

M E M O R A N D U M

DATE: 06/20/2025

TO: Faculty and Students

FROM: Professor(s) Jonathan Montaña
Chair/Co-Chairs of Sudipta Das
Defense for the PhD in Mathematics
Committee Members
Andrzej Czygrinow
Florian Sprung
Julien Paupert
Susanna Fishel
Zilin Jiang

DEFENSE ANNOUNCEMENT

Candidate: Sudipta Das

Defense Date: 07-03-2025 (Thursday)

Defense Time: 9:00 AM

Virtual Meeting Link: <https://asu.zoom.us/j/86432577178>

Tempe Campus: PSF 101

Title: Asymptotic colength for families of ideals, and summation formula for rational powers.

Please share this information with colleagues and other students, especially those studying in similar fields. Faculty and students are encouraged to attend. The defending candidate will give a 40 minute talk, after which the committee members will ask questions. There may be time for questions from those in attendance. But, guests are primarily invited to attend as observers and will be excused when the committee begins its deliberations or if the committee wishes to question the candidate privately.

ABSTRACT
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Abstract

This thesis delves into the asymptotic and combinatorial foundations of families of ideals in Noetherian local rings (R, m) , illuminating new connections between algebra, geometry, and combinatorics. In collaboration with Cheng Meng, we advance the classical theory by analyzing weakly graded families, weakly p -families, and weakly inverse p -families of m -primary ideals. Through the development of a unified analytic framework, we establish the existence of asymptotic colength limits for these broad classes of families and prove Brunn-Minkowski-type inequalities, positivity criteria, and volume = multiplicity formulas. These results generalize and strengthen previous work on p -families, revealing deep connections between algebraic growth, convex geometry, and singularity theory.

In a separate collaboration with Sankhaneel Bisui, Tàì Huy Hà, and Jonathan Montaña, we present explicit polyhedral and lattice-point models for the rational powers and Rees valuations of ideals invariant under torus and general linear group actions. This combinatorial viewpoint enables us to prove a Mustața-Takagi-type summation formula for rational powers in these equivariant settings, and to establish a weaker version of the formula for arbitrary ideals over the complex numbers when the exponent is sufficiently large. Together, these investigations deepen our understanding of asymptotic invariants and multiplicity theory, and reveal the rich interplay between algebraic structures, geometric intuition, and combinatorial techniques in commutative algebra and invariant theory.