NAME:______________________

MAT 271

Instructor

TEST 1

Directions: **This text has 13 questions worth a total of 100 points.**

- **Questions 1-10 are Multiple Choice worth 7 points each.**
  Please circle the correct letter.

- **Questions 11, 12, 13 are Free Response worth 10 points each.**
  For these questions:
  You must SHOW YOUR WORK.
  Box your answer.
  Leave your answer in exact form unless otherwise noted.

Read all the questions carefully. Always indicate how a calculator was used (i.e. sketch graph, etc. …). No calculators with QWERTY keyboards or ones like TI-89 or TI-92 that do symbolic algebra may be used.

Honor Statement:

By signing below you confirm that you have neither given nor received any unauthorized assistance on this exam. This includes any use of a graphing calculator beyond those uses specifically authorized by the Mathematics Department and your instructor. Furthermore, you agree not to discuss this exam with anyone until the exam testing period is over. In addition, your calculator's program memory and menus may be checked at any time and cleared by any testing center proctor or Mathematics Department instructor.

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Signature Date

MULTIPLE CHOICE

1. Evaluate

$$\int \frac{5x}{x^2+1}\,dx$$

a. $\frac{5}{2}\ln(x^2+1)+C$ b. $5\tan^{-1}x + C$ c. $\ln 5x - \ln(x^2+1)+C$

d. $\sqrt{x^2+1} + C$ e. None of these

2. Evaluate. $\int x^3 \ln(2x)\,dx$

a. $\frac{1}{4}x^4\ln(2x) - \frac{1}{4}x^3 + C$ b. $\frac{1}{2}x^3\ln(2x) - \frac{1}{16}x^3 + C$ c. $\frac{1}{4}\ln(2x) - \frac{1}{16}x^4 + C$

d. $\frac{1}{4}x^4\ln(2x) - \frac{1}{16}x^4 + C$ e. None of these

3. Find $\int \sin^4 x\,dx$ using a reduction formula:

Reduction Formulas

Assume n is a positive integer.

1. $\displaystyle\int \sin^n x\,dx = -\frac{\sin^{n-1}x\cos x}{n} + \frac{n-1}{n}\int \sin^{n-2}x\,dx$

2. $\displaystyle\int \cos^n x\,dx = \frac{\cos^{n-1}x\sin x}{n} + \frac{n-1}{n}\int \cos^{n-2}x\,dx$

3. $\displaystyle\int \tan^n x\,dx = \frac{\tan^{n-1}x}{n-1} - \int \tan^{n-2}x\,dx,\ n\neq 1$

4. $\displaystyle\int \sec^n x\,dx = \frac{\sec^{n-2}x\tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2}x\,dx,\ n\neq 1$

a. $-\frac{\sin^3 x\cos x}{4} - \frac{3\sin x\cos x}{8} + \frac{3}{8}x + C$ b. $-\frac{\sin^3 x\cos^2 x}{4} - \frac{3\sin^2 x\cos x}{8} + C$

c. $-\frac{\sin x\cos x}{4} - \frac{3\sin x\cos x}{8} + \frac{3}{8}x + C$ d. $-\frac{\sin^3 x\cos x}{4} + \frac{3\sin x\cos x}{8} - \frac{3}{8}x + C$

e. None of these

4. Evaluate

$$\int \sin^4 x \cos^3 x \, dx$$

a. $\frac{1}{5}sin^5x - \frac{1}{7}sin^7x + C$ b. $\frac{1}{7}cos^7x - \frac{1}{5}cos^5x + C$ c. $\frac{1}{35}sin^5x \, cos^7x + C$

d. . $\frac{1}{8}sin^8x - \frac{1}{6}sin^6x + C$ e. None of these

5. For what values of p is the following integral improper and convergent?

$$\int_0^1 \frac{1}{x^p} \, dx$$

a. $p \geq 1$ b. $0 < p < 1$ c. $p \leq 0$ d. $p < 1$ e. None of these

6. Evaluate

$$\int_0^3 \frac{3}{\sqrt{81-9x^2}} \, dx$$

a. π b. $\frac{\pi}{2}$ c. $\frac{\pi}{6}$ d. ∞ e. None of these

7. Evaluate $\int \tan^{11}x \; \sec^2 x \; dx$

a. $\frac{1}{14}\sec^{14}x - \frac{1}{12}\sec^{12}x + C$ b. $\frac{1}{14}\tan^{14}x + C$ c. $\frac{1}{14}\tan^{14}x + \frac{1}{12}\tan^{12}x + C$

d. $\frac{1}{12}\tan^{12}x + C$ e. None of these

8. The form of the partial fraction decomposition for $\int \frac{1}{(x-1)^2(x^2+1)} dx$ is

a. $\int \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+1} \, dx$ b. $\int \frac{Ax+B}{(x-1)^2} + \frac{Cx+D}{x^2+1} \, dx$

c. $\int \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{Cx}{x^2+1} \, dx$ d. $\int \frac{A}{x-1} + \frac{Bx+C}{(x-1)^2} + \frac{Dx+E}{x^2+1} \, dx$

e. None of these

9. What trig substitution would you use for

$$\int \frac{1}{\sqrt{x^2 - 25}}\,dx$$

?

a. $x = 5\tan\theta$ b. $x = 5\sec\theta$ c. $x = 5\sin\theta$

d. $x = 5\cos\theta$ e. None of these

10. Using the trig-substitution from Problem #9 above, the resulting trigonometric integral in its simplified form is:

a. $\int \sec\theta\, d\theta$ b. $\int 5\sec\theta\, d\theta$ c. $\int 5\tan\theta\, d\theta$

d. $\int 5\cot\theta\, d\theta$ e. None of these

Free Response

11. Determine whether the integral converges or diverges. Find the value of the integral if it converges.

$$\int_0^{\infty} xe^{-2x}\,dx$$

Answer:________________________________

12. Evaluate

$$\int \frac{1}{(x-5)(x^2+1)}\,dx$$

Answer:________________________________

13. Find $\int e^x \sin x \, dx$

Answer:________________________________

MAT 271 TEST 1 KEY

Multiple Choice (7 pts each, no partial credit.)

1. a. $\frac{5}{2}\ln(x^2+1)+C$

2. d. $\frac{1}{4}x^4\ln(2x)-\frac{1}{16}x^4+C$

3. a. $-\frac{\sin^3 x\cos x}{4}-\frac{3\sin x\cos x}{8}+\frac{3}{8}x+C$

4. a. $\frac{1}{5}sin^5 x-\frac{1}{7}sin^7 x+C$

5. b. $0<p<1$

6. b. $\frac{\pi}{2}$

7. d. $\frac{1}{12}\tan^{12}x+C$

8. a. $\int \frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{Cx+D}{x^2+1}\,dx$

9. b. $x=5\sec\theta$

10. a. $\int \sec\theta\, d\theta$

Free Response

11. [10 points] Determine whether the integral converges or diverges. Find the value of the integral if it converges.

$$\int_0^\infty xe^{-2x}\,dx$$

ANSWER: (Students answers may vary. Below is a rough rubric.)

(i) [1 pt] First we write $\int_0^{\infty} xe^{-2x}\,dx = \lim_{b\to\infty}\int_0^b xe^{-2x}dx$

(ii) [4 pts] Next we want to find $\int xe^{-2x}dx$ We use integration by parts:

Let $u = x$ and $dv = e^{-2x}dx$
Then $du = dx$ and $v = -\frac{1}{2}e^{-2x}$
Thus

$$\int xe^{-2x}dx = -\tfrac{1}{2}xe^{-2x} - \int -\tfrac{1}{2}e^{-2x}dx$$

$$\int xe^{-2x}dx = -\frac{1}{2}xe^{-2x} + \frac{1}{2}\int e^{-2x}dx$$

So $\int xe^{-2x}dx = -\frac{1}{2}xe^{-2x} + \frac{1}{2}\left(-\frac{1}{2}e^{-2x}\right) + C$

$$\int xe^{-2x}dx = -\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + C$$

(iii) [2 pts] Now we want to find $\int_0^b xe^{-2x}dx$

$$\int_0^b xe^{-2x}dx = \left[-\frac{1}{2}be^{-2b} - \frac{1}{4}e^{-2b}\right] - \left[-\frac{1}{2}0e^{-0} - \frac{1}{4}e^{-0}\right]$$

$$= -\frac{b}{e^{2b}} - \frac{1}{4e^{2b}} + \frac{1}{4}$$

(iv) [3 pts] Finally we take the limit as $b \to \infty$

$$\int_0^{\infty} xe^{-2x}\,dx = \lim_{b\to\infty}\int_0^b xe^{-2x}dx$$

$$= \lim_{b\to\infty}\left[-\tfrac{b}{e^{2b}} - \tfrac{1}{4e^{2b}} + \tfrac{1}{4}\right]$$

(But both $-\frac{b}{e^{2b}}$ and $-\frac{1}{4e^{2b}}$ approach 0 as $b \to \infty$ by L'Hospitals' Rule)

so $\lim_{b\to\infty}\left[-\frac{b}{e^{2b}} - \frac{1}{4e^{2b}} + \frac{1}{4}\right] = \lim_{b\to\infty}\left[-\frac{b}{e^{2b}}\right] + \lim_{b\to\infty}\left[-\frac{1}{4e^{2b}}\right] + \lim_{b\to\infty}\left[\frac{1}{4}\right]$

$$= -0 - 0 + \tfrac{1}{4}$$

$$= \tfrac{1}{4}$$

12. [10 pts total] Evaluate

$$\int \frac{1}{(x-5)(x^2+1)}\,dx$$

ANSWER:

(i) [3 pts] First we note that the integrand is a rational function with the numerator having smaller degree than the denominator. Thus the method of partial fractions applies. We write the form of the decomposition first:

$$\frac{1}{(x-5)(x^2+1)} = \frac{A}{x-5} + \frac{Bx+C}{x^2+1}$$

(ii) [4 pts] Then we want to find suitable *A, B,* and *C* we multiply both sides of the equation above by $(x-5)(x^2+1)$

to get

$$1 = A(x^2+1) + (Bx+C)(x-5)$$

we multiply the right hand side to get

$$1 = Ax^2 + A + Bx^2 - 5Bx + Cx - 5C$$

$$1 = (A+B)x^2 + (-5B+C)x + (A-5C)$$

We can write the left hand side with 0 coefficients in front of missing powers:

$$0x^2 + 0x + 1 = (A+B)x^2 + (-5B+C)x + (A-5C)$$

As this is true for all x the coefficients of each side must match. That is

$$A + B = 0$$
$$-5B + C = 0$$
$$A - 5C = 1$$

This is a linear system of 3 equations in 3 unknowns. There are many ways of solving This. We can use row reduction, matrix inverses, substitutions, or even our calculators.

In any case we get $A = \frac{1}{26}$, $B = -\frac{1}{26}$, $C = -\frac{5}{26}$.

We do a little algebra to get

$$\frac{1}{(x-5)(x^2+1)} = \frac{1}{26}\left(\frac{1}{x-5}\right) - \frac{1}{26}\left(\frac{x}{x^2+1}\right) - \frac{5}{26}\left(\frac{1}{x^2+1}\right)$$

So

$$\int \frac{1}{(x-5)(x^2+1)}\,dx = \frac{1}{26}\int \frac{1}{x-5}\,dx - \frac{1}{26}\int \frac{x}{x^2+1}\,dx - \frac{5}{26}\int \frac{1}{x^2+1}\,dx$$

(iii) [3 pts] Finally we do each of the integrals on the right to get

$$\int \frac{1}{(x-5)(x^2+1)}\,dx = \frac{1}{26}\ln|x-5| - \frac{1}{52}\ln\,|x^2+1| - \frac{5}{26}\tan^{-1} x + C$$

