Posted

MAT 267 – Calculus for Engineers-III	Student Name : KEY
Instructor :XXXX	Student ID :
TEST 3 Form A	Class Time :

Honor Statement

By signing below I confirm that I have neither given nor received any unauthorized assistance on this exam. This includes any use of a graphing calculator beyond those uses specifically authorized by the School of Mathematical and Statistical Sciences and my instructor. Furthermore, I agree not to discuss this exam with anyone until the exam testing period is over. In addition, my calculator's memory and menus may be checked at any time and cleared by any testing center proctor or School of Mathematical and Statistical Sciences instructor.

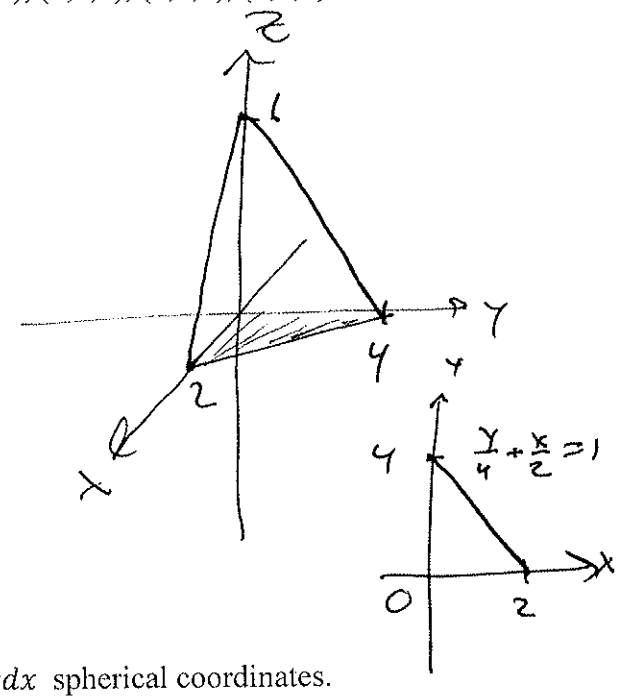
Signature:_____ Date:_____

Instructions:

1. The exam consists of two parts: multiple choice, worth 49%, and free response (show your work), worth 51%. Please read each problem carefully.
2. There are 5 multiple choice questions worth 7 points each. Please fill in the table provided on the last page
3. There are 2 matching problems worth 7 points each.
4. There are 3 free response problems worth 17 points each.
5. Provide complete and well-organized answers in the free response section.
6. Answers in the free response section without supporting work will be given zero credit. Partial credit is granted only if work is shown.
7. No calculators with Qwerty keyboards or ones like the Casio FX-2, TI-89, TI-92, or TI-*n*spire that do symbolic algebra may be used.
8. Proctors reserve the right to check calculators.
9. Please request scratch paper from me if you need it.
10. The use of cell phones is prohibited. TURN YOUR CELL PHONE OFF! Do not allow your cell phone to ring while you are taking the exam. Do not use the calculator on your cell phone. If a proctor sees you using a cell phone, they will take your exam and you will be reported to the Dean of Students for cheating.
11. *PLEASE NOTE:* "Any student who accesses a phone or any internet-capable device during an exam for any reason automatically receives a score of zero on the exam. All such devices must be turned off and put away and made inaccessible during the exam.

1. PART I– Multiple choice. Circle the correct answer.

1. [7 pts] Consider the solid tetrahedron with vertices (0,0,0), (2,0,0), (0,4,0), (0,0,6)
The appropriate limits of integration for
$\iiint_E f(x,y,z)dzdydx$ are



a) $\int_0^2 \int_0^{4(1+2x)} \int_0^{(1-6x-6y)} f(x,y,z)dzdydx$

b) $\int_0^2 \int_0^{4(1-x/2)} \int_0^{\frac{x}{2}+\frac{y}{4}} f(x,y,z)dzdydx$

c) $\int_0^4 \int_0^{(1+2x)} \int_0^{6z+4y} f(x,y,z)dzdydx$

d) $\int_0^2 \int_0^{4\left(1-\frac{x}{2}\right)} \int_0^{6\left(1-\frac{x}{2}-\frac{y}{4}\right)} f(x,y,z)dzdydx$

e) none of these

2. [7 pts] Change $\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} e^{x^2+y^2+z^2} dzdydx$ spherical coordinates.

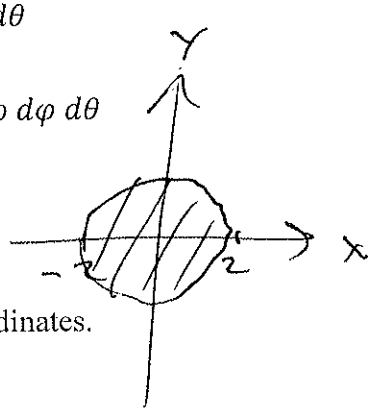
a) $\int_0^{2\pi} \int_0^{\pi/2} \int_0^2 e^{\rho^2} d\rho\, d\varphi\, d\theta$

b) $\int_0^{2\pi} \int_{-\pi/2}^{\pi/2} \int_0^2 e^{\rho^2} \rho^2 \sin\varphi\, d\rho\, d\varphi\, d\theta$

c) $\int_0^{\pi} \int_{-\pi/2}^{\pi/2} \int_0^2 e^{\rho^2} \rho^2 \sin\varphi\, d\rho\, d\varphi\, d\theta$

d) $\int_0^{2\pi} \int_0^{\pi/2} \int_0^2 e^{\rho^2} \rho^2 \sin\varphi\, d\rho\, d\varphi\, d\theta$

e) none of these



3 [7 pts] Change $\int_{-1}^{0} \int_0^{\sqrt{1-x^2}} \int_{x^2+y^2}^{2-x^2-y^2} \sqrt{x^2+y^2} dzdydx$ to cylindrical coordinates.

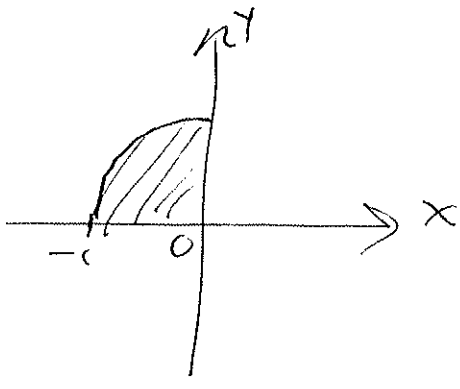
a) $\int_0^{\frac{\pi}{2}} \int_0^1 \int_{r^2}^{2-r^2} r^2 dzdrd\theta$

b) $\int_{\frac{\pi}{2}}^{\pi} \int_0^1 \int_r^{2-r} r dzdrd\theta$

c) $\int_{\frac{\pi}{2}}^{\pi} \int_0^1 \int_{r^2}^{2-r^2} r^2 dzdrd\theta$

d) $\int_{\frac{\pi}{2}}^{\pi} \int_0^{\sqrt{2}} \int_{r^2}^{2-r^2} r^2 dzdrd\theta$

e) none of the above



$m = \sqrt{3}$ $\tan\theta = \sqrt{3}$ $\theta = \pi/3$

4. [7 pts] Find an equation for the plane $y = (\sqrt{3})x$ in cylindrical coordinates.

a) $\theta = \frac{\pi}{3}$ b) $r = \sqrt{3}\tan\theta$ c) $3\tan\theta = 1$ d) $r = \sqrt{3}$ e) None of these

5. [7 pts] Consider the line integral $\int_C \boldsymbol{F}\cdot d\boldsymbol{r}$ where $\boldsymbol{F}(x,y,z) = 2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$ and C is a closed, simple, piecewise smooth, curve. Which of the following is true: $f = 2x+3y+z$ $F = \nabla f$

a) $\int_C \boldsymbol{F}\cdot d\boldsymbol{r} = 0$ b) $\int_C \boldsymbol{F}\cdot d\boldsymbol{r} > 0$ c) $\int_C \boldsymbol{F}\cdot d\boldsymbol{r} < 0$ d) None of these

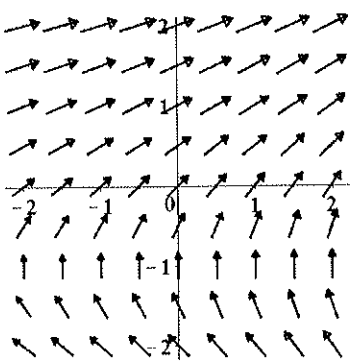
Matching Section:

6 [7 pts] Match the vector fields with the plots below. One vector field will not match.

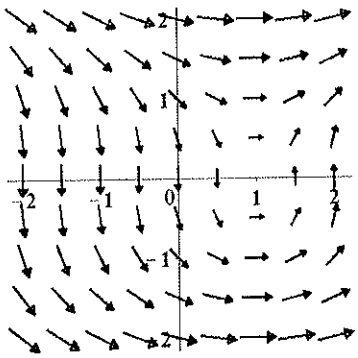
a) $\mathbf{F} = \frac{x\mathbf{i}+y\mathbf{j}}{\sqrt{x^2+y^2}}$ b) $\mathbf{F} = y^2\mathbf{i} + (x-1)\mathbf{j}$ c) $\mathbf{F} = -x\mathbf{i} + y\mathbf{j}$

d) $\mathbf{F} = (y+1)\mathbf{i} + e^{x/6}\mathbf{j}$ e) $\mathbf{F} = y\mathbf{i} + x\mathbf{j}$

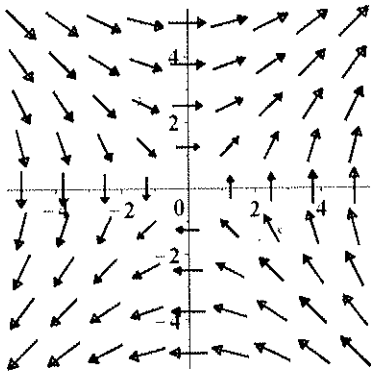
I Matches d



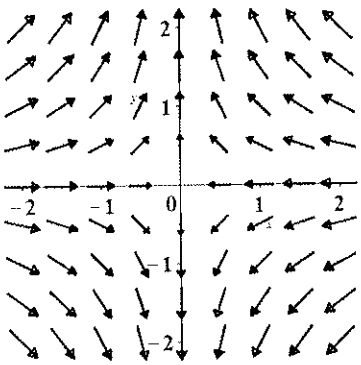
II Matches b



III Matches e

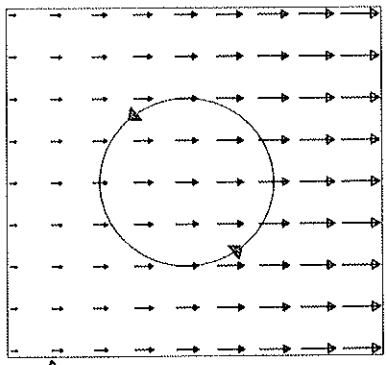


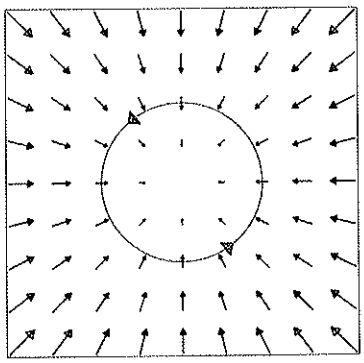
IV Matches c

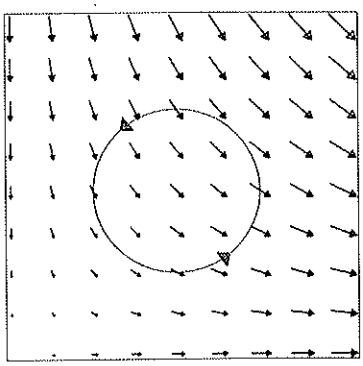


7. [7 pts]
Determine whether the line integral of each vector field along the oriented path (a circle oriented counterclockwise) is positive, negative or zero.

a) Zero b) Zero c) ~~Negative~~ Zero



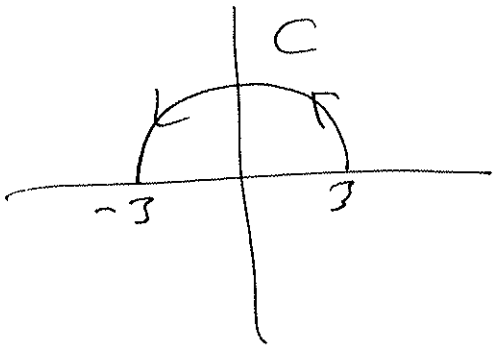




Example $f = x\vec{i}$ Let $F = -x\vec{i} - y\vec{j}$ Let $F = x\vec{i} - y\vec{j}$

Free Response:

1. [17 pts] Let the curve C be the upper half of the circle $x^2 + y^2 = 9$ from right to left. That is from (3,0) as initial point to terminal point (-3,0) A force field is given by $F = \langle -x^2, xy \rangle$. Calculate the work done by the force in moving a particle along the curve C.



$x = 3\cos t$

$y = 3\sin t$

$\vec{F} = \langle -9\cos^2 t, 9\cos t \sin t \rangle$

$\vec{r} = \langle 3\cos t, 3\sin t \rangle$

$d\vec{r} = \langle -3\sin t, 3\cos t \rangle$

Answer: _____

$$\int_C \vec{F} \cdot d\vec{r} = \int_0^{\pi} 27\cos^2 t \sin t + 27\cos^2 t \sin t \, dt$$

$u = \cos t \quad -du = \sin t \, dt$

$$= 54 \int_0^{\pi} \cos^2 t \sin t \, dt = 54 \int_1^{-1} u^2 (-du)$$

$$= 54 \cdot \frac{1}{3} u^3 \Big|_{-1}^{1} = 18 \cdot 2 = 36$$

2. [17 pts] Use Green's theorem to evaluate $\int_C (e^{x^2} - \frac{1}{2}y)dx + (\frac{1}{2}x + (\sin y)^2)dy$, where C is the rectangle with corners (0,0), (2,0), (2,3) and (0,3) oriented in positive direction.

See attached

Answer:_____

3. [17 pts] Let $\boldsymbol{F}(x, y, z) = (2x + 5y + 7z)\boldsymbol{i} + (5x)\boldsymbol{j} + (7x + 2z)\boldsymbol{k}$

a) Find a potential function for $\vec{F}$.

See Attached.

Answer:_____

b) Evaluate $\int_C \vec{F} \cdot d\vec{r}$, where C is a curve from (1, -1, 1) to (1, 2, 3)

Answer:_____

2)

$$\oint_C \underbrace{\left(e^{x^2} - \frac{1}{2}y\right)}_{P} dx + \underbrace{\left(\frac{1}{2}x + \sin^2 y\right)}_{Q} dy$$

$$= \iint_R Q_x - P_y \, dA$$

$$= \iint_R \frac{1}{2} - \left(-\frac{1}{2}\right) dA$$

$$= \iint_R 1 \, dA = \text{Area of } R$$

$$= 2 \cdot 3$$

$$= 6$$

3) a) $F(x,y,z) = \underbrace{(2x+5y+7z)}_{f_x}\vec{i} + \underbrace{(5x)}_{f_y}\vec{j} + \underbrace{(7x+2z)}_{f_z}\vec{k}$

$f_x = 2x + 5y + 7z$

$f = x^2 + 5xy + 7xz + g(y,z)$

$f_y = 5x + g_y(y,z) =$

$g_y(y,z) = 0$

$g(y,z) = 0 + h(z)$

$f = x^2 + 5xy + 7xz + h(z)$

$f_z = 7x + h'(z)$

$h'(z) = 2z$

$h(z) = z^2 + C$

$$\boxed{f = x^2 + 5xy + 7xz + z^2 + C}$$

b) $\int_C \vec{F} \cdot d\vec{r} = f(r(b)) - f(r(a))$

$= f(1,2,3) - f(1,-1,1) = \boxed{37}$