Honor Statement:
By signing below you confirm that you have neither given nor received any unauthorized assistance on this exam. This includes any use of a graphing calculator beyond those uses specifically authorized by the Mathematics Department and your instructor. Furthermore, you agree not to discuss this exam with anyone until the exam testing period is over and this exam has been returned to you.

_____________________________________________                _________________

Signature                                                                                   Date

Test Information:
• This test has 11 questions worth 100 points total.
• Multiple choice are x pts each, Free response are as marked
• You have 50 minutes from the start of the exam in which to complete the items.
• For the 6 multiple choice questions, list your answers in the answer grid is at the end of the exam.
• For free response questions, show all work in the area provided by the questions.
• Do not round or approximate your answers unless otherwise indicated.

• You may use a calculator so long as it does NOT perform symbolic algebra. For example, the TI-89 or TI-nSpire CAS are not permitted.
• Sharing of calculators is not permitted
• Cell phones may not be accessed during the exam and must be turned to silent
• The use of bilingual dictionaries is permitted on this test.
I. Multiple Choice

List your answers on the back of the exam in the provided grid.

1. Calculate **without calculator** the **exact value** of the following definite integral. 
\[ \int_{0}^{2} 2e^{x+5} \, dx. \]

A. \(2e^{7} - 2e^{5}\)  
B. \(2e^{7}\)  
C. \(\frac{e^{7}}{2} - \frac{e^{5}}{2}\)  
D. 1896.43999  
E. None of these.

\[ \int_{0}^{2} 2e^{x+5} \, dx = 2e^{x+5}\bigg|_{0}^{2} = 2e^{7} - 2e^{5} \]

2. Calculate **without calculator** the **exact value** of the following definite integral. 
\[ \int_{1}^{3} \frac{4}{x} \, dx. \]

A. \(-\frac{32}{9}\)  
B. 4.39444  
C. \(\ln 12\)  
D. \(4 \ln 3\)  
E. None of these.

\[ \int_{1}^{3} \frac{4}{x} \, dx = 4 \ln x\bigg|_{1}^{3} = 4 \ln 3 - 4 \ln 1 = 4 \ln 3 \text{ since } \ln 1 = 0 \]

3. Decide whether or not the given integral converges.

\[ \int_{1}^{\infty} \frac{8}{x^2} \, dx \]

If it converges determine the value it converges to.

A. \(\frac{8}{3}\)  
B. Diverges  
C. 8  
D. 4  
E. None of these.

\[ \int_{1}^{\infty} \frac{8}{x^2} \, dx = \lim_{t \to \infty} \int_{1}^{t} \frac{8}{x^2} \, dx = \lim_{t \to \infty} \left( -\frac{8}{x} \right)_{1}^{t} = \lim_{t \to \infty} \left( -\frac{8}{t} + 8 \right) = 8 \]
4. Use integration by **substitution** to evaluate the integral \( \int x(x^2 + 1)^{10} \, dx \).

A. \( xu^{10} \frac{1}{2x} \, du + C \)
B. \( \frac{x^2}{2} \left( \frac{x^3}{3} + x \right)^{10} \, + C \)
C. \( \frac{1}{22} \cdot (x^2 + 1)^{11} \, + C \)
D. \( \frac{1}{22} u^{11} \, + C \)
E. None of these.

\[
\begin{align*}
t &= x^2 + 1, \quad du = 2xdx, \quad \frac{du}{2x} = dx \\
\int x(x^2 + 1)^{10} \, dx &= \int xu^{10} \, \frac{du}{2x} = \frac{1}{2} \int u^{10} \, du = \frac{1}{2} \cdot \frac{1}{11} u^{11} + c = \frac{1}{22} (x^2 + 1)^{11} + c
\end{align*}
\]

5. A ball thrown in the air has a velocity of \( v(t) = 94 - 32t \) feet/sec. Which formula would calculate the displacement of the ball between times \( t = 1 \) second and \( t = 6 \) seconds.

A. \( \int_{1}^{6} (94 - 32t) \, dt \)  
B. \( \int_{1}^{6} (94t - 16t^2) \, dt \)  
C. \( \frac{1}{5} \int_{1}^{6} (94 - 32t) \, dt \)  
D. \( \frac{1}{5} \int_{1}^{6} (94t - 16t^2) \, dt \)  
E. None of these.

6. Evaluate the following integral \( \int \left( \frac{2}{x^2} - 5\sqrt{x} \right) dx \)

A. \( -4x^{-3} - 2.5x^{-0.5} + C \)
B. \( \frac{2}{x^3} - 5x^{1.5} + C \)
C. \( 2 \ln |x^2| - 5x^{1.5} \frac{1}{1.5} + C \)
D. \( -\frac{2}{x} \cdot 5x^{1.5} \frac{1}{1.5} + C \)
E. None of these.

\[
\int \left( \frac{2}{x^2} - 5\sqrt{x} \right) dx = \int (2x^{-2} - 5x^{0.5}) \, dx = -\frac{2}{x} - 5x^{1.5} \frac{1}{1.5} + C
\]
II. Free Response.
Write your final answers in the spaces provided. Show your work and circle your answer. Remember to include units where applicable.

7. Calculate the left Riemann sum for the given function over the given interval, using the given value of $n$. Show your work.

$$f(x) = 3x^2 + 2x - 3$$ over $[1, 3]$ with $n = 5$.

$$\Delta x = \frac{3 - 1}{5} = 0.4$$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$x_0 = 1$</th>
<th>$x_1 = 1.4$</th>
<th>$x_2 = 1.8$</th>
<th>$x_3 = 2.2$</th>
<th>$x_4 = 2.6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>$f(1) = 2$</td>
<td>$f(1.4) = 5.68$</td>
<td>$f(1.8) = 10.32$</td>
<td>$f(2.2) = 15.92$</td>
<td>$f(2.6) = 22.48$</td>
</tr>
</tbody>
</table>

$$LRS = \Delta x(f(x_0) + f(x_1) + f(x_2) + f(x_3) + f(x_4)) = 0.4(2 + 5.68 + 10.32 + 15.92 + 22.48) = 22.56$$

8. Algebraically find the area enclosed by the curves $y = -x^2 + 6x + 2$ and $y = 2x^2 + 9x - 4$. Hint: graph the functions. Show your work.

Find the point of intersections:

$$-x^2 + 6x + 2 = 2x^2 + 9x - 4$$
$$0 = 3x^2 + 3x - 6$$
$$0 = 3(x + 2)(x - 1)$$
$$x = -2 \text{ and } x = 1$$

$$\int_{-2}^{1}((-x^2 + 6x + 2) - (2x^2 + 9x - 4)) \, dx =$$

$$\int_{-2}^{1}(-3x^2 - 3x + 6) \, dx = -x^3 - \frac{3}{2}x^2 + 6x \bigg|_{-2}^{1} =$$

$$\left(-1^3 - \frac{3}{2}1^2 + 6 \cdot 1\right) - \left(-(-2)^3 - \frac{3}{2}(-2)^2 + 6 \cdot (-2)\right) = 13.5$$
9. The rate at which the revenue earned by a company for fiscal years 2004 through 2010 can be approximated by $R(t) = 5000 + 200t$ million dollars per year, $0 \leq t \leq 6$, where $t$ is the time in years and $t = 0$ represents the beginning of fiscal year 2004.

a. Write a definite integral formula which would calculate the total revenue from the beginning of fiscal year 2008 to the beginning of fiscal year 2010.

$$\int_{4}^{6} (5000 + 200t)\,dt$$

b. Calculate the total revenue from the beginning of fiscal year 2008 to the beginning of fiscal year 2010. Show your work.

$$\int_{4}^{6} (5000 + 200t)\,dt = 5000t + 100t^2 \bigg|_{4}^{6} =$$

$$(5000 \cdot 6 + 100 \cdot 6^2) - (5000 \cdot 4 + 100 \cdot 4^2) = 12000$ dollars.$$
10. Use integration by parts to evaluate the integral \( \int (-2x + 1)e^{-x} \, dx \). Show your work. Recall: \( \int uv = uv - \int vu \).

\[
\begin{align*}
  u &= -2x + 1 & dv &= e^{-x} \, dx \\
  du &= -2 \, dx & v &= -e^{-x} \\
  \int (-2x + 1)e^{-x} \, dx &= (-2x + 1)(-e^{-x}) - \int (-e^{-x})(-2 \, dx) = \\
  &= (-2x + 1)(-e^{-x}) - 2 \int e^{-x} \, dx = (-2x + 1)(-e^{-x}) + 2e^{-x} + C = e^{-x}(2x + 1) + C
\end{align*}
\]

11. The marginal cost of producing the xth box of tissues is

\( M(x) = 8 + x^2 \) dollars. The cost of producing \( x = 3 \) boxes is $10000. Write the function \( C(x) \) which gives you the total cost for producing \( x \) boxes of tissues.

\[
\begin{align*}
  C(x) &= \int (8 + x^2) \, dx = 8x + \frac{x^3}{3} + B \\
  C(3) &= 10000. \text{ Thus, } C(3) = 8 \cdot 3 + \frac{3^3}{3} + B = 10000 \text{ and solve for } B. \\
  33 + B &= 1000, B = 9967 \\
  C(x) &= 8x + \frac{x^3}{3} + 9967
\end{align*}
\]