REAL ANALYSIS
QUALIFIER EXAMINATION SYLLABUS
MAT 570 – MAT 571

Topics:

- Metric space analysis:
  - metric spaces, open sets, closed sets
  - sequences and subsequences, convergence
  - compactness, completeness, connectedness
  - continuity, uniform continuity
  - pointwise convergence, uniform convergence
  - normed vector spaces, bounded linear maps
  - Arzelà-Ascoli theorem
  - Stone-Weierstrass theorem
- Measure and integration:
  - σ-algebras, Borel sets, measures
  - Cantor set, Cantor function, fat Cantor sets
  - Carathéodory's theorem
  - Lebesgue-Stieltjes measures, Lebesgue measure
  - existence of nonmeasurable sets
  - measurable functions
  - elementary properties of integration
  - Monotone and Dominated Convergence theorems, Fatou's lemma,
  - convergence almost everywhere, in measure, in L^p
  - product measures, Tonelli and Fubini theorems
  - real and complex measures
  - Radon-Nikodym and Lebesgue Decomposition theorems
  - bounded variation, absolute continuity
  - Hölder and Minkowski inequalities, L^p spaces and duality

References:

- G. Folland, *Real Analysis*, chapters 0–3, parts of sections 4.6–4.7, 5.1–5.2, 6.1–6.2
- W. Rudin, *Principles of Mathematical Analysis*, chapters 1–8

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