MAT 210 Brief Calculus
Sample Exam 1 (Practice Version A)

Instructor	XXXXXXX
Full Name (Print clearly!)	
Student ID	

Important Disclaimer for Students:

- This is a **sample exam**, not the actual in-class exam. Problems are of similar difficulty but may differ in type, numbers, or context.
- Success in the course depends on practicing concepts beyond this sample: lecture notes, Edfinity, Group Work, quizzes, and homework.
- You should complete this exam as if you were in the real exam room: no notes, no internet, no calculators beyond what is allowed, and an approximately 60-minute time limit.
- After finishing, check your answers against the provided solutions and grade yourself.
- If you could not complete within the time limit, practice again and attempt another sample exam.
- Treat every problem in your coursework (notes, quizzes, assignments) as fair game for the actual exam.

Part I - Multiple Choice: Circle the response that best completes the statement or answers the question. Then write the letter of your answer in the table after Problem 7.

1. (5 points) The table below shows the median monthly electricity bill (in tens of dollars) for households in a city from January 2015 to January 2021, where t is years since the start of 2015. Compute the average rate of change of the bill from the start of 2017 to the start of 2021.

Year t (years after 2015)	0	1	2	3	4	5	6
Bill (tens of US \$)	12	13.2	14.5	15.5	16.4	17.5	18.1

A. From the start of 2017 to the start of 2021, the bill increased by 0.5 tens of dollars per year, on average.

B. From the start of 2017 to the start of 2021, the bill decreased by 0.6 tens of dollars per year, on average.

C. From the start of 2017 to the start of 2021, the bill increased by 0.75 tens of dollars per year, on average.

D. From the start of 2017 to the start of 2021, the bill increased by 0.9 tens of dollars per year, on average.

E. From the start of 2017 to the start of 2021, the bill decreased by 1.2 tens of dollars per year, on average.

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2. (5 points) A bacterial culture (in millions) is modeled by the function

$$B(t) = 12 + 4\ln(t),$$

where t is measured in hours after the culture is started. Find the instantaneous rate of change of the culture size at $t = 4$ hours. Round your answer to the nearest whole number.

A. 4 million cells per hour

B. 1 million cells per hour

C. 12 million cells per hour

D. 16 million cells per hour

E. None of the above

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3. (5 points) Determine the constant k such that $\lim_{x\to 2} g(x)$ exists, where

$$g(x) = \begin{cases} 3x + k, & x < 2, \\ 2x^2 - 1, & x > 2 \end{cases}$$

A. $k = \frac{7}{6}$

B. $k = 6$

C. $k = -1$

D. $k = 1$

E. None of the above

4. (5 points) At the age of t solaks, the height of a child is given by $M(t)$ (measured in molarks). Which of the following calculations gives the *average rate of change* of the child's height from age 3 solaks to 9 solaks?

   A. $\dfrac{M(6)}{6}$

   B. $\dfrac{M'(9) - M'(3)}{9-3}$

   C. $\dfrac{M(9) - M(3)}{9-3}$

   D. $\dfrac{M'(6)}{6}$

   E. $\dfrac{M(9) - M(3)}{9}$

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5. (5 points) Which of the following functions is *not continuous* at $x = 3$? There is only **one** correct answer.

   A. $\dfrac{x-3}{x+3}$

   B. $\dfrac{x+3}{x-3}$

   C. $\ln(x+3)$

   D. $(3x-9)^2$

   E. None of the above

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6. (5 points) The number of bicycles in a city from 1990 to 2010 can be modeled by the function

$$B(t) = 80t^2 - 400t + 5000$$

where t is the number of years since 1990. Interpret the meaning of $B(15) = 17000$ and $B'(15) = 2000$.

   A. In 2005, there were 17000 bicycles, and the number of bicycles was increasing at a rate of 2000 bicycles per year at that time.

   B. In 2005, there were 17000 bicycles, and the total increase over the 15 years since 1990 was 2000 bicycles.

   C. In 2005, there were 17000 bicycles, and the number of bicycles was increasing at an average of 2000 bicycles per year between 1990 and 2005.

   D. In 2005, there were 17000 bicycles, and the number of bicycles was increasing at a rate of 2000 bicycles every 15 years after 1990.

   E. None of the above

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7. (5 points) The graph of $f(x)$ is shown in Figure 1. Fill in the blanks: The function has a ______ discontinuity at $x =$ ______ and a ______ discontinuity at $x =$ ______.

A. removable, 1; infinite, -1

B. jump, -1; removable, 4

C. removable, -1; jump, 1

D. infinite 4; removable 1

E. None of the above

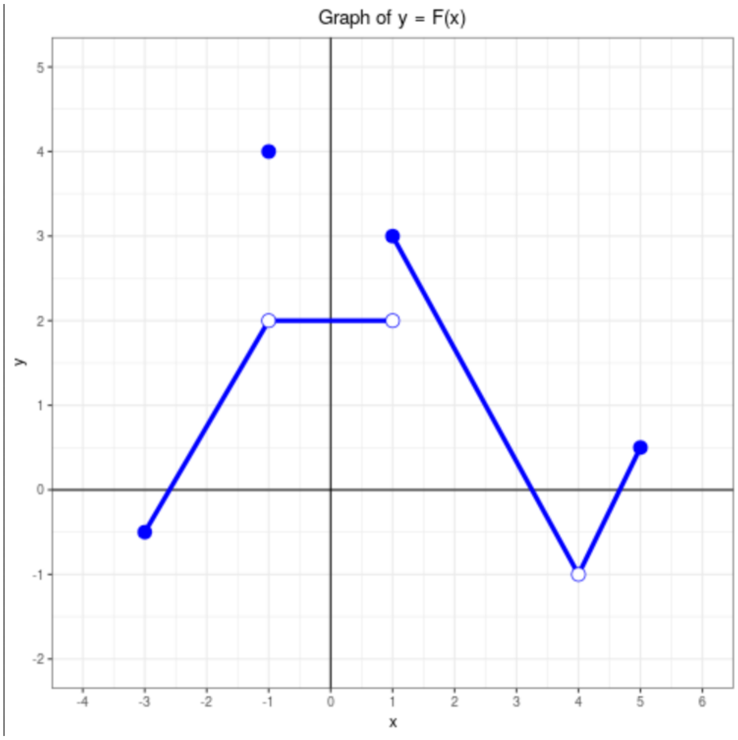


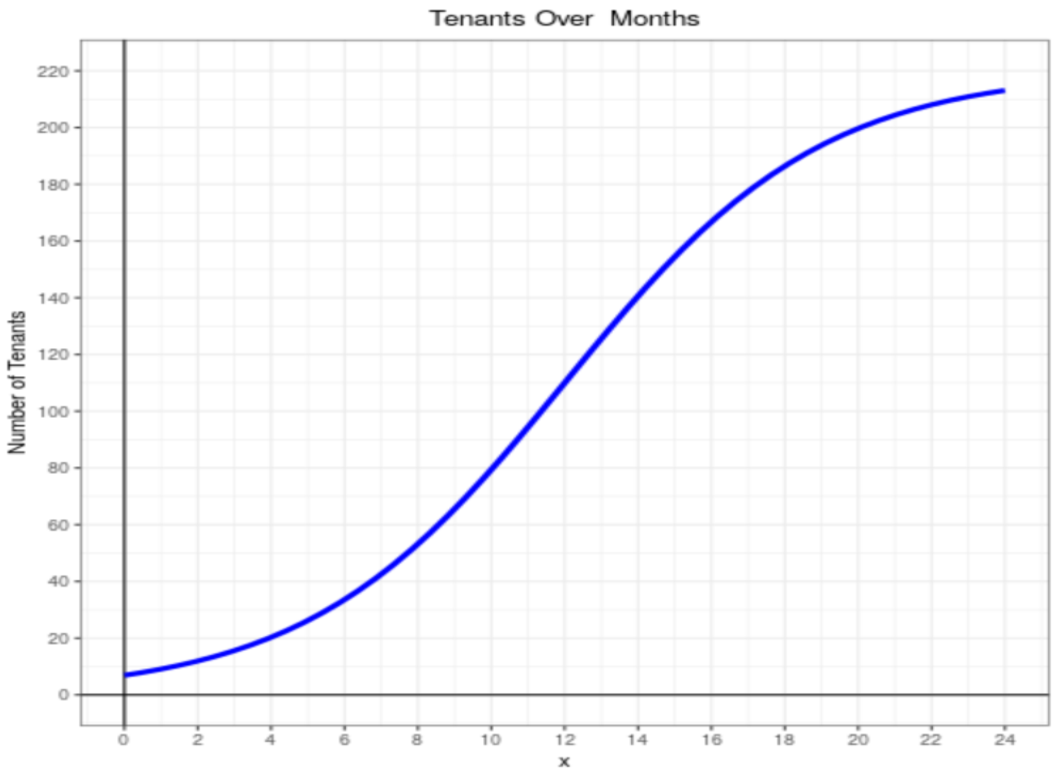
Figure 1: Sample discontinuities

PART I Answers: Please write the letter of your selected answers to multiple choice questions 1 through 7 in the following table.

Problem	1	2	3	4	5	6	7
Answer							

Part II - Fill in the blanks

8. (5 points) (1.25 points each) This graph is a model of the number of tenants in a particular apartment complex in its first 24 months. Answer the questions below.



Circle the correct answer to complete the following fill-in-the-blank questions:

A. From month 1 to month 11, the total population is **(increasing / decreasing)**.

B. From month 1 to month 11, the average rate of change of the population is **(greater / less)** than the instantaneous rate of change in month 20.

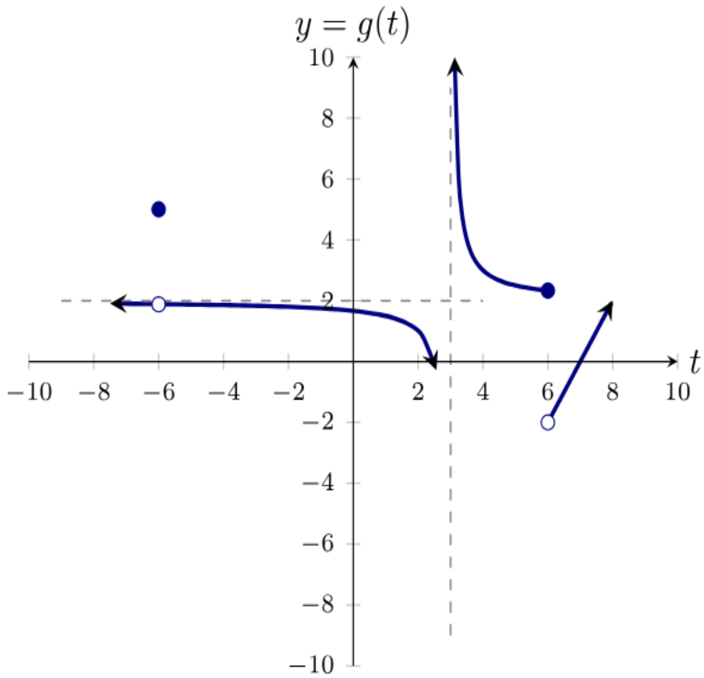
C. The instantaneous rate of change in month 22 is **(greater / less)** than the average rate of change from month 1 to 24.

D. The instantaneous rate of change in month 4 is **(greater / less)** than the instantaneous rate of change in month 14.

Part III - Free Response: You must show your work in the space provided to receive full credit. Correct final answers without sufficient supporting work may receive zero credit.

Please put a box around your final answers.

9. (5 points) (0.5 point each) Use the graph to calculate the following limits.



a) $\lim_{x \to -6^-} f(x) =$

b) $\lim_{x \to -6^+} f(x) =$

c) $\lim_{x \to -6} f(x) =$

d) $f(-6) =$

e) $\lim_{x \to 3^-} f(x) =$

f) $\lim_{x \to 3^+} f(x) =$

g) $\lim_{x \to 3} f(x) =$

h) $f(3) =$

i) $\lim_{x \to 6} f(x) =$

j) $f(6) =$

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10. (10 points) Let $t \geq 1$ denote the number of weeks since a species of insects (*in hundreds*) were introduced in a greenhouse. The number of insects in the greenhouse at week t is modeled by

$$P(t) = 150 + 40 \ln t,$$

.

a) (4 points) Find $P'(t)$.

b) (4 points) Evaluate $P'(8)$, round to 2 decimal places, include units.

c) (2 points) Interpret the meaning of your answer in part (b).

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11. (10 points) Suppose an object is thrown upward with an initial velocity of 49 feet per second from a height of 135 feet. The height of the object t seconds after it is thrown is given by

$$h(t) = -2t^2 + 49t + 135.$$

a) (7 points) Find the average rate of change of the position of the object for the time period $t = 5$ to $t = 10$. Include units.

b) (3 points) Interpret the meaning of your answer in part (a).

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12. (15 points) (5 points each) Find the derivative of the following functions. Show your work. You **do NOT** need to simplify your results.

a) $p(y) = 14y^{\frac{9}{2}} - \dfrac{8}{y^3} - \dfrac{13}{9y^{-6}} + 121.7$

b) $m(z) = \sqrt[5]{z^{13}} - \sqrt[7]{z^{18}}$

c) $r(s) = 6\ln(s) + 11e^{21s}$

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13. (10 points) The number of bacteria in a lab culture from the start of the experiment can be modeled by

$$B(t) = 2e^{0.1t} + 4e^{0.3t} + 0.5t \quad \text{thousands of bacteria,}$$

where t is the number of hours since the start of the experiment ($t = 0$ represents the beginning of the experiment).

a) (4 points) Find $B'(t)$.

b) (4 points) Calculate the instantaneous rate of change of total number of bacteria after **10 hours**. Include units. (Round your answer to the nearest whole number.)

c) (2 points) Interpret your answer from part (b).

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14. (10 points) Let $f(x) = 5x^2 - 6x$. Answer the following:

    (a) (3 points) Find $f(x + h)$.

    (b) (4 points) Find the difference quotient.

    (c) (3 points) Use the limit definition of the derivative to find $f'(x)$.
