FINAL REVIEW

13.4: Green's Theorem

1. Use Green's Theorem to evaluate the line integral along the given positively oriented curve. $\int_C (10xy)\, dx + (10x^2)\, dy$ C consists of the line segment from (-3, 0) to (3, 0) and the top half of the circle $x^2 + y^2 = 9$.
2. Use Green's Theorem to evaluate the line integral along the given positively oriented curve. $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y) = \langle x^2 - y^3, xy^2 \rangle$ and C consists of the part of the circle $x^2 + y^2 = 16$ from (4, 0) to (0, 4) and the line segments from (0, 4) to (0, 0) and from (0, 0) to (4, 0).
3. Use Green's Theorem to find the work done by the force $\mathbf{F}(x, y) = x(x + 5y)\mathbf{i} + 4xy^2\mathbf{j}$ in moving a particle from the origin along the x-axis to (4, 0) then along the line segment to (0, 4) and then back to the origin along the y-axis.
4. A particle starts at the point (-3, 0), moves along the x-axis to (3, 0) and then along the semicircle $y = \sqrt{9 - x^2}$ to the starting point. Use Green's Theorem to find the work done on this particle by the force field $\mathbf{F}(x, y) = \langle 24x,\ 8x^3 + 24xy^2 \rangle$.
5. Use Green's Theorem to evaluate the line integral along the given positively oriented curve: $\int_C (7.5y^2 - \tan^{-1} x)dx + (12x + \sin y)dy$ and C is the boundary of the region enclosed by the parabola $y = x^2$ and the line $y = 49$.

13.5: Curl & Divergence

1. Let $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ and $r = |\mathbf{r}|$. Find $\nabla \cdot (4\mathbf{r})$.
2. Find the curl of $x^2 z\mathbf{i} + 0\mathbf{j} + xz^2\mathbf{k}$
3. Find the curl of the vector field: $\mathbf{F}(x, y, z) = 5e^x \sin(y)\mathbf{i} + 3e^x \cos(y)\mathbf{j} + 8z\mathbf{k}$
4. Let $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ and $r = |\mathbf{r}|$. Find $\nabla \cdot (9\ r\ \mathbf{r})$.
5. Find the div F if $F(x, y, z) = e^{xz}(\cos yz\,\mathbf{i} + \sin yz\,\mathbf{j} - \mathbf{k})$.
6. Find the correct identity, if *f* is a scalar field, **F** and **G** are vector fields.

   Select the correct answer.

   a. $\text{div}(\mathbf{F} + \mathbf{G}) = \text{div}\,\mathbf{F} + \text{div}\,\mathbf{G}$

   b. $\text{div}(\mathbf{F} + \mathbf{G}) = \text{curl}\,\mathbf{F} + \text{div}\,\mathbf{G}$

   c. $\text{div}(\mathbf{F} + \mathbf{G}) = \text{curl}\,\mathbf{F} + \text{curl}\,\mathbf{G}$

   d. $\text{curl}(\mathbf{F} + \mathbf{G}) = \text{div}\,\mathbf{F} + \text{curl}\,\mathbf{G}$

   e. none of these

7. Find the correct identity, if f is a scalar field, $\mathbf{F}$ and $\mathbf{G}$ are vector fields.

Select the correct answer.

a. $\operatorname{div}(f\mathbf{F}) = f\operatorname{curl}(\mathbf{F}) + (\nabla f)\times\mathbf{F}$

b. $\operatorname{div}(f\mathbf{F}) = f\operatorname{div}(\mathbf{F}) + \mathbf{F}\cdot\nabla f$

c. $\operatorname{curl}(f\mathbf{F}) = f\operatorname{div}(\mathbf{F}) + \mathbf{F}\cdot\nabla f$

d. none of these

13.6: Parametric Surfaces & their areas

1. Find a parametric representation for the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies above the cone $z = \sqrt{x^2 + y^2}$.
2. Which of the equations below is an equation of a plane? Select the correct answer.

   a. $r(u,v) = (5 + 10u)\mathbf{i} + (-u + 9v)\mathbf{j} + (2 + 6u + 4v)\mathbf{k}$ b. $r(u,v) = u\cos v\mathbf{i} + u\sin v\mathbf{j} + u^2\mathbf{k}$
3. Find a parametric representation for the part of the elliptic paraboloid $x + y^2 + 6z^2 = 9$ that lies in front of the plane $x = 0$. Select the correct answer.

   a. $x = x,\ y = \pm\sqrt{9 - x + 6z^2},\ z = z$

   b. $x = x,\ y = \sqrt{9 - x + 6z^2},\ z = z$

   c. $x = 9 - y^2 - 6z^2,\ y = y,\ z = y,\ 0 \le y^2 + 6z^2 \le 3$

   d. $x = 9 - y^2 - 6z^2,\ y = y,\ z = y,\ y^2 + 6z^2 \ge 9$

   e. $x = 9 - y^2 - 6z^2,\ y = y,\ z = z,\ y^2 + 6z^2 \le 9$

4. Which of the equations below is an equation of a cylinder?

a. $r(x, \theta) = \langle x, \cos 7\theta, \sin 7\theta \rangle$ b. $r(x, \theta) = \langle x, \cos 5\theta, \sin 5\theta \rangle$

5. Find the area of the part of the paraboloid $z = x^2 + y^2$ that lies under the plane $z = 9$

13.7: Surface Integrals

1. Evaluate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$ for the given vector field **F** and the positively oriented surface S.

   In other words, find the flux of **F** across S. $\mathbf{F}(x, y, z) = 4x\mathbf{i} + 4y\mathbf{j} + 4z\mathbf{k}$, S is the sphere $x^2 + y^2 + z^2 = 2$.

2. Evaluate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$ for the given vector field **F** and the positively oriented surface S.

   In other words, find the flux of **F** across S. $\mathbf{F}(x,y,z) = 9x\mathbf{i} + 2y\mathbf{j} + 6z\mathbf{k}$ and S is the cube with vertices $(\pm 1, \pm 1, \pm 1)$.

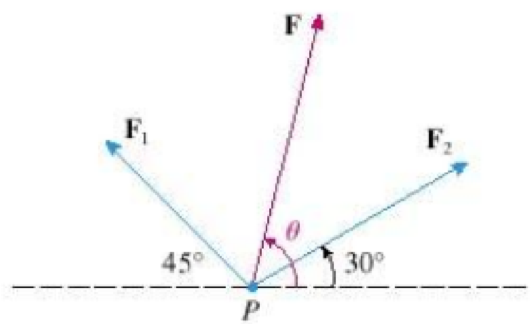
3. Evaluate the surface integral. $\iint_S 4(x^2 y + z^2)\, dS$ where S is the part of the cylinder $x^2 + y^2 = 9$ between the planes $z = 0$ and $z = 4$.

4. Evaluate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x,y,z) = x\mathbf{i}$, and S is the positively oriented part of the plane $x + y + z = 1$ in the first octant.

TOPICS FROM TEST 1

10.3: Dot Product

1. Find the values of x such that the vectors $\langle 3, 2, x \rangle$ and $\langle 2x, 4, x \rangle$ are orthogonal
2. A woman walks due west on the deck of a ship at 4 mi/h. The ship is moving north at a speed of 20 mi/h. Find the speed of the woman relative to the surface of the water. Round the result to the nearest tenth.
3. Find the angle between the vectors, if $\mathbf{a} = \langle 6, 0 \rangle$ and $\mathbf{b} = \langle 6, 6 \rangle$.
4. Two forces F_1 and F_2 with magnitudes 10 lb and 18 lb act on an object at a point P as shown in the figure. Find the magnitude of the resultant force F acting at P. Round the result to the nearest tenth.



5. Find the scalar projection of **b** onto **a**: $\mathbf{a} = \langle 4, 2 \rangle$ & $\mathbf{b} = \langle 9, 10 \rangle$. Round your answer to the nearest hundredth

10.4: Cross Product/Scalar Triple Product

1. Find a unit vector that is orthogonal to both $9\mathbf{i} + 9\mathbf{j}$ and $9\mathbf{i} + 9\mathbf{k}$.
2. Let $\mathbf{v} = 7\,\mathbf{j}$ and let $\mathbf{u}$ be a vector with length 5 that starts at the origin and rotates in the xy - plane. Find the maximum value of the length of the vector $|\mathbf{u} \times \mathbf{v}|$.
3. Find a unit vector that is orthogonal to both $9\mathbf{i} + 9\mathbf{j}$ and $9\mathbf{i} + 9\mathbf{k}$.
4. Find the volume of the parallelepiped with adjacent edges PQ, PR, and PS:
   $P(1, 2, 3)$, $Q(3, 5, 4)$, $R(3, 2, 5)$, $S(4, 2, 3)$

10.5: Equations of Lines & Planes

1. Find the distance between the planes $5x-2y+z-1=0$, $5x-2y+z+4=0$
2. Find an equation of the plane through $(1, 1, -2)$, $(-3, -4, 2)$ and $(-3, 4, 1)$.
3. Find the equation of the line through $(2, -2, 4)$ and perpendicular to the plane $-x+2y+5z=12$.
4. Find parametric equations for the line through $(-2, 1, 1)$ and $(2, 3, 5)$.
5. Find an equation of the plane that passes through the point $(4, 0, -2)$ and contains the line

   $x=10-3t,\ \ y=10+8t,\ \ z=6+7t$
6. Find an equation of the plane that passes through the line of intersection of the planes $x-z=2$ and $y+3z=7$, and is perpendicular to the plane $5x+3y-2z=8$.
7. Find a parametric equation for the line through the point $(-6, 9, 3)$ and parallel to the vector $\langle 7, 3, -7\rangle$.
8. Find the point of intersection of

   $L_1 : \dfrac{x-17}{3}=\dfrac{y-58}{8}=\dfrac{z-23}{2}$ and

   $L_2 : \dfrac{x-49}{7}=\dfrac{y-26}{4}=z-15$
9. Find an equation of the plane with x-intercept = 9, y-intercept = 1, and z-intercept = -3.

10.9: Motion in space

1. Find $r'(t)$ for the function given by $r(t) = 2\,\mathbf{i} + \sin t\,\mathbf{j} + \cos t\,\mathbf{k}$
2. If $\mathbf{r}(t) = (t + t^2)\mathbf{i} + (2 + t^3)\mathbf{j} + t^4\mathbf{k}$, evaluate $\int_0^1 \mathbf{r}(t)\,dt$.
3. A particle moves with position function $r(t) = 2\sqrt{2}\,t\,\mathbf{i} + e^{2t}\,\mathbf{j} + e^{-2t}\mathbf{k}$. Find the acceleration of the particle.
4. Find the velocity of a particle with the given position function $\mathbf{r}(t) = 13e^{15t}\mathbf{i} + 10e^{-18t}\mathbf{j}$

TOPICS FROM TEST 2

11.6: Directional Derivatives & The gradient vector

1. Find the direction in which the maximum rate of change of f at the given point occurs:

$$f(x,y) = \sin(xy), \ (1,\ 0)$$

2. Find the gradient of the function.

$$f(x,y,z) = x^2 e^y \sqrt{z}$$

3. If $f(x,y) = x^2 + 9y^2$, use the gradient vector $\nabla f(10,\ 2)$ to find the tangent line to the level curve $f(x,y) = 136$ at the point (10, 2).
4. Find the equation of the tangent plane to the given surface at the specified point

$$5x^2 + 3y^2 + 8z^2 = 353, \ (3,\ 6,\ 5)$$

5. Suppose that over a certain region of space the electrical potential V is given by

$$V(x,y,z) = 2x^2 - 2xy + 8xyz$$

Find the rate of change of the potential at $(-4,\ -6,\ -1)$ in the direction of the vector $v = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$.

6. Find the directional derivative of the function at the given point in the direction of the vector ***v***.

$$f(x,y,z) = x\tan^{-1}\left(\frac{y}{z}\right), \ (-8,\ -8,\ -8), \ v = -10\mathbf{i} + 7\mathbf{j} + 7\mathbf{k}$$

11.7: Maximum & minimum values

1. Find the critical points of the function.

$$f(x,y) = 5 + 76xy + 38x^2 + 240y + \frac{y^4}{4}$$

2. Find the local maximum, and minimum value and saddle points of the function.

$$f(x,y) = x^2 - xy + y^2 - 9x + 6y + 10$$

3. Find the dimensions of a rectangular box of maximum volume such that the sum of the lengths of its 12 edges is 24.
4. Find the minimum of the function.

$$f(x,y) = x^2 + 2y^2 + 2xy + 2x + 3y \quad \text{subject to} \quad x^2 - y = 1$$

5. Suppose (1, 1) is a critical point of a function f with continuous second derivatives. In the case of

$$f_{xx}(1,\ 1) = 7,\quad f_{xy}(1,\ 1) = 8,\quad f_{yy}(1,\ 1) = 10$$ what can you say about f ?

6. Find all the saddle points of the function $f(x,y) = x \sin \frac{y}{3}$
7. Find the absolute maximum value of the function f on the set D.

$$f(x,y) = 3x^2 + 8y^2 + 10x^2y + 9,\quad D = \{(x,y) \mid |x| \le 1,\ |y| \le 1$$

12.2: Double Integrals Over general regions

1. Find the volume of the solid in the first octant bounded by the cylinder $z = 9 - y^2$ and the plane $x = 1$.

2. Evaluate the iterated integral.

$$\int_1^5 \int_y^5 xy \; dxdy$$

3. Evaluate $\iint_D x^2 y^2 dA$ where D is the figure bounded by $y = 1,\ y = 2,\ x = 0,\ and\ x = y$.

3. Evaluate the double integral $\iint_D y^3 \; dA$, where D is the triangular region with vertices (0, 1), (7, 0) and (1, 1).

4. Evaluate the integral by reversing the order of integration.

$$\int_0^1 \int_{4y}^4 e^{x^2} dxdy$$

4. Compute $\iint_D \sqrt{4 - x^2 - y^2} \; dA$, where D is the disk $x^2 + y^2 \le 4$, by first identifying the integral as the volume of a solid.

12.3: Double Integrals in Polar coordinates

1. Evaluate the integral by changing to polar coordinates: $\iint_D e^{-x^2-y^2}\, dA$, where D is the region bounded by the semicircle $x=\sqrt{4-y^2}$ and the y-axis.
2. Use polar coordinates to find the volume of the solid inside the cylinder $x^2+y^2=9$ and the ellipsoid $2x^2+2y^2+z^2=36$.
3. Use polar coordinates to find the volume of the solid under the paraboloid $z=x^2+y^2$ and above the disk $x^2+y^2\le 9$.
4. Use polar coordinates to find the volume of the solid bounded by the paraboloid $z=7-6x^2-6y^2$ and the plane $z=1$.
5. Evaluate the iterated integral by converting to polar coordinates.

$$\int_{-2}^{2}\int_{0}^{\sqrt{4-y^2}} (x^2+y^2)^{3/2}\, dx\, dy$$

TOPICS FROM TEST 3

12.6: Triple Integrals in Cylindrical Coordinates

1. Use cylindrical coordinates to evaluate $\iiint_E \sqrt{x^2+y^2}\, dV$, where E is the region that lies inside the cylinder $x^2+y^2=25$ and between the planes $z=-6$ and $z=5$.

2. Use cylindrical coordinates to find the volume of the solid that the cylinder $r=3\cos\theta$ cuts out of the sphere of radius 3 centered at the origin.

3. Use cylindrical coordinates to evaluate the triple integral $\iiint_E y\, dV$ where E is the solid that lies between the cylinders $x^2+y^2=3$ and $x^2+y^2=7$ above the xy-plane and below the plane $z=x+4$.
4. Using an appropriate coordinate system, evaluate the integral $\iiint_Q ze^{x^2}e^{y^2}\, dV$ where Q is the region that lies inside $y=\sqrt{2-x^2}$ and $y=0,$ between the planes $z=1$ and $z=0$.

12.7: Triple Integrals in Spherical Coordinates

1. Use spherical coordinates to evaluate $\iiint_E xe^{(x^2+y^2+z^2)^2}\,dV$, where E is the solid that lies between the spheres $x^2+y^2+z^2=4$ and $x^2+y^2+z^2=25$ in the first octant.
2. Use spherical coordinates to find the volume above the cone $z=\sqrt{x^2+y^2}$ and inside the sphere $x^2+y^2+z^2=2az$ where a is a positive constant.

3. Use cylindrical or spherical coordinates, whichever seems more appropriate, to find the volume of the solid E that lies above the cone $z=\sqrt{x^2+y^2}$ and below the sphere $x^2+y^2+z^2=9$.

4. Use spherical coordinates to find the volume of the solid that lies within the sphere $x^2+y^2+z^2=9$ above the xy-plane and below the cone $z=\sqrt{x^2+y^2}$.

5. Use cylindrical or spherical coordinates, whichever seems more appropriate, to evaluate $\iiint_E z\,dV$ where E lies above the paraboloid $z=x^2+y^2$ and below the plane $z=4y$.

13.2 Line Integrals

1. Evaluate $\int_C xy^4 ds$, where C is the right half of the circle $x^2 + y^2 = 9$.

2. Evaluate $\int_C yz\,dy + xy\,dz$, where C is given by $x = 4\sqrt{t},\ \ y = 5t,\ \ z = 2t^2,\ \ 0 \le t \le 1$.

3. Evaluate the line integral $\int_C (x^2 - 3xy + y^2)dx$, where C is the arc $y = 2x^2,\ \ 0 \le x \le 2$.

4. Evaluate $\int_C 5x^4 ds$, where C is the line segment from $(6,6)$ to $(7,8)$

5. Find the work done by the force field $\mathbf{F}(x, y) = xz\mathbf{i} + yx\mathbf{j} + zy\mathbf{k}$ on a particle that moves along the curve $\mathbf{r}(t) = t^2\mathbf{i} - t^3\mathbf{j} + t^4\mathbf{k},\ \ 0 \le t \le 1$.

6. Evaluate the line integral $\int \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y) = (x - y)\mathbf{i} + (xy)\mathbf{j}$ and C is the arc of the circle $x^2 + y^2 = 9$ traversed counterclockwise from (3, 0) to (0, -3).

7. Find the work done by the force field $\mathbf{F}(x, y) = x\sin(y)\mathbf{i} + y\mathbf{j}$ on a particle that moves along the parabola $y = x^2$ from (1, 1) to (2, 4)

13.3: The Fundamental Theorem for Line Integrals

1. Show that the line integral $\int_C 5x^4 y\, dx + \left(x^5 - 8\right) dy$ is independent of the path.

2. Determine whether or not **F** is a conservative vector field. If it is, find a function f such that $\mathbf{F} = \nabla f$.

$$\mathbf{F} = (14x + 8y)\mathbf{i} + (8x + 18y)\mathbf{j}$$

3. Let $\mathbf{F} = (8x\cos y - y\cos x)\mathbf{i} + (-4x^2 \sin y - \sin x)\mathbf{j}$

   a) Show that $\boldsymbol{F}$ is a conservative vector field and find a function f such that $\mathbf{F} = \nabla f$.

   b) Use the potential function f to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the part of the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ in the first quadrat, traced in the clockwise direction.

4. Let $\mathbf{F}(x, y) = x^5 y^6 \mathbf{i} + y^5 x^6 \mathbf{j}$

   a) Find a function f such that $\mathbf{F} = \nabla f$

   a) Use the potential function f to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ along the given curve

   $C.C : \mathbf{r}(t) = \sqrt{t}\mathbf{i} + (1 + t^3)\mathbf{j}\ ,\ \ 0 \le t \le 1$

5. Determine whether or not the vector field is conservative. If it is conservative, find a function f such that $\mathbf{F} = \nabla f$.

$$\mathbf{F}(x, y, z) = 10x\mathbf{i} + 4y\mathbf{j} + 6z\mathbf{k}$$

6. Determine whether or not the vector field is conservative. If it is conservative, find a function f such that $\mathbf{F} = \nabla f$.

$$\mathbf{F}(x, y, z) = 35yze^{7xz}\mathbf{i} + 5e^{7xz}\mathbf{j} + 35xye^{7xz}\mathbf{k}$$

Answers

13.4 Green's Theorem

1. 0
2. 64π
3. 32
4. 486π
5. -196196

13.5: Curl & Divergence

1. 12
2. $(x^2 - z^2)\mathbf{j}$
3. $-2e^x \cos(y)\mathbf{k}$
4. $36r$
5. $e^{xz}(2z\cos yz - x)$
6. a
7. b

13.6: Parametric Surfaces & their areas

1. $x = x,\ z = \sqrt{4 - x^2 - y^2},\ y = y,\ x^2 + y^2 \le 2$
2. a
3. e
4. both
5. $\frac{\pi}{6}(37\sqrt{37} - 1)$

13.7: Surface Integrals

1. $32\sqrt{2}\pi$
2. 136
3. 512π
4. $\frac{1}{6}$

10.3: Dot Product

1. $x = -2,\ x = -4$
2. 20.4
3. $\frac{\pi}{4}$

4. 18.2 lb
5. 12.52

10.4: Cross Product/Scalar Triple Product

1. $\frac{\mathbf{i}}{\sqrt{3}} - \frac{\mathbf{j}}{\sqrt{3}} - \frac{\mathbf{k}}{\sqrt{3}}$
2. $|\mathbf{u} \times \mathbf{v}| = 35$
3. $\frac{\mathbf{i}}{\sqrt{3}} - \frac{\mathbf{j}}{\sqrt{3}} - \frac{\mathbf{k}}{\sqrt{3}}$
4. 18

10.5: Equations of Lines & Planes

1. $\frac{\sqrt{30}}{6}$
2. $27x + 4y + 32z = -33$
3. $x = 2 - t,\ y = -2 + 2t,\ z = 4 + 5t$
4. $x = -2 + 4t,\ y = 1 + 2t,\ z = 1 + 4t$
5. $6x - 66y + 78z = -132$
6. $3x + 7y + 18z = 55$
7. $x = -6 + 7t,\ y = 9 + 3t,\ z = 3 - 7t$
8. $(-7,\ -6,\ 7)$
9. $-3x - 27y + 9z = -27$

8. $(-7, -6, 7)$
9. $-3x - 27y + 9z = -27$

10.9: Motion in space

1. $r'(t) = \cos t\,\mathbf{j} - \sin t\,\mathbf{k}$
2. $\frac{5}{6}\mathbf{i} + \frac{9}{4}\mathbf{j} + \frac{1}{5}\mathbf{k}$
3. $a(t) = 4e^{2t}\mathbf{j} + 4e^{-2t}\mathbf{k}$
4. $v = 195e^{15t}\mathbf{i} - 180e^{-18t}\mathbf{j}$

11.6: Directional Derivatives & The gradient vector

1. $\langle 0, 1 \rangle$
2. $\left\langle 2xe^{y}\sqrt{z},\ x^{2}e^{y}\sqrt{z},\ \frac{x^{2}e^{y}}{2\sqrt{z}} \right\rangle$
3. $10x + 18y = 136$
4. $15x + 18y + 40z = 353$
5. $-64 / \sqrt{6}$
6. $-5\pi / (2\sqrt{198})$

11.7: Maximum & minimum values

1. (-6, 6), (-4, 4), (10, -10)
2. Min at $(4,-1)$, $f(4,-1)=-11$, Max point - none, Saddle point - none
3. 2, 2, 2
4. Min at $(-3/4,-7/16)$
5. $f(1,1)$ is a local minimum
6. $(0,\ 3\pi n)$
7. 30

12.2: Double Integrals over general regions

1. 18
2. 72
3. 7/2
4. 1/5
5. $\dfrac{e^{16}-1}{8}$
6. $\dfrac{16}{3}\pi$

12.3: Double Integrals in Polar coordinates

1. $\frac{(-e^{-4})\pi}{2}$
2. 292.45
3. 40.5π
4. 3π
5. 20.11

12.6: Triple Integrals in Cylindrical Coordinates

1. $\frac{2750}{3}\pi$
2. 32.55
3. 0
4. $\frac{\pi}{4}\ e^2 - 1$

12.7: Triple Integrals in Spherical Coordinates

1. $\frac{\pi}{16}(e^{625} - e^{16})$
2. πa^3
3. 16.56
4. 39.99
5. 167.55

13.2: Line Integrals

1. 291.6
2. 35.36
3. 64/15
4. $9031\sqrt{5}$
5. 23/88
6. $(9/2)(1+(3\pi/2))$
7. $(15 + \cos 1 - \cos 4)/2$

13.3: The Fundamental Theorem for Line Integrals

1.

$$\int_C 5x^4 y\,dx + \left(x^5 - 8\right) dy = \int_C M(x,y)\,dx + N(x,y)\,dy$$

with $M(x,y) = 5x^4 y$ and $N(x,y) = x^5 - 8$.

$$M_y = \frac{\partial M}{\partial y} = 5x^4 \text{ and}$$

$$N_x = \frac{\partial N}{\partial x} = 5x^4$$

Since $M_y = N_x$, the line integral is independent of the path.

2. $f(x,y) = 7x^2 + 8yx + 9y^2 + K$

3. a) $\mathbf{F}(x,y) = P(x,y)\mathbf{i} + Q(x.y)\mathbf{j}$ with $P(x,y) = 8x\cos y - y\cos x$ and $Q(x,y) = -4x^2\sin y - \sin x$. Since $P_y = Q_x$, the vector field is conservative. The potential function is $f(x,y) = 4x^2\cos y - y\sin x + K$

   b) $\int_C \mathbf{F}\cdot d\mathbf{r} = f(2,0) - f(0,3) = 16 - 0 = 16$

4. a) $f(x,y) = \frac{1}{6}x^6 y^6$ b) $\int_C \mathbf{F}\cdot d\mathbf{r} = f(1,2) - f(0,0) = \frac{32}{3}$

5. $5x^2 + 2y^2 + 3z^2 + K$

6. $5ye^{7xz} + K$