MAT 210

Sample Exam 2

Instructor:

NAME: (PRINT!)

SOLUTION

Honor Statement:
By signing below you confirm that you have neither given nor received any unauthorized assistance on this exam. This includes any use of a graphing calculator beyond those uses specifically authorized by the Mathematics Department and your instructor. Furthermore, you agree not to discuss this exam with anyone until the exam testing period is over and this exam has been returned to you.

_____________________________________________                _________________
Signature                                                                                   Date

Test Information:
• This test has 11 questions worth 100 points total.
• Multiple choice are x pts each, Free response are as marked
• You have 50 minutes from the start of the exam in which to complete the items.
• For the 6 multiple choice questions, list your answers in the answer grid following #11.
• For free response questions, show all work in the area provided by the questions.
• Do not round or approximate your answers unless otherwise indicated.

• You may use a calculator so long as it does NOT perform symbolic algebra. For example, the TI-89 or TI-nSpire CAS are not permitted.
• Sharing of calculators is not permitted
• Cell phones may not be accessed during the exam and must be turned to silent
• The use of bilingual dictionaries is permitted on this test.
1. The graph of the function \( y = f(x) \) is given below. At what x-value(s) does it appear that \( f'(x) = 0 \)?

   A. \( f'(-1) = 0 \) and \( f'(1) = 0 \)
   B. Only \( f'(-1) = 0 \)
   C. Only \( f'(1) = 0 \)
   D. Only \( f'(0) = 0 \)
   E. There is no x-value for which \( f'(x) = 0 \).

2. The graph of the function \( y = f(x) \) is given below. The function \( f(x) \) appears to have a point of inflection

   A. At \( x = -5 \) and \( x = 5 \)
   B. Only at \( x = -5 \)
   C. Only at \( x = 5 \)
   D. Only at \( x = 0 \)
   E. \( f(x) \) does not have a point of inflection.

3. Determine the x-coordinate of all points of inflection \( f(x) = \frac{x^3}{3} - 4x \) on \([-5, 5]\).

   A. \( x = 2 \)    B. \( x = -2 \)    C. \( x = -2 \) and \( x = 2 \)    D. \( x = 0 \)    E. There are no points of inflection.

   \( f'(x) = x^2 - 4 \) and \( f''(x) = 2x. f''(x) = 2x = 0 \) when \( x = 0 \).

   Test points: when \( x = -1, f(-1) = -2 < 0 \). Thus \( f(x) \) is concave down on \((-\infty, 0)\).
   When \( x = 1, f(1) = 2 > 0 \). Thus \( f(x) \) is concave up on \((0, \infty)\).

   The concavity changes at \( x = 0 \).
4. A car’s distance from the intersection, in feet, is given by the function $s(t) = \sqrt{t} + 4t^2$, where $t$ measures time in seconds since the car left the intersection. Which of the following functions of $t$ gives the car’s acceleration in ft/sec$^2$ as function of time $t$?

A. $a(t) = \frac{1}{2} t^{-1/2} + 8t$
B. $a(t) = -\frac{1}{4} t^{-3/2} + 8t$
C. $a(t) = -\frac{1}{4} t^{-3/2} + 8$
D. $a(t) = \frac{1}{4} t^{-3/2} + 8$
E. None of these

$s(t) = \sqrt{t} + 4t^2 = t^{1/2} + 4t^2$, $v(t) = s'(t) = \frac{1}{2} t^{-1/2} + 8t$, $a(t) = v'(t) = s''(t) = -\frac{1}{4} t^{-3/2} + 8$

5. Harry, Ginny, Hermione, Ron and Neville would like to create a rectangular vegetable patch to grow mandrake. The fencing for the east and west sides costs $4 per foot, and the fencing for the north and south sides costs only $2 per foot. They have a budget of $80 for the project and they want to largest possible area they can enclose to grow mandrake. Harry would like to use magic to create the garden but Hermione insists on using calculus. She said let the variable $x$ denote the length of the east and west sides of the garden and let the variable $y$ denote the length of the north and south sides of the garden. The friends individually set up a solution for the optimization problem using these variables. Who set up the problem properly?

A. Harry: Maximize the area $8x + 4y$ subject to the constraint $32xy = 80$.
B. Ginny: Maximize the area $xy$ subject to the constraint $8x + 4y = 80$
C. Hermione: Maximize the area $32xy$ subject to the constraint $8x + 4y = 80$
D. Ron: Maximize the area $8xy$ subject to the constraint $4x + 2y = 80$
E. Neville: Maximize the area $4x + 2y$ subject to the constraint $8xy = 80$.

The length of the east and west side is $x$ feet and the length of the north and south side is $y$ feet. Thus, the area $A$ of the rectangular garden is $A = xy$. This area needs to be the largest possible to subject to the cost constraint $80$ for the fencing.

The cost of the fencing of the east side is $4x$ dollars and the cost of the west side fencing is $4x$ dollars too. The cost of the fencing of the north side is $2y$ dollars and the cost of the south side fencing is $2y$ dollars too. Thus, the total cost the fencing can be represented by the following equation $80 = 8x + 4y$.

6. Assume the price elasticity of demand $E = 1.02$, when the unit price of the product is set at $p = $25. Interpret the meaning of this result.

A. When the price of the item is $25 the demand is increasing 1.02% per 1% increase in unit price.
B. When the price of the item is $25 the demand is dropping 1.02% per 1% increase in unit price.
C. The unit price of the item is dropping $25 per 1% increase in demand.
D. The demand is dropping 1.02% per every $25 increase in unit price.
E. None of the above.
II. Free Response.

Write your final answers in the spaces provided. Show your work. Remember to include units where applicable.

7. You are running a small business selling homemade bread. You know that your weekly revenue from the sale of \( q \) loaves bread is \( R(q) = 68q - 0.1q^2 \). Assuming your weekly costs of producing \( q \) loaves of bread is given by the function \( C(q) = 23 + 20q \).

a. Write the weekly profit function as a function of \( q \) the number of loaves of bread.

Profit = Revenue - Cost

\[
P(q) = (68q - 0.1q^2) - (23 + 20q) = -0.1q^2 - 48q - 23 \text{ dollars}
\]

b. How many loaves of bread do you have to sell to maximize the weekly profit?

\[
P'(q) = -0.2q + 48, \ -0.2q + 48 = 0 \text{ when } q = 240 \text{ loaves of bread.}
\]

First derivative test:

\[
P'(239) = 0.2 > 0. \text{ The profit function increases when } q < 240.
\]

\[
P'(241) = -0.2 < 0. \text{ The profit function decreases when } q > 240.
\]

Therefore the profit function has the abs. max when \( q = 240 \).

240 loaves of bread do you have to sell to maximize the weekly profit.

c. Find the maximum profit.

Maximum profit is \( P(240) = -0.1 \cdot 240^2 - 48 \cdot 240 - 23 = 5737 \text{ dollars} \)
Consider the function \( y = f(x) = -2.495x^3 + 7.485x + 5.01 \) on the interval \([-1.9, 1.99]\).

a. Algebraically find the \( x \) and \( y \) coordinates of all the critical points of \( f(x) \).

\[
f'(x) = -7.485x^2 + 7.485
= -7.485x^2 + 7.485 = -7.485(x^2 - 1) = -7.485(x + 1)(x - 1) = 0 \text{ when } x = -1 \text{ or } x = 1
\]

Critical points: \( x = -1, y = 0.02 \) and \( x = 1, y = 10 \)

b. Algebraically find the exact location of all relative and absolute extrema of \( f(x) \) on the interval \([-1.9, 1.99]\).

Check the end points and the critical points.

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-1.9)</th>
<th>(-1)</th>
<th>(1)</th>
<th>(1.99)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = f(x) )</td>
<td>(7.9017)</td>
<td>(0.02)</td>
<td>(10)</td>
<td>(0.24306)</td>
</tr>
<tr>
<td>Classification</td>
<td>Relative maximum</td>
<td>Absolute minimum</td>
<td>Absolute maximum</td>
<td>Relative minimum</td>
</tr>
</tbody>
</table>

It is extremely important to apply your calculus knowledge when you solve optimization problem and not just rely on the graph that your graphing calculator shows. For example, on the graph it seems like the absolute minimum is at \((-1,0)\) but the absolute minimum is at \((-1, 0.02)\).
9. The weekly sales of ASU “I love MAT 210” t-shirts is given by the function \( q = 1080 - 18p \), where the variable \( q \) represents the number of t-shirts sold when the unit price is \( p \).

a) Calculate the price elasticity of demand when the price is $20 per t-shirt. 

Recall \( E = -\frac{dq}{dp} \cdot \frac{p}{q} \) (Round to 2 decimal places)

\[
\frac{dq}{dp} = -18. \text{ Thus, } E = -(-18) \cdot \frac{p}{1080-18p} = \frac{18p}{1080-18p}
\]

When \( p = 20 \) dollars \( E = \frac{18 \cdot 20}{1080-18 \cdot 20} = 0.5 \) (no units)

b) Is the demand at the price \( p = 20 \) elastic or inelastic? ELASTIC INELASTIC

(Circle the correct answer)

\( 0.5 < 1. \)

c) What would you need to do to increase revenue? LOWER RAISE

(Circle the correct answer)

d) What price for a t-shirt will maximize revenue? Round to the nearest cent.

Solve \( E = 1. \) \( \frac{18p}{1080-18p} = 1, \) \( 18p = 1080 - 18p, \) \( 36p = 1080, \) \( p = \frac{1080}{36} = 30 \) dollars.
10. The average cost function for the weekly manufacture of portable CD players is given by

\[ C(x) = 180000x^{-1} + 50 + 0.0006x \]
dollars per player, where \( x \) is the number of CD players manufactured that week. Weekly production is currently \( x = 5,000 \) players and is increasing at a rate of 300 players per week. Let the variable \( t \) represent the time in weeks. At what rate does the average cost change with respect to time when the weekly production is \( x = 5000 \) players?

Include units.

Use the chain rule to differentiate the average cost function with respect the time \( t \).

\[
\frac{dC}{dt} = \frac{dC}{dx} \cdot \frac{dx}{dt} = (-180000x^{-2} + 0.0006) \cdot 300, \text{ where } \frac{dC}{dx} = -180000x^{-2} + 0.0006 \text{ (dollars per player per player)} \text{ and } \frac{dx}{dt} = 300 \text{ (players per week)}.\]

When the weekly production is \( x = 5000 \) players, then \( \frac{dC}{dt} = (-180000 \cdot 5000^{-2} + 0.0006) \cdot 300 = -1.98 \text{ dollars per player per week}.\)

When the weekly production is \( x = 5000 \) players, then the average cost of portable CD players decreasing at a rate of \( -1.98 \text{ dollars per player per week}.\)

11. Find the derivative of the following functions.

a) \( f(x) = 5e^{3x^2-6x} \)

\[ f'(x) = 5e^{3x^2-6x}(6x - 6) = 30e^{3x^2-6x}(x - 1) \]

b) \( f(x) = \ln|3x^3 - 5x^2 + 4| \)

\[ f'(x) = \frac{9x^2 - 10x}{3x^3 - 5x^2 + 4} \]

c) \( 5x^2y - y^2 = 17x \). Find \( \frac{dy}{dx} \).

hint: use implicit differentiation.

\[
10xy + 5x^2 \cdot \frac{dy}{dx} - 2y \cdot \frac{dy}{dx} = 17, \quad 5x^2 \cdot \frac{dy}{dx} - 2y \cdot \frac{dy}{dx} = 17 - 10xy, \quad (5x^2 - 2y) \cdot \frac{dy}{dx} = 17 - 10xy
\]

\[ \frac{dy}{dx} = \frac{17 - 10xy}{5x^2 - 2y} \]