MAT 272

SPRING 2015

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TEST 3

SoMSS, ASU

Directions:

1. **There are 8 questions worth a total of 100 points.**
2. **Read all the questions carefully.**
3. **You must show all work in order to receive credit for the free response questions!!**
4. **When possible, box your answer, which must be complete, organized, and exact unless otherwise directed.**
5. **Always indicate how a calculator was used (i.e. sketch graph, etc. ...).**
6. **No calculators with QWERTY keyboards or ones like TI-89 or TI-92 that do symbolic algebra may be used.**

Honor Statement:

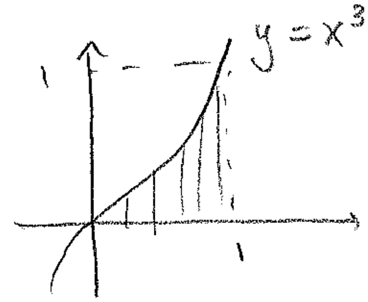
By signing below you confirm that you have neither given nor received any unauthorized assistance on this exam. This includes any use of a graphing calculator beyond those uses specifically authorized by the Mathematics Department and your instructor. Furthermore, you agree not to discuss this exam with anyone until the exam testing period is over. In addition, your calculator's program memory and menus may be checked at any time and cleared by any testing center proctor or Mathematics Department instructor.

______________________ Signature   ____________ Date

PRINT NAME: SOLUTION - Rodrigo Platte

RECITATION (TUESDAY OR THURSDAY):______________

1. **[15 pts]** Evaluate $\int_0^1 \int_{\sqrt[3]{y}}^1 e^{x^4}\, dxdy$.



Change order of integration.

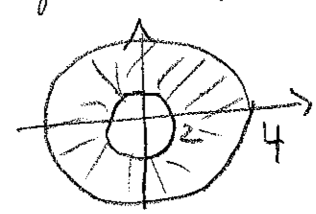
$$\int_0^1 \int_0^{x^3} e^{x^4} dy\,dx = \int_0^1 \left[y e^{x^4} \right]_0^{x^3} dx = \int_0^1 x^3 e^{x^4} dx$$

$u = x^4$

$$= \frac{1}{4} \int_0^1 e^u du = \boxed{\frac{1}{4}(e-1)}$$

2. **[15 pts]** Evaluate $\iiint_V (x^2 + y^2)\, dV$ where V is the volume **outside** of the cylinder $x^2 + y^2 = 4$, above the plane $z = 0$, and **inside** the paraboloid $z = 16 - x^2 - y^2$.

The paraboloid intersects the xy-plane when $16 = x^2 + y^2$. Hence the projected region of integration in the xy-plane is



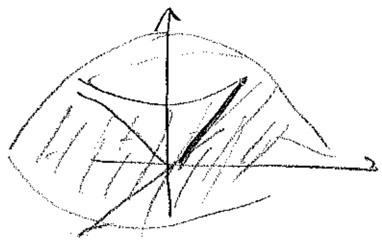
Using Cylindrical coordinates:

$$\int_0^{2\pi} \int_2^4 \underbrace{\int_0^{16-r^2} r^3\, dz}_{r^3(16-r^2)}\, dr\, d\theta = \int_0^{2\pi} \int_2^4 16r^3 - r^5\, dr\, d\theta$$

$$= 2\pi \left[4r^4 - \frac{1}{6} r^6 \right]_2^4 = 2\pi \left(4\, 4^4 - \frac{4^6}{6} - 4\, 2^4 + \frac{2^6}{6} \right)$$

$$= 2\pi (960 - 672) = \boxed{576\pi}$$

3. **[15 pts]** Find the volume of the solid **inside** the sphere $x^2 + y^2 + z^2 = 4$, **outside** the cone $z = \sqrt{x^2 + y^2}$, and **above** the plane $z = 0$ using a triple integral in spherical coordinates.



$$V = \int_0^{2\pi} \int_{\pi/4}^{\pi/2} \int_0^2 \rho^2 \sin\phi \, d\rho\, d\phi\, d\theta$$

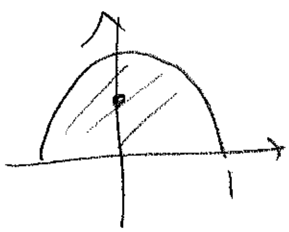
$$V = \frac{16}{3}\pi \int_{\pi/4}^{\pi/2} \sin\phi \, d\phi$$

$$= \frac{16}{3}\pi \left(-\cos\phi\right)\Big|_{\pi/4}^{\pi/2}$$

$$= \frac{16\pi}{3}\frac{\sqrt{2}}{2} = \boxed{\frac{8}{3}\sqrt{2}\,\pi}$$

4. **[15 pts]** Find the centroid (center of mass with density=1) of the upper half disk in $\mathbb{R}^2$ given by the set $\{x^2 + y^2 \le 1, y \ge 0\}$.

Because of symmetry we only need to find the y-component of the centroid.



$$\bar{y} = \frac{1}{m}\iint_R y \, dA = \frac{1}{m}\int_0^{\pi}\int_0^1 r^2 \sin\theta \, dr\, d\theta$$

$$= \frac{1}{m}\int_0^{\pi} \frac{1}{3}\sin\theta \, d\theta$$

$$= \frac{1}{m}\left[-\frac{\cos\theta}{3}\right]_0^{\pi} = \frac{1}{m}\frac{2}{3}$$

$m = \text{area} = \frac{\pi}{2}$, Hence the centroid is

$$\boxed{\left(0, \frac{4}{3\pi}\right)}$$

$\frac{4}{3\pi} \approx 0.4244$

5. **[12 pts]** Let V be the volume of the solid bounded by the cylinder $y = x^2$, the plane $z = 0$ and the plane $y + z = 2$ (sketched below). Then

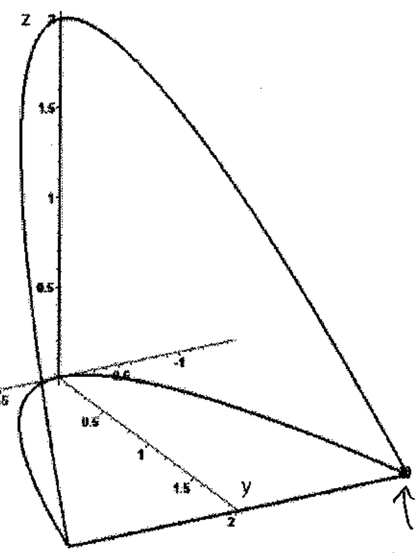
$$V = \int_0^2 \int_0^{2-z} \int_{-\sqrt{y}}^{\sqrt{y}} dx\, dy\, dz.$$

Change the order of integration

of this integral to $\int \int \int dz\, dy\, dx$.

Do not evaluate.

$$\int_{-\sqrt{2}}^{\sqrt{2}} \int_{x^2}^{2} \int_0^{2-y} dz\, dy\, dx$$



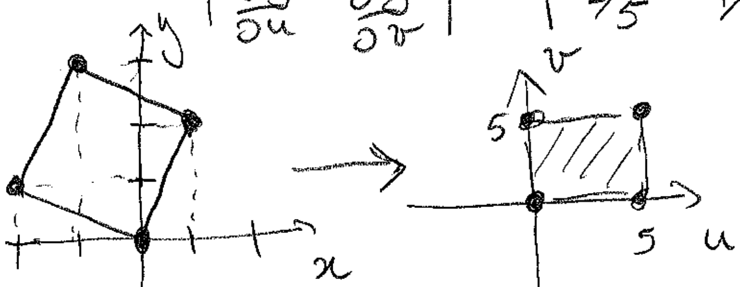
Intersection $y=2 \Rightarrow x = \pm\sqrt{2}$

6. **[12 pts]** Let R be the square with vertices (0,0), (-2,1) and (-1,3) and (1,2). Use the change of variables $u = x + 2y$, $v = y - 2x$ to rewrite the double integral $\iint_R (x+2y)(y-2x)\; dA$, as a double integral over a rectangle in the uv-plane. **Do not evaluate.**

$$u = x + 2y,\quad v = y - 2x \Rightarrow u - 2v = 5x \Rightarrow x = \frac{1}{5}(u - 2v)$$

$$2u + v = 5y \Rightarrow y = \frac{1}{5}(2u + v)$$

$$J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 1/5 & -2/5 \\ 2/5 & 1/5 \end{vmatrix} = \frac{1}{25}(1+4) = \boxed{\frac{1}{5}}$$



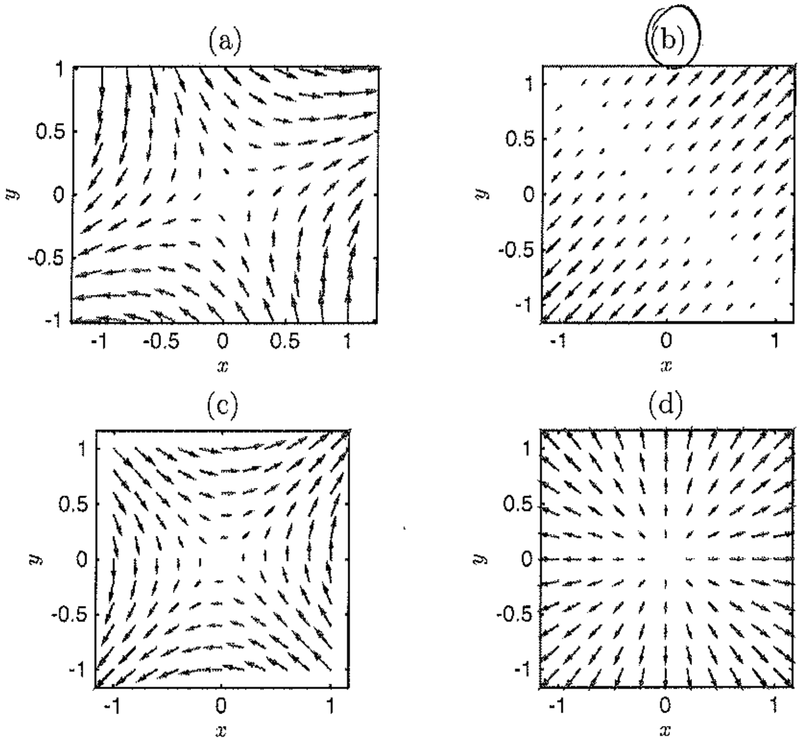
$$\int_0^5 \int_0^5 u\, v\, \frac{1}{5}\, du\, dv$$

(0,0) → (0,0) (-1,3) → (5,5)

(-2,1) → (0,5) (1,2) → (5,0)

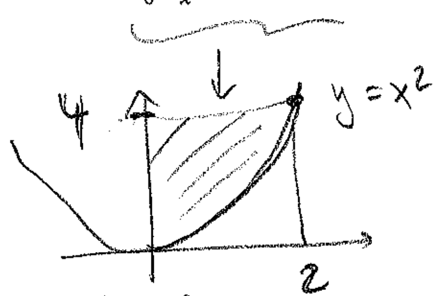
7. **[4 pts]** Which of the following is a graphical representation of the vector field

$\boldsymbol{F} = \langle x+y, x+y\rangle$?



8. **[12 pts]** State whether the following statements are TRUE or FALSE. If FALSE, correct the statement.

a. Reversing the order of integration on the double integral $\int_0^2 \int_{x^2}^4 f(x,y)\, dydx$ gives the double integral $\int_0^{x^2} \int_0^2 f(x,y)\, dxdy$.



False.

$$\int_0^2 \int_{x^2}^4 f(x,y)\, dydx = \int_0^4 \int_0^{\sqrt{y}} f(x,y)\, dxdy$$

b. Assuming g and f are integrable functions of one variable,

$$\int_0^1 \int_{-1}^2 f(x)g(y)dxdy = \left(\int_{-1}^2 f(x)dx\right)\left(\int_0^1 g(y)dy\right).$$

True: $\int_0^1 \int_{-1}^2 f(x)g(y)\,dx\,dy = \int_0^1 g(y) \underbrace{\int_{-1}^2 f(x)dx}\, dy$

$= \left(\int_{-1}^2 f(x)dx\right)\left(\int_0^1 g(y)dy\right)$

this is a number and can be moved out of the integral

c. In spherical coordinates, the set $\left\{(\rho, \varphi, \theta) : \theta = \frac{\pi}{4}\right\}$ is a cone.

False.

$\theta = \pi/4$ is a semi-plane.