1. (65 points) Let $X_1, \ldots, X_n$ be a random sample from pdf $f(x; \theta) = \frac{1}{\theta} \exp(-x/\theta)$ for $x \in (0, \infty)$ and zero otherwise

(a) (6 points) Find the MLE (Maximum Likelihood Estimator) of $\theta$.
(b) (9 points) Find the CDF of $X_{(1:n)} = \min\{X_i, i = 1, \ldots, n\}$.
(c) (9 points) Find an exact 90% two-tailed confidence interval for $\theta$ using $X_{(1:n)} = \min\{X_i, i = 1, \ldots, n\}$.
(d) (9 points) Find a minimal sufficient statistic for $\theta$.
(e) (12 points) Find the UMVUE for $\theta$.
(f) (4 points) Find the MLE for $1/\theta$.
(g) (4 points) Find the MME (Method of Moments Estimator) of $1/\theta$. (Hint: Consider $\beta = 1/\theta$ so the problem is to find the MME of $\beta$)
(h) (12 points) Find the UMVUE of $1/\theta$

2. (35 points) Let $X_1, \ldots, X_n$ be a random sample from pdf

$$f(x; \theta) = \frac{1}{\theta} I_{[0, \theta]}(x) \quad \theta > 0$$

(a) (5 points) Show that $X_{(n:n)} = \max\{X_i, i = 1, \ldots, n\}$ is a sufficient statistic.
(b) (12 points) Is $X_{n:n}$ complete? If "yes", then prove it. If "no", then find an counterexample.
(c) (13 points) Find the MP test for testing the following hypotheses at level .05.

$$H_0 : \theta = 2 \quad H_\alpha : \theta = 1$$

Express your test in terms of a sufficient statistic with the exact "k" (i.e. find the distribution of the sufficient statistic and find the "cutoff point").
(d) (5 points) Find the UMP test for testing

$$H_0 : \theta = 2 \quad H_\alpha : \theta < 2$$