1. (a) In how many ways may we pass out \( k \) distinct pieces of candy to \( n \) children so that each gets at most one? (Assume \( k \leq n \).)

(b) In how many ways may we pass out \( k \) identical pieces of candy to \( n \) children so that each gets at least one? (Assume \( k \geq n \).)

(c) In how many ways may we pass out \( k \) identical pieces of candy to \( n \) children so that each gets at most one? (Assume \( k \leq n \).)

(d) In how many ways may we pass out \( k \) distinct pieces of candy to \( n \) children so that each gets at least one? (Assume \( k \geq n \).)

2. Given a sequence of \( p \) integers \( a_1, a_2, \ldots, a_p \), show that there exist consecutive terms in the sequence whose sum is divisible by \( p \). That is, show that there are \( i \) and \( j \), with \( 1 \leq i \leq j \leq p \), such that \( a_i + a_{i+1} + \cdots + a_j \) is divisible by \( p \).

3. Let \( S(n, k) \) be the Stirling numbers of the 2nd kind.

(a) Show that \( S(n + 1, k + 1) = \sum_{j=k}^{n} S(j, k) \binom{n}{j} \).

(b) Show that \( S(n + 1 + k, k) = \sum_{j=0}^{k} j S(n + j, j) \).

4. (a) Find the number of \( k \)-element subsets of \( \{2, \ldots, 2n\} \) that contain no consecutive integers.

(b) Consider colorings of \( \{1, \ldots, 2n\} \) with red and blue with the property that if \( i \) is red then \( i - 1 \) is red to show:

\[
\sum_{k=0}^{n} (-1)^k \binom{2n-k}{k} 2^{2n-2k} = 2n + 1.
\]

5. Let \( Q \) be the 3-dimensional cube with faces \( F \) and let \( G = G(F) \) be the symmetry group of \( F \) in \( \mathbb{R}^3 \).

(a) Find the cycle index of the group of \( G \).

(b) Find the number of distinct colorings of \( F \) with \( R, B, G \) such that \( R \) is used at most twice.