\frac{1}{3}, \frac{50}{9}\right)$, Local maximum: $(3, 34)$.

c) Find the intervals of concavity.

Concave up: $f''(x) > 0 \implies -18x + 30 > 0 \implies x < \frac{5}{3}$ or $(-\infty, \frac{5}{3})$.

Concave down: $f''(x) < 0 \implies x > \frac{5}{3}$ or $(\frac{5}{3}, \infty)$.

d) Find inflection points. Write your answers in $(x, f(x))$ form.

Inflection points occur where $f''(x) = -18x + 30 = 0 \implies x = \frac{30}{18} = \frac{5}{3}$.
Corresponding function value $f(x)$: $f(5/3) = -3(125/27) + 15(25/9) - 9(5/3) + 7 = 178/9 \approx 19.78$.
Inflection point: $\left(\frac{5}{3}, \frac{178}{9}\right)$.

e) Interpret what concavity reveals about the graph.

Concave up: the graph is "cup-shaped" and slopes are increasing; the function is accelerating upward.
Concave down: the graph is "cap-shaped" and slopes are decreasing; the function is accelerating downward.
The inflection point at $x = 5/3$ is where the graph changes concavity.

14. (15 points) Determine the derivative of the following functions. Show your work. You **do NOT need** to simplify your results.

a) $h(x) = \dfrac{4x^3 - 5\sqrt{x}}{e^{x^4}}$

Solution: Use the quotient rule:

$$h'(x) = \frac{(12x^2 - \frac{5}{2}x^{-1/2})e^{x^4} - (4x^3 - 5\sqrt{x})(4x^3 e^{x^4})}{(e^{x^4})^2}$$

b) $g(x) = 2\left(3x^3 - \frac{8}{x}\right)^{-7}$

Solution: Use the chain rule:

$$g'(x) = 2 \cdot (-7)\left(3x^3 - \frac{8}{x}\right)^{-8} \cdot \left(9x^2 + \frac{8}{x^2}\right) = -14\left(3x^3 - \frac{8}{x}\right)^{-8}\left(9x^2 + \frac{8}{x^2}\right)$$

c) $k(x) = \ln(3x^4 - 8x + 5)$

Solution: Use the derivative of $\ln(u)$:

$$k'(x) = \frac{d}{dx}[3x^4 - 8x + 5] \cdot \frac{1}{3x^4 - 8x + 5} = \frac{12x^3 - 8}{3x^4 - 8x + 5}$$

d) $m(x) = \frac{5}{3x^4} \cdot \sqrt{x^2 + 4}$

Solution: Use the product rule: Let $u = \frac{5}{3x^4}$, $v = (x^2 + 4)^{1/2}$. Then

$$u' = -\frac{20}{3}x^{-5}, \quad v' = \frac{x}{\sqrt{x^2 + 4}}$$

$$m'(x) = u'v + uv' = \left(-\frac{20}{3}x^{-5}\right)(\sqrt{x^2 + 4}) + \left(\frac{5}{3x^4}\right)\left(\frac{x}{\sqrt{x^2 + 4}}\right)$$

e) $f(x) = \left(5\sqrt{x} + \frac{4}{x^3}\right)^4 \ln(2x^2 + 3x)$

Solution: Use the product rule: Let $u = \left(5\sqrt{x} + \frac{4}{x^3}\right)^4$, $v = \ln(2x^2 + 3x)$. Then

$$u' = 4\left(5\sqrt{x} + \frac{4}{x^3}\right)^3\left(\frac{5}{2}x^{-1/2} - \frac{12}{x^4}\right), \quad v' = \frac{4x + 3}{2x^2 + 3x}$$

$$f'(x) = u'v + uv' = 4\left(5\sqrt{x} + \frac{4}{x^3}\right)^3\left(\frac{5}{2}x^{-1/2} - \frac{12}{x^4}\right)\ln(2x^2 + 3x) + \left(5\sqrt{x} + \frac{4}{x^3}\right)^4 \frac{4x + 3}{2x^2 + 3x}$$

15. (10 points) A bakery produces x loaves of bread each day. The total cost (in dollars) to produce x loaves and the total revenue (in dollars) from selling x loaves are modeled by

$$C(x) = 150 + 2x, \quad R(x) = 3x - 0.002x^2.$$

a) Write the profit function $P(x)$.

Solution: Profit is revenue minus cost:

$$P(x) = R(x) - C(x) = \left(3x - 0.002x^2\right) - (150 + 2x) = -0.002x^2 + x - 150$$

b) Find the number of loaves, x, that will maximize the profit function.

Solution: To maximize profit, set $P'(x) = 0$:

$$P'(x) = -0.004x + 1$$

Set derivative to zero:

$$-0.004x + 1 = 0 \quad \Rightarrow \quad x = \frac{1}{0.004} = 250$$

$\Rightarrow$ Maximum profit occurs at 250 loaves.

c) Interpret your result from part (b).

Solution: The bakery maximizes its profit by producing and selling 250 loaves of bread per day. Producing more or fewer loaves would result in lower profit.

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16. (3 points) A car travels along a straight road with velocity $v(t) = 4t - 1$ m/s for $0 \le t \le 5$ seconds. Find the total distance traveled by evaluating the definite integral

$$\int_0^5 v(t)\, dt.$$

Solution: Compute the integral.

$$\int_0^5 (4t - 1)\, dt = \Big[2t^2 - t\Big]_0^5 = (2 \cdot 5^2 - 5) - (0) = (50 - 5) = 45.$$

Answer: The car travels $\boxed{45}$ meters over the interval $[0, 5]$.

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17. (6 points) The population of a small town is growing at a rate approximated by

$$P'(t) = 500e^{0.1t} \quad \text{people per year}, \quad 0 \le t \le 10,$$

where $t = 0$ corresponds to the start of the year 2020.

a) (3 points) Set up a definite integral that represents the total population increase from the start of 2021 to the start of 2023. Clearly indicate limits and the function to be integrated.
Solution:

The start of 2021 corresponds to $t = 1$ and the start of 2023 corresponds to $t = 3$. The total population increase is

$$\int_1^3 500e^{0.1t}\,dt.$$

b) (3 points) Evaluate the integral and provide the total population increase. Include appropriate units.
Solution: Evaluate the integral:

$$\int_1^3 500e^{0.1t}\,dt = 500\int_1^3 e^{0.1t}\,dt = 500\cdot\frac{1}{0.1}\Big[e^{0.1t}\Big]_1^3 = 5000\big(e^{0.3}-e^{0.1}\big).$$

This is the exact value (people).
Answer: The population increases by $5000\big(e^{0.3}-e^{0.1}\big)$ people, approximately $\boxed{\mathbf{1223}}$ people (rounded).

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18. (10 points) Consider $f(x) = x^2 - 4x + 3$ and $g(x) = -x^2 + 6x - 5$.

a) (2 points) Solve algebraically for the intersection points.
Solution:
Solve $f(x) = g(x)$:

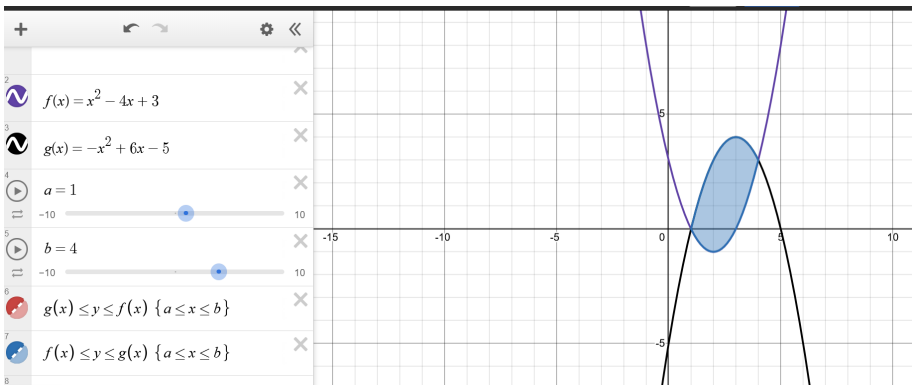
$$x^2 - 4x + 3 = -x^2 + 6x - 5 \quad \Rightarrow \quad 2x^2 - 10x + 8 = 0.$$

Divide by 2 and factor:

$$x^2 - 5x + 4 = 0 \quad \Rightarrow \quad (x-1)(x-4) = 0,$$

so the intersection points are $\boxed{x = 1}$ and $\boxed{x = 4}$.

b) (3 points) Sketch the graphs of f and g and shade the region enclosed.
Solution: You can use Desmos calculator to learn and practice for how to sketch functions.
Click to sketch this graph on Desmos.
Note: Desmos is NOT allowed on any in-person exams. The picture below is provided only to help you understand the setup and to practice.



c) (5 points) Write the definite integral corresponding to the area between f and g, then compute the area.
Solution: Observe the positions of functions in part (b). Since $g(x) > f(x)$ in this interval, the area is

$$\int_1^4 \big(g(x)-f(x)\big)\,dx = \int_1^4 \Big[(-x^2+6x-5)-(x^2-4x+3)\Big]\,dx = \int_1^4 (-2x^2+10x-8)\,dx.$$

Evaluate from 1 to 4:

$$\left(-\tfrac{2}{3}x^3+5x^2-8x\right)\Big|_1^4=\left(-\tfrac{2}{3}\cdot 64+5\cdot 16-8\cdot 4\right)-\left(-\tfrac{2}{3}\cdot 1+5\cdot 1-8\cdot 1\right).$$

The area between the curves is $\boxed{9}$ square units.