MAT 267 – Calculus for Engineers-III	Student Name: KEY
Instructor :	Student ID :
TEST 2	Class Time :

Honor Statement

By signing below I confirm that I have neither given nor received any unauthorized assistance on this exam. This includes any use of a graphing calculator beyond those uses specifically authorized by the School of Mathematical and Statistical Sciences and my instructor. Furthermore, I agree not to discuss this exam with anyone until the exam testing period is over. In addition, my calculator's memory and menus may be checked at any time and cleared by any testing center proctor or School of Mathematical and Statistical Sciences instructor.

Signature:______________________________ Date:_________________

Instructions:

1. The exam consists of two parts: multiple choice, worth 42%, and free response (show your work), worth 58%. Please read each problem carefully.
2. There are 7 multiple choice questions worth 6 points each. Please fill in the table provided on the last page
3. Provide complete and well-organized answers in the free response section.
4. Answers in the free response section without supporting work will be given zero credit. Partial credit is granted only if work is shown.
5. No calculators with Qwerty keyboards or ones like the Casio FX-2, TI-89, TI-92, or TI-*n*spire that do symbolic algebra may be used.
6. Proctors reserve the right to check calculators.
7. Please request scratch paper from me if you need it.
8. The use of cell phones is prohibited. TURN YOUR CELL PHONE OFF! Do not allow your cell phone to ring while you are taking the exam. Do not use the calculator on your cell phone. If a proctor sees you using a cell phone, they will take your exam and you will be reported to the Dean of Students for cheating.
9. *PLEASE NOTE:* "Any student who accesses a phone or any internet-capable device during an exam for any reason automatically receives a score of zero on the exam. All such devices must be turned off and put away and made inaccessible during the exam.

PART I Multiple Choice

1. [6 pts] Suppose $z = x\cos y \quad x = s + t,$ and $y = \sin(st)$. Find $\dfrac{\partial z}{\partial s}$
(Do not simplify.)

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$$

$$= (\cos y)\cdot 1 + (-x\sin y)\cos(st) +$$

A. $\cos y - x\,(\sin y)(\cos(st)t)$
B. $\sin y - x\,(\cos y)(\cos(st)t)$
C. $\cos y - x\,(\sin y)(\cos(st))$
D. $\cos y - x\,(\sin y)(\cos(st)s)$
E. None of these

2. [6 pts] Let $f(x,y) = \frac{1}{4}x^3y^2$ Which direction gives the maximum rate of change in $f(x,y)$ at the point (1,2)

A) $< 12,4 >$ B) $< 3,2 >$ C) $< \frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}} >$ D) $< \frac{3}{5}, \frac{4}{5} >$

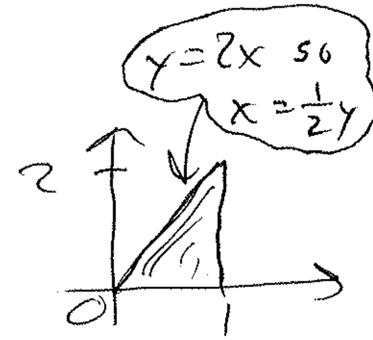
E) none of these

$$\nabla f = \left\langle \frac{3}{4}x^2y^2, \frac{2}{4}x^3y \right\rangle = \langle 3, 1 \rangle, \quad |\nabla f| = \sqrt{10}$$

3. [6 pts] Which is the reversing order of integration of $\int_0^1\int_0^{2x} f(x,y)\,dydx$.

A) $\int_0^2\int_0^{y/2} f(x,y)\,dxdy$ B) $\int_0^2\int_{y/2}^{1} f(x,y)\,dxdy$ C) $\int_0^1\int_0^{2y} f(x,y)\,dxdy$

D) $\int_0^{2x}\int_0^1 f(x,y)\,dxdy$ E) none of these

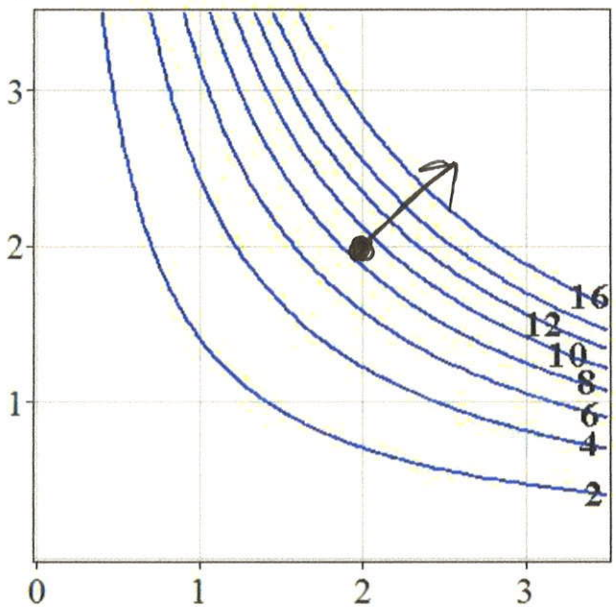


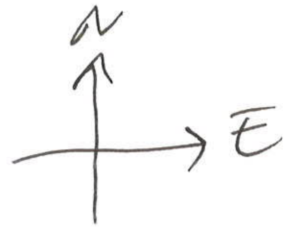
4. [6 pts] Find the domain of $f(x,y) = \ln(4 - x^2 - y^2)$.

A) $\{(x,y) \mid x^2 + y^2 < 4\}$
B) $\{(x,y) \mid x^2 + y^2 > 4\}$
C) $\{(x,y) \mid x^2 + y^2 \le 2\}$
D) $\{(x,y) \mid x^2 + y^2 \ge 4\}$
E) none of these

$$4 - x^2 - y^2 > 0$$
$$x^2 + y^2 < 4$$

5. [6 pts]





Consider the contour plot of the function $z = f(x,y)$ above. Which of the following has the greatest value?

A. $f_x(2,2)$

B. $f_y(2,2)$

C. The rate of ascent if we started at $(2,2)$ and traveled northeast

D. The rate of ascent if we started at $(2,2)$ and traveled west.

6. [6 pts] Which of the following are true

I. ✓ If f is a differentiable function of two variables, and has a local minimum at (a,b), then $\nabla f(a,b) =< 0,0 >$

~~II.~~ If $f(x,y) = x^2$, then $\nabla f(x,y) = 2x$.

~~III.~~ $\int_0^1 \int_0^x f(x,y)dydx = \int_0^x \int_0^1 f(x,y)dxdy$

IV. ✓ The gradient is always orthogonal to the level curves or surfaces.

A. II and IV B. I. III and IV C. II and III D. I and IV E. They are all true

$$A = \frac{1}{2}BH$$

$$\frac{dA}{dt} = \frac{1}{2}\frac{dB}{dt}H + \frac{1}{2}B\frac{dH}{dt}$$

7. [6 pts] The base B, and height H of a triangle change with time t, at a certain instant the dimensions are B = 4cm H=5 cm and B is increasing at a rate of 2 cm/s while H is decreasing at a rate of 1cm/s. At that instant the rate of change of the area of the triangle is

A) 3 cm^2/s B) 4 cm^2/s C) 5 cm^2/s D) 20 cm^2/s E) none of these.

PART II – Free response. Show all your work.

8. [15 pts] Find the directional derivative of the function $f(x,y) = x^2y^3 - 4y$ at the point (2,-1) in the direction of the vector $\boldsymbol{v} = 3\boldsymbol{i} + 4\boldsymbol{j}$. **Give an exact, not decimal, answer.**

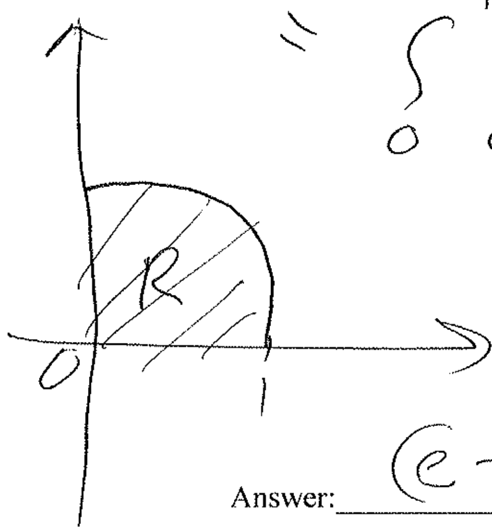
$$u = \left\langle \frac{3}{5}, \frac{4}{5} \right\rangle$$

$$\nabla f = \langle 2xy^3, 3x^2y^2 - 4\rangle \text{ at } (2,-1) = \langle -4, 8\rangle$$

$$D_u f = \nabla f \cdot u = \langle -4, 8\rangle \cdot \left\langle \frac{3}{5}, \frac{4}{5}\right\rangle = -\frac{12}{5} + \frac{32}{5} = \frac{20}{5} = 4$$

Answer: 4

9. [14 pts] Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} 2e^{x^2+y^2}\,dy\,dx$ by converting to polar coordinates.



$$= \int_0^{\pi/2}\int_0^1 2e^{r^2} r \, dr\, d\theta$$

$$\left(\int_0^1 2re^{r^2} dr = e^{r^2}\Big|_0^1 = e^1 - e^0 = e - 1 \right)$$

Answer: $(e-1)\pi/2$

$F(x,y,z)$

10. [14 pts] Find an equation of the tangent plane to the surface $x + y^2 + z^3 - e^z + 1 = 0$ at the point $(-4,2,0)$.

$$\frac{\partial z}{\partial x} = \frac{-F_x}{F_z} = \frac{-1}{3z^2 - e^z} = 1 \text{ at } (-4,3,0)$$

$$\frac{\partial z}{\partial y} = \frac{-F_y}{F_z} = \frac{-2y}{3z^2 - e^z} = \frac{-4}{-1} = 4$$

$$z = 0 + 1\cdot(x-(-4)) + 4(y-2)$$

$$z = x + 4 + 4y - 8$$

Answer: $z = x + 4y - 4$

11. [15 pts] Find the local maximum and minimum values and saddle point(s), if any of the function $f(x, y) = x^2 + 2y^2 + 2xy - 6x + 8y$

$$\begin{cases} f_x = 2x + 2y - 6 = 0 \\ f_y = 4y + 2x + 8 = 0 \end{cases}$$

System $\begin{cases} 2x + 2y = 6 \\ 2x + 4y = -8 \end{cases}$

$-2y = 14$

$y = -7$

$x = 10$

Critical Pt $(10, -7)$

$f_{xx} = 2 \quad f_{yy} = 4$

$f_{xy} = 2$

$D = 2 \cdot 4 - 2^2 = 8 - 4 = 4$

$D > 0, \; f_{xx} > 0$

Answer: so $(10, -7)$ is a local minimum