GRAPH THEORY QUALIFYING EXAM
SPRING 2017

Directions. Solve the five problems below. Assume that all graphs are simple.

(1) (Thomassen) Prove that every planar graph is 5-choosable.

(2) (König-Egerváry) Show that in a bipartite graph the size of the maximum matching is equal to the size of the minimum vertex cover.

(3) (Euler's Formula) Prove that a connected, plane multigraph with \( n \) vertices, \( m \) edges, and \( f \) faces satisfies \( n - m + f = 2 \).

(4) Let \( G \) be a graph with \( n \) vertices. Let \( u, v \) be distinct non-adjacent vertices of \( G \) with \( d(u) + d(v) \geq n \). Show that \( G \) is Hamiltonian if and only if \( G + uv \) is Hamiltonian.

(5) Prove that every bridgeless cubic graph has a 1-factor.