MAT 210  

Sample Exam 1

Instructor:

**NAME: (PRINT!!) SOLUTION**

**Honor Statement:**
By signing below you confirm that you have neither given nor received any unauthorized assistance on this exam. This includes any use of a graphing calculator beyond those uses specifically authorized by the Mathematics Department and your instructor. Furthermore, you agree not to discuss this exam with anyone until the exam testing period is over and this exam has been returned to you.

_____________________________________________                _________________
Signature                                          Date

Test Information:
- This test has 12 questions worth 100 points total.
- Multiple choice are x pts each, free response are as marked.
- You have 50 minutes from the start of class in which to complete the exam.
- For the 6 multiple choice questions, list your answers in the answer grid at the end of the exam.
- For free response questions, show all work in the area provided by the questions.
- Do not round or approximate your answers unless otherwise indicated.

- You may use a calculator so long as it does NOT perform symbolic algebra. **For example, the TI-89 or TI-nSpire CAS are not permitted.**
- Sharing of calculators is not permitted
- Cell phones *may not* be accessed during the exam and must be turned to silent
- The use of bilingual dictionaries is permitted on this test.
I. Multiple Choice.  List your answers on the next sheet in the provided grid.

1. Calculate the following limits: 1. $\lim_{x \to \infty} \frac{2x^2-3}{7x^2-x-2}$ and 2. $\lim_{x \to 1} \frac{2x^2-2}{x^2+x-2}$, DNE means does not exist.
   A. 1. $\frac{3}{2}$ and 2. 2   B. 1. $\frac{2}{7}$ and 2. $\frac{4}{3}$   C. 1. DNE and 2. $\frac{4}{3}$   D. 1. $\infty$ and 2. $-1$
   E. None of these

   1. $\lim_{x \to \infty} \frac{2x^2-3}{7x^2-x-2} = \lim_{x \to \infty} \frac{2 - \frac{3}{x^2}}{7 - \frac{1}{x}} = \frac{2}{7}$, as $x \to \infty$ each purple term approaches 0.
   2. $\lim_{x \to 1} \frac{2x^2-2}{x^2+x-2} = \lim_{x \to 1} \frac{2(x-1)(x+1)}{(x-1)(x+2)} = \lim_{x \to 1} \frac{2(x+1)}{x+2} = \frac{4}{3}$

2. The following curve is a model of a certain country's total prison population as a function of time in years ($t = 0$ represents 2000). Using the graph, which of the following statements is true?

   A. Over the period [5, 10] the instantaneous rate of change of $N$ is increasing
   B. The instantaneous rate of change of prison population at $t = 4$ was less than the average rate of change over the interval [0, 10].
   C. Over the period [0, 10] the instantaneous rate of change of $N$ is decreasing, then increasing.
   D. The instantaneous rate of change of prison population at $t = 4$ was greater than the average rate of change over the interval [0, 10].
   E. None of these statements are true.
The distance, \( s \) (in feet), traveled by a car moving in a straight line is given by the function, \( s(t) = t^2 + t \), where \( t \) is measured in seconds. Use this problem to answer question 3 and 4.

\( s(1) \) represents the distance traveled by the car from its initial position after 1 second.

\( s(4) \) represents the distance traveled by the car from its initial position after 4 seconds.

\( \Delta s = s(4) - s(1) \) represents the distance traveled by the car as the time changes from \( t = 1 \) seconds to \( t = 4 \) seconds, during the time \( \Delta t = 4 - 1 = 3 \) seconds. The average velocity during this 3 second time interval is

\[
\frac{\Delta s}{\Delta t} = \frac{s(4) - s(1)}{4 - 1} = \frac{20 - 2}{3} = 6
\]

3. Find the **average velocity** for the time period from \( t = 1 \) to \( t = 4 \).

A. 5 ft/sec  
B. 6 ft/sec  
C. 9 ft/sec  
D. 10 ft/sec  
E. None of these

4. Find the **velocity** of the car at \( t = 4 \) second. The velocity of the car at the moment \( t = 4 \) second is the instantaneous rate of change of the distance function with respect to the time when \( t = 4 \). \( s'(t) = 2t + 1 \), \( s'(4) = 9 \)

A. 20 ft/sec  
B. 5 ft/sec  
C. 9 ft/sec  
D. 2 ft/sec  
E. None of these

5. Suppose the function \( P(t) \) represents the daily oil production by Pemex in million barrels, \( 1 \leq t \leq 9 \), where \( t \) is time in years since the start of 2000. Assume \( P(2) = 3 \) and \( P'(2) = 0.112 \). Interpret the meaning of \( P(2) = 3 \) and \( P'(2) = 0.112 \).

A. In 2002 the daily oil production is 3 million barrels and the daily oil production is increasing 0.112 million barrels in every 2 years after 2002.
B. In 2002 the price of selling a barrel of oil is 3 million dollars and this is increasing at a rate of 0.112 million dollars per barrel.
C. In 2002 the daily oil production is 3 million barrels and the daily oil production is increasing at a rate of 0.112 million barrels per year.
D. We can only conclude that the daily oil production is 3 million barrels and \( P'(2) = 0.112 \) does not give us any further information.
E. None of these

6. The cost of producing \( x \) copies of an edition of your newspaper is given by \( C(x) = 30 + 0.10x + 0.001x^2 \) dollars. Calculate the marginal cost if you have produced and sold 500 copies.

A. 1.1 dollars per additional day  
B. 1.1 dollars per additional copy  
C. 1.1 dollars  
D. 1.1 dollars per additional 500 copies  
E. None of these

Note that the marginal cost is the derivative of the cost function \( C(x) \) **with respect to the number of items \( x \)**. **(not with respect to time.)**
Answer grid for multiple choice answers is on the last page of the exam.

II. Free Response. Write your final answers on the lines provided. Show work. Remember units.

1. Use the graph of function $f(x)$ to determine the requested limit values. Write DNE if the value does not exist.

- a) $\lim_{x \to 2} f(x) =$ __________ 2 __________
- b) $\lim_{x \to 2} f(x) =$ ____DNE______
- c) $f(2) =$ __________ 0 __________
- d) $\lim_{x \to 4} f(x) =$ ______ 2 ______
- e) Is the function $f(x)$ continuous at $x=4$. YES or NO

Because $\lim_{x \to 4} f(x) = 2$ and $f(4) = 3$. The limit at 4 is not the same as the function value at 4.

2. The following graph shows the approximate number (in thousands) of people who emigrated from Ireland during the period 2006–2010

- a. Calculate the average rate of change of people who emigrated from Ireland during the period 2006-2010. Include units.

$$\Delta E = 30 - 38 = \frac{-8}{4} = -2.$$ That is $-2,000$ people/year.

- b. Interpret your answer obtained in part a. Between 2006 and 2010 the number of people emigrated for Ireland decreasing at an average rate of 2,000 people per year.

Wrong Interpretation: “The average rate of change of people who emigrated from Ireland is decreasing 2000 people per year between 2006-2010”. The average rate of change is not changing, that is always -2,000 people/year between 2006-2010. It means that the average of the yearly change of the number of people who emigrated from Ireland between 2006-2010 is 2,000 people per year. The average of the yearly change between 2006-2010 is $\frac{2+5+15+(-30)}{4} = -2$, -2,000 people per year.
Wrong Interpretation: “The average rate of change of people who emigrated from Ireland is 2000 people per year between 2006-2010”. This is a self-referral explanation, which does not interpret the result. For example: explain “what an apple is”. You cannot say that “apple is an apple”.

3. Determine the following derivative functions. Show your work. **You need not simplify your results.** (5 pts each)
   
   a) \( f(x) = \frac{2}{\sqrt[3]{x}} - \frac{1}{x} = 2x^{-\frac{1}{2}} - x^{-1} \)
   
   \( f'(x) = -x^{-\frac{3}{2}} + x^{-2} = \frac{-1}{x\sqrt{x}} + \frac{1}{x^2} \)

   b) \( g(x) = (2x^3 + 5x^2)(x^2 - 3.1) \)
   
   Use the product rule.

   Let \( h(x) = 2x^3 + 5x^2 \) and \( k(x) = x^{-2} - 3.1 \)

   \( h'(x) = 6x^2 + 10x \) and \( k'(x) = (-2)x^{-3} \)

   \( g'(x) = h'(x)k(x) + h(x)k'(x) = (6x^2 + 10x)(x^{-2} - 3.1) + (2x^3 + 5x^2)(-2)x^{-3} \)

   c) \( h(x) = \frac{2x^2 - 3x + 5}{x - 1} \)
   
   Use the quotient rule.

   Let \( f(x) = 2x^2 - 3x + 5 \) and \( g(x) = x - 1 \)

   \( f'(x) = 4x - 3 \) and \( g'(x) = 1 \)

   \( h'(x) = \frac{(4x - 3)(x - 1) - 2x^2 - 3x + 5}{(x - 1)^2} = \frac{2x^2 - 4x + 2}{(x - 1)^2} \)

4. Find the equation of the tangent line to the graph of \( y = f(x) = 3x^2 + 5 \) at the point \( x = 1 \). Show your work.

   A point on the tangent line: \( x = 1 \) and \( y = f(1) = 8 \).

   Find the derivative of \( f(x) \). \( f'(x) = 6x \).

   The slope of the tangent line at \( x = 1 \): \( f'(1) = 6 \).

   Equation of the tangent line: \( y - 8 = 6(x - 1) \)

   Slope intercept form: \( y = 6x + 2 \)

5. Let \( f(x) = 2x^2 - 3x + 7 \). Use the limit definition of the derivative, \( \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} \), to find the derivative \( f'(x) \).

   \[ \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h \to 0} \frac{[2(x+h)^2 - 3(x+h) + 7] - [2x^2 - 3x + 7]}{h} = \lim_{h \to 0} \frac{[2x^2 + 4xh + 2h^2 - 3x - 3h + 7 - 2x^2 + 3x - 7]}{h} = \lim_{h \to 0} \frac{4xh + 2h^2 - 3h}{h} = \lim_{h \to 0} (4x + 2h + 3) = 4x - 3 \]
6. Your business’ total cost for producing a newspaper is given by \( C(x) = 50 + 0.15x + 0.001x^2 \), where \( x \) is the number of copies produced and cost is given in dollars ($). The revenue from selling \( x \) copies of this newspaper is \( R(x) = 0.75x \).

a) Write a function \( P(x) \) for your profit producing and selling \( x \) copies of this newspaper.

\[
P(x) = R(x) - C(x) = 0.75x - (50 + 0.15x + 0.001x^2) = -50 + 0.6x - 0.001x^2
\]

b) Calculate \( P(100) \). Include units.

\[
P(100) = -50 + 0.6 \cdot 100 - 0.001 \cdot 100^2 = 0 \text{ dollars.}
\]

When 100 copies are sold, there is no profit. (Break-even)

c) Write a function for your marginal profit.

\[
P'(x) = 0.6 - 0.002x
\]

d) Calculate your marginal profit if you produce and sell 100 copies of the newspaper. Include units.

\[
P'(100) = 0.6 - 0.002 \cdot 100 = 0.4
\]

0.4 dollars per copy or 40 cents per copy.

e) Interpret the meaning of your result obtained in part d)

When 100 copies are sold there is no profit but the profit is increasing at a rate of 40 cents per additional copy sold. The approximate profit from the sale of the 101st copy is 40 cents.