Homework 01: Integration by Substitution

Instructor: Joseph Wells
Arizona State University

Due: (Wed) January 22, 2014/
(Fri) January 24, 2014

Instructions: Complete ALL the problems on this worksheet (and staple on any additional pages used). Show ALL your work in the spaces provided. If you do not show your work, you will not receive credit for this assignment. This assignment is due at the *beginning* of the recitation period on the date above. Group work is allowed and encouraged, but each member must write up his/her own solutions.

1. If the change of variables $u = x^2 - 4$ is used to evaluate the definite integral $\int_2^4 f(x)\,dx$, what are the new limits of integration?

$$\int_{x=2}^{x=4} f(x)\,dx = \int_{u(2)}^{u(4)} g(u)\,du = \int_0^{12} g(u)\,du$$

2. Use a change of variables to find the following indefinite integrals. Check your work by differentiation.

a. $\displaystyle\int \frac{(\sqrt{x}+1)^4}{2\sqrt{x}}\,dx$

Let $u = \sqrt{x} + 1$. Then $du = \frac{dx}{2\sqrt{x}}$, so

$$\begin{aligned}&\int \frac{(\sqrt{x}+1)^4}{2\sqrt{x}}\,dx\\ =&\int u^4\,du\\ =&\frac{u^5}{5} + C\\ =&\frac{\sqrt{x}+1}{5} + C.\end{aligned}$$

b. $\displaystyle\int \sin^{10}(\theta)\cos(\theta)\,d\theta$

Let $u = \sin(\theta)$. Then $du = \cos(\theta)\,d\theta$, so

$$\begin{aligned}&\int \sin^{10}(\theta)\cos(\theta)\,d\theta\\ =&\int u^{10}\,du\\ =&\frac{u^{11}}{11} + C\\ =&\frac{\sin^{11}(\theta)}{11} + C\end{aligned}$$

3. Use a change of variables to evaluate the following definite integrals. Give exact answers (i.e. evaluate by hand, not with a calculator)

a. $\displaystyle\int_0^4 \frac{p}{\sqrt{9+p^2}}\,dp$

Let $u=\sqrt{9+p^2}$. Then $du=\frac{p}{\sqrt{9+p^2}}\,dp$, so

$$\begin{aligned}&\int_{p=0}^{p=4} \frac{p}{\sqrt{9+p^2}}\,dp\\ =&\int_{u=3}^{u=5} du\\ =&u\Big|_3^5\\ =&2.\end{aligned}$$

b. $\displaystyle\int_0^6 \frac{dx}{x^2+36}$

Let $u=\frac{x}{6}$. Then $du=\frac{dx}{6}$, so

$$\begin{aligned}&\int_{x=0}^{x=6} \frac{dx}{x^2+36}\\ =&\frac{1}{36}\int_{x=0}^{x=6} \frac{dx}{\left(\frac{x}{6}\right)^2+1}\\ =&\frac{6}{36}\int_{u=0}^{u=1} \frac{du}{u^2+1}\\ =&\frac{1}{6}\arctan(u)\Big|_0^1\\ =&\frac{\pi}{24}.\end{aligned}$$

4. Evaluate the following indefinite integral. This can be done with only one substitution, but may be easier to approach with two. Hint: Use $u=x^2$ for the first substitution, rewrite the integral in terms of u, and then find a substitution $v=f(u)$.

$$\int x\sin(x^2)\cos^8(x^2)\,dx$$

Let $v=\cos(u)=\cos(x^2)$. Then $dv=2x\sin(x^2)\,dx$, so

$$\begin{aligned}&\int x\sin(x^2)\cos^8(x^2)\,dx\\ =&\frac{1}{2}\int v^8\,du\\ =&\frac{v^9}{18}+C\\ =&-\frac{\cos^9(x^2)}{18}+C.\end{aligned}$$