TEST 3 REVIEW

12.5: Triple Integrals.

1. Evaluate the integral $\int_0^2 \int_1^3 \int_0^{1-y} 5ze^{3y}\, dxdzdy$
2. Evaluate the triple integral.:

$$\iiint_E 5x\ dV\ ,\ E = \ (x, y, z)|0 \le y \le 3,\ 0 \le x \le \sqrt{9-y^2}, 0 \le z \le y$$

3. Express the integral $\iiint_E f(x,y,z)\ dV$ as an iterated integral of the form $\int_a^b \int_{u(x)}^{v(x)} \int_{c(x,y)}^{d(x,y)} f\ dzdydx$ where E is the solid bounded by the surfaces $x^2 = 1 - y$, $z = 0$, and $z = y$.

4. Express the integral $\int_0^2 \int_y^2 \int_0^y f(x,y,z)\ dzdxdy$ in the form $\int_a^b \int_{u(y)}^{v(y)} \int_{c(y,z)}^{d(y,z)} f\ dxdzdy$.
5. Find the mass of the solid E, if E is the tetrahedron bounded by $6x + 2y - z = 6$, $z = 0$, $x = 0$, $y = 0$ and the density function ρ is $\rho(x, y, z) = 2$.

6. Find the region E for which the triple integral $\iiint_E (1 - 2x^2 - 7y^2 - 2z^2)\ dV$ is a maximum.

7. Compute the volume of the solid bounded by the given surfaces.
$$z = 16 - x^2 - y^2, z = 0, x = 0, \text{ and } x^2 + y^2 \ge 1$$
8. Find the mass of the solid with density $\rho\ \ x, y, z$ and the given shape.
$\rho\ \ x, y, z\ = x + 6y$, tetrahedron bounded by $x + y + 8z = 8$ and the coordinate planes.

12.6: Triple Integrals in Cylindrical Coordinates

1. Use cylindrical coordinates to evaluate $\iiint_E \sqrt{x^2 + y^2}\ dV$, where E is the region that lies inside the cylinder $x^2 + y^2 = 25$ and between the planes $z = -6$ and $z = 5$.

2. Use cylindrical coordinates to find the volume of the solid that the cylinder $r = 3\cos\theta$ cuts out of the sphere of radius 3 centered at the origin.

3. Use cylindrical coordinates to evaluate the triple integral $\iiint_E y\ dV$ where E is the solid that lies between the cylinders $x^2 + y^2 = 3$ and $x^2 + y^2 = 7$ above the xy-plane and below the plane $z = x + 4$.

4. Using an appropriate coordinate system, evaluate the integral $\iiint_Q ze^{x^2} e^{y^2}\, dV$ where Q is the region that lies inside $y = \sqrt{2 - x^2}$ and $y = 0$, between the planes $z = 1$ and $z = 0$.

12.7: Triple Integrals in Spherical Coordinates

1. Use spherical coordinates to evaluate $\iiint_E xe^{(x^2+y^2+z^2)^2}\,dV$, where E is the solid that lies between the spheres $x^2+y^2+z^2=4$ and $x^2+y^2+z^2=25$ in the first octant.
2. Use spherical coordinates to find the volume above the cone $z=\sqrt{x^2+y^2}$ and inside the sphere $x^2+y^2+z^2=2az$ where a is a positive constant.
3. Use cylindrical or spherical coordinates, whichever seems more appropriate, to find the volume of the solid E that lies above the cone $z=\sqrt{x^2+y^2}$ and below the sphere $x^2+y^2+z^2=9$.
4. Use spherical coordinates to find the volume of the solid that lies within the sphere $x^2+y^2+z^2=9$ above the xy-plane and below the cone $z=\sqrt{x^2+y^2}$.
5. Use cylindrical or spherical coordinates, whichever seems more appropriate, to evaluate $\iiint_E z\,dV$ where E lies above the paraboloid $z=x^2+y^2$ and below the plane $z=4y$.

13.2 Line Integrals

1. Evaluate $\int_C xy^4\,ds$, where C is the right half of the circle $x^2+y^2=9$.
2. Evaluate $\int_C yz\,dy+xy\,dz$, where C is given by $x=4\sqrt{t},\ y=5t,\ z=2t^2,\ 0\le t\le 1$.
3. Evaluate the line integral $\int_C (x^2-3xy+y^2)dx$, where C is the arc $y=2x^2,\ 0\le x\le 2$.
4. Evaluate $\int_C 5x^4\,ds$, where C is the line segment from $(6,6)$ to $(7,8)$
5. Find the work done by the force field $\mathbf{F}(x,y)=xz\mathbf{i}+yx\mathbf{j}+zy\mathbf{k}$ on a particle that moves along the curve $\mathbf{r}(t)=t^2\mathbf{i}-t^3\mathbf{j}+t^4\mathbf{k},\ 0\le t\le 1$.
6. Evaluate the line integral $\int \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F}(x,y)=(x-y)\mathbf{i}+(xy)\mathbf{j}$ and C is the arc of the circle $x^2+y^2=9$ traversed counterclockwise from (3, 0) to (0, -3).
7. Find the work done by the force field $\mathbf{F}(x,y)=x\sin(y)\mathbf{i}+y\mathbf{j}$ on a particle that moves along the parabola $y=x^2$ from (1, 1) to (2, 4)

13.3: The Fundamental Theorem for Line Integrals

1. Show that the line integral $\int_C 5x^4 y\,dx + x^5 - 8\ dy$ is independent of the path.

2. Determine whether or not **F** is a conservative vector field. If it is, find a function f such that $\mathbf{F} = \nabla f$.

$$\mathbf{F} = (14x + 8y)\mathbf{i} + (8x + 18y)\mathbf{j}$$

3. Let $\mathbf{F} = (8x\cos y - y\cos x)\mathbf{i} + (-4x^2\sin y - \sin x)\mathbf{j}$
   a) Show that ***F*** is a conservative vector field and find a function f such that $\mathbf{F} = \nabla f$.
   b) Use the potential function f to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the part of the ellipse

   $\frac{x^2}{4} + \frac{y^2}{9} = 1$ in the first quadrat, traced in the clockwise direction.

4. Let $\mathbf{F}(x,y) = x^5y^6\mathbf{i} + y^5x^6\mathbf{j}$

   a) Find a function f such that $\mathbf{F} = \nabla f$

   a) Use the potential function f to evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$ along the given curve

   $C.C: \mathrm{r}(t) = \sqrt{t}\mathrm{i} + (1+t^3)\mathrm{j}\ ,\ 0 \le t \le 1$

5. Determine whether or not the vector field is conservative. If it is conservative, find a function f such that $\mathbf{F} = \nabla f$.

$$\mathbf{F}(x,y,z) = 10x\mathbf{i} + 4y\mathbf{j} + 6z\mathbf{k}$$

6. Determine whether or not the vector field is conservative. If it is conservative, find a function f such that $\mathbf{F} = \nabla f$.

$$\mathbf{F}(x,y,z) = 35yze^{7xz}\mathbf{i} + 5e^{7xz}\mathbf{j} + 35xye^{7xz}\mathbf{k}$$

Note: Also know the relations between the Cartesian, Cylindrical, and Spherical Coordinates and conversions from one to the other.

13.4: Green's Theorem

1. Use Green's Theorem to evaluate the line integral along the given positively oriented curve.
   $\int_C (10xy)\ dx + (10x^2)\ dy$ C consists of the line segment from (-3, 0) to (3, 0) and the top half of the circle $x^2 + y^2 = 9$.
2. Use Green's Theorem to evaluate the line integral along the given positively oriented curve. $\int_C \mathbf{F}\cdot d\mathbf{r}$,
   where $\mathbf{F}(x,y) = \langle x^2 - y^3, xy^2 \rangle$ and C consists of the part of the circle $x^2 + y^2 = 16$ from (4, 0) to (0, 4) and the line segments from (0, 4) to (0, 0) and from (0, 0) to (4, 0).
3. Use Green's Theorem to find the work done by the force $\mathbf{F}(x,y) = x(x+5y)\mathbf{i} + 4xy^2\mathbf{j}$ in moving a particle from the origin along the x-axis to (4, 0) then along the line segment to (0, 4) and then back to the origin along the y-axis.
4. A particle starts at the point (-3, 0), moves along the x-axis to (3, 0) and then along the semicircle $y = \sqrt{9-x^2}$ to the starting point. Use Green's Theorem to find the work done on this particle by the force field $\mathbf{F}(x,y) = \langle 24x,\ 8x^3 + 24xy^2 \rangle$.
5. Use Green's Theorem to evaluate the line integral along the given positively oriented curve:
   $\int_C (7.5y^2 - \tan^{-1} x)dx + (12x + \sin y)dy$ and C is the boundary of the region enclosed by the parabola $y = x^2$ and the line $y = 49$.

Answers

12.5: Triple Integrals.

1. $-\dfrac{40}{9}\ e^6 + 2$
2. 50.625

3. $\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}}\int_{0}^{y} f \ dzdydx$
4. $\int_{0}^{2}\int_{0}^{y}\int_{y}^{2} f \ dxdzdy$
5. 6
6. $2x^2+7y^2+2z^2\le 1$
7. $\frac{225}{4}\pi$
8. $\frac{448}{3}$

12.6: Triple Integrals in Cylindrical Coordinates

1. $\frac{2750}{3}\pi$
2. 32.55
3. 0
4. $\frac{\pi}{4}\ e^2-1$

12.7: Triple Integrals in Spherical Coordinates

1. $\frac{\pi}{16}(e^{625}-e^{16})$
2. πa^3
3. 16.56
4. 39.99
5. 167.55

13.2: Line Integrals

1. 291.6
2. 35.36
3. 64/15
4. $9031\sqrt{5}$
5. 23/88
6. $(9/2)(1+(3\pi/2))$
7. $(15+\cos 1-\cos 4)/2$

13.3: The Fundamental Theorem for Line Integrals

1.

$$\int_C 5x^4 y\,dx + \left(x^5 - 8\right) dy = \int_C M\left(x, y\right) dx + N\left(x, y\right) dy$$

with $M\left(x, y\right) = 5x^4 y$ and $N\left(x, y\right) = x^5 - 8$.

$M_y = \dfrac{\partial M}{\partial y} = 5x^4$ and

$N_x = \dfrac{\partial N}{\partial x} = 5x^4$

Since $M_y = N_x$, the line integral is independent of the path.

2. $f(x, y) = 7x^2 + 8yx + 9y^2 + K$
3. a) $\mathbf{F}(x, y) = P(x, y)\mathbf{i} + Q(x.y)\mathbf{j}$ with $P(x, y) = 8x\cos y - y\cos x$ and $Q(x, y) = -4x^2 \sin y - \sin x$. Since $P_y = Q_x$, the vector field is conservative. The potential function is $f(x, y) = 4x^2 \cos y - y\sin x + K$

   b) $\int_C \mathbf{F}\cdot d\mathbf{r} = f(2,0) - f(0,3) = 16 - 0 = 16$
4. a) $f(x, y) = \frac{1}{6}x^6 y^6$ b) $\int_C \mathbf{F}\cdot d\mathbf{r} = f(1,2) - f(0,0) = \frac{32}{3}$
5. $5x^2 + 2y^2 + 3z^2 + K$
6. $5ye^{7xz} + K$

13.4 Green's Theorem

1. 0
2. 64π
3. 32
4. 486π
5. -196196