TEST 2 REVIEWS

11.3: Partial Derivatives

1. Find f_x for $f(x,y)=3x^5+2x^3y^2+9xy^4$
2. Find the first partial derivatives of the function of $c=\ln(a+\sqrt{a^2+b^2})$
3. Find $\frac{\partial^2 z}{\partial x^2}$ for $z=y\tan 8x$
4. Find f_x for $f(x,y)=\int_y^x \cos(t^8)\,dt$
5. Find all the second partial derivatives of $f(x,y)=4x^3y-7xy^2$

11.4: Tangent Planes & Linear Approximations

1. Find the linearization $L(x, y)$ of the function at the given point: $f(x,y)=x\sqrt{y},\ (-5,\ 4)$. Round the answers to the nearest hundredth.
2. Find the differential of the function.

$$u=e^{5t}\sin 3x$$

3. Find the equation of the tangent plane and the normal line to the given surface at the specified point.

$$f(x,y)=4x^3y-7xy^2\ ,\ (1,\ 1,\ -3)$$

4. Find an equation of the tangent plane to the given surface at the specified point.

$$z=\sqrt{18-x^2-2y^2},\ (3,\ 2,\ 1)$$

5. Use differentials to estimate the amount of metal in a closed cylindrical can that is 13 cm high and 6 cm in diameter if the metal in the top and bottom is 0.09 cm thick and the metal in the sides is 0.01 cm thick. (rounded to the nearest hundredth.)

11.5: The Chain Rule

1. Use the Chain Rule to find $\frac{\partial w}{\partial s}$ where s = 3, t = 0.

$$w=x^2+y^2+z^2,\qquad x=st,\qquad y=s\cos t,\qquad z=s\sin t$$

2. Use the Chain Rule to find $\frac{\partial u}{\partial p}$:

$$u=\frac{x+y}{y+z},$$

$$x=p+6r+5t,\quad y=p-6r+5t,\quad z=p+6r-5t$$

11.6: Directional Derivatives & The gradient vector

1. Find the direction in which the maximum rate of change of f at the given point occurs:

$$f(x, y) = \sin(xy), \quad (1, \; 0)$$

2. Find the gradient of the function.

$$f(x, y, z) = x^2 e^y \sqrt{z}$$

3. If $f(x, y) = x^2 + 9y^2$, use the gradient vector $\nabla f(10, \; 2)$ to find the tangent line to the level curve $f(x, y) = 136$ at the point (10, 2).
4. Find the equation of the tangent plane to the given surface at the specified point

$$5x^2 + 3y^2 + 8z^2 = 353, \; (3, \; 6, \; 5)$$

5. The temperature of a gas at the point (x, y, z) is given by $G(x, y, z) = x^2 - 5xy + y^2 z$.

(a) What is the rate of change in the temperature at the point (3, 2, 1) in the direction $\mathbf{v} = \langle 2, \; 1, \; -4 \rangle$

(b) What is the direction of the maximum rate of change of temperature at the point (3, 2, 1)?

(c) What is the maximum rate of change at the point (3, 2, 1)?

6. Find the directional derivative of the function at the given point in the direction of the vector $\boldsymbol{v}$.

$$f(x, y, z) = x \tan^{-1}\left(\frac{y}{z}\right), \quad (-8, \; -8, \; -8), \quad v = -10\mathbf{i} + 7\mathbf{j} + 7\mathbf{k}$$

11.7: Maximum & minimum values

1. Find the critical points of the function.

$$f(x, y) = 5 + 76xy + 38x^2 + 240y + \frac{y^4}{4}$$

2. Find the local maximum, and minimum value and saddle points of the function.

$$f(x, y) = x^2 - xy + y^2 - 9x + 6y + 10$$

3. Find the dimensions of a rectangular box of maximum volume such that the sum of the lengths of its 12 edges is 24.
4. Find the minimum of the function.

$$f(x, y) = x^2 + 2y^2 + 2xy + 2x + 3y \quad \text{subject to} \quad x^2 - y = 1$$

5. Suppose (1, 1) is a critical point of a function f with continuous second derivatives. In the case of

$f_{xx}(1,\ 1)=7,\quad f_{xy}(1,\ 1)=8,\quad f_{yy}(1,\ 1)=10$ what can you say about f ?

6. Find all the saddle points of the function $f(x,y)=x\sin\dfrac{y}{3}$
7. Find the absolute maximum value of the function f on the set D.

$$f(x,y)=3x^2+8y^2+10x^2y+9,\ D=\{\ (x,y)\mid |x|\le 1,\ |y|\le 1\ \}$$

12.1: Double Integrals over Rectangles

1. Calculate the double integral $\iint_R x\sin(x+y)\ dA,\ \ R=\left[0,\ \dfrac{\pi}{6}\right]\times\left[0,\ \dfrac{\pi}{2}\right]$
2. Calculate the iterated integral $\int_0^6\int_0^3 \sqrt{x+y}\ dxdy$
3. Calculate the iterated integral $\int_0^{\ln 4}\int_0^{\ln 5} e^{5x-y}\ dxdy$
4. Calculate the double integral.

$$\iint_R (3x^2y^3-5x^4)\,dA,\quad R=\{(x,y)\mid 0\le x\le 1,\ 0\le y\le 4\}$$

5. Calculate the double integral.

$$\iint_R \frac{xy^2}{x^2+4}\ dA,\ \ R=\{\ (x,y)|0\le x\le 2,\ -1\le y\le 1\ \}$$

6. Calculate the double integral.

$$\iint_R xye^y\ dA,\ \ R=\{(x,y)|0\le x\le 2, 0\le y\le 1\}$$

12.2: Double Integrals Over general regions

1. Find the volume of the solid in the first octant bounded by the cylinder $z=9-y^2$ and the plane $x = 1$.
2. Evaluate the iterated integral.

$$\int_1^5\int_y^5 xy\ \ dxdy$$

3. Evaluate $\iint_D x^2 y^2 dA$ where D is the figure bounded by $y=1,\ y=2,\ x=0,\ and\ x=y$.
4. Evaluate the double integral $\iint_D y^3\ dA$, where D is the triangular region with vertices (0, 1), (7, 0) and (1, 1).
5. Evaluate the integral by reversing the order of integration.

$$\int_0^1 \int_{4y}^4 e^{x^2}\, dxdy$$

4. Compute $\iint_D \sqrt{4-x^2-y^2}\ dA$, where D is the disk $x^2+y^2 \le 4$, by first identifying the integral as the volume of a solid.

12.3: Double Integrals in Polar coordinates

1. Evaluate the integral by changing to polar coordinates: $\iint_D e^{-x^2-y^2}\ dA$, where D is the region bounded by the semicircle $x=\sqrt{4-y^2}$ and the y-axis.
2. Use polar coordinates to find the volume of the solid inside the cylinder $x^2+y^2=9$ and the ellipsoid $2x^2+2y^2+z^2=36$.
3. Use polar coordinates to find the volume of the solid under the paraboloid $z=x^2+y^2$ and above the disk $x^2+y^2 \le 9$.
4. Use polar coordinates to find the volume of the solid bounded by the paraboloid $z=7-6x^2-6y^2$ and the plane z = 1.
5. Evaluate the iterated integral by converting to polar coordinates.

$$\int_{-2}^{2} \int_0^{\sqrt{4-y^2}} (x^2+y^2)^{3/2}\, dxdy$$

ANSWERS

11.3: Partial Derivatives

1. $15x^4 + 6x^2y^2 + 9y^4$
2. $\dfrac{1}{\sqrt{a^2+b^2}}$, $\dfrac{b}{a\sqrt{a^2+b^2}+a^2+b^2}$
3. $128y\sec^2 8x\tan 8x$
4. $\cos(x^8)$
5. $f_{xy} = 12x^2 - 14y$, $f_{yx} = 12x^2 - 14y$, $f_{xx} = 24xy$, $f_{yy} = -14x$

11.4: Tangent Planes /Linear Approximations/Differentials

1. $L = 2x - 1.25y + 5$
2. $du = 5e^{5t}\sin(3x)dt + 3e^{5t}\cos(3x)dx$
3. $-5x + 10y + z = 2$, $\dfrac{x-1}{-5} = \dfrac{y-1}{10} = \dfrac{z+3}{1}$
4. $3x + 4y + z = 18$
5. $7.54\ \text{cm}^3$

11.5: The Chain Rule

1. 6
2. $\dfrac{\partial u}{\partial p} = -\dfrac{5t}{p^2}$

11.6: Directional Derivatives & The gradient vector

1. $\langle 0, 1\rangle$
2. $\left\langle 2xe^y\sqrt{z},\ x^2e^y\sqrt{z},\ \dfrac{x^2e^y}{2\sqrt{z}}\right\rangle$
3. $10x + 18y = 136$
4. $15x + 18y + 40z = 353$
5. a) $-35/\sqrt{21}$ b) $< -4, -11, 4 >$ c) $\sqrt{153}$
6. $-5\pi/(2\sqrt{198})$

11.7: Maximum & minimum values

1. (-6, 6), (-4, 4), (10, -10)
2. Min at $(4,-1)$, $f(4,-1) = -11$, Max point - none, Saddle point - none
3. 2, 2, 2

4. Min at $(-3/4, -7/16)$
5. $f(1,1)$ is a local minimum
6. $(0,\ 3\pi n)$
7. 30

12.1: Double Integrals over Rectangles

1. 0.17
2. 37.13
3. 468.6
4. 60
5. 0.23
6. 2

12.2: Double Integrals over general regions

1. 18
2. 72
3. 7/2
4. 1/5
5. $\frac{e^{16}-1}{8}$
6. $\frac{16}{3}\pi$

12.3: Double Integrals in Polar coordinates

1. $\frac{(1-e^{-4})\pi}{2}$
2. 292.45
3. 40.5π
4. 3π
5. 20.11