FINAL EXAM PRACTICE

Trig Review

1. Complete the table:

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin(\theta)$					
$\cos(\theta)$					

2. Find all values of θ, $0 \leq \theta < 2\pi$ so that

(a) $\cos(\theta) = \frac{1}{2}$ (b) $\sin(\theta) = \frac{-1}{2}$ (c) $\cos(3\theta) = 0$

I. Tangent lines to parametric curves.

1. Find an equation of the tangent line to the curve at the point corresponding to the value of the parameter:

(a) $x = e^{\sqrt{t}}, y = t - \ln(t^9);\ t = 1$

(b) $x = t\sin(t),\ y = t\cos(t);\ t = 6\pi$

2. Given the parametric curve $x = e^{t/2}\sin(t), y = e^{t/2}\cos(t)$, find $\frac{dy}{dx}$ at the point corresponding to $t = \frac{\pi}{6}$.

3. Find the points on the curve $x = 2\cos(t), y = \sin(2t)$ where the tangent is horizontal or vertical.

II. Areas of regions bounded by parametric curves.

1. Use parametric equations of the ellipse, $\frac{x^2}{16} + \frac{y^2}{9} = 1$, to find the area that it encloses in the first quadrant.

2. Find the exact area below the parametric curve $x = t + \sqrt{t},\ y = 2 + 3t - t^3,\ 0 \leq t \leq 2$, and above the x-axis.

III. Length of a curve given in parametric form

1. Find the exact length of the curve $x = 3t^2, y = 2t^3, 0 \leq t \leq 1$.

2. Find the exact length of the parametric curve: $x = e^t\cos(t), y = e^t\sin(t), 0 \leq t \leq \frac{\pi}{5}$.

3. Set up the integral to find the length of the curve $x = \frac{1}{t}, y = \ln(t), 1 \leq t \leq 2$.

IV. Polar coordinates and cartesian coordinates of a point

1. The polar coordinates of a point are given. Find the Cartesian coordinates of the point.

(a) $\left(-1, \frac{\pi}{2}\right)$ **(b)** $\left(3, \frac{2\pi}{3}\right)$ **(c)** $\left(2\sqrt{2}, \frac{5\pi}{4}\right)$

2. The Cartesian coordinates of a point are given.

(i) Find polar coordinates (r, θ) of the point with $r>0$ and $0\leq\theta<2\pi$

(ii) Find polar coordinates (r, θ) of the point with $r<0$ and $0\leq\theta<2\pi$

(a) $(-1, 1)$ **(b)** $(-\sqrt{3}, -1)$ **(c)** $(-2, -3)$

V. Cartesian equation for a curve given in polar form and vice versa.

1. Find a Cartesian equation for the curve described by the given polar equation: $r = 3\sin(\theta)$

2. Find the polar equation for the curve represented by the given Cartesian equation: $x + \sqrt{3}y = 4$

3. Find a Cartesian equation for the curve $\theta = \frac{\pi}{4}$

4. Find a Cartesian equation for the curve $r = \frac{6}{\sin(\theta) - 2\cos(\theta)}$

VI. Area of the region bounded by polar curves

1. Find the exact area of the inner loop of the polar curve $r = 1 + 2\cos(\theta)$.

2. Find the area of the region that lies inside both curves: $r = 4\sin(\theta)\,, r = 4\cos(\theta)$.

3. Find the area enclosed by one loop of the curve $r = \sin(2\theta)$.

4. Find the exact area of that part of the polar curve $r = \frac{1}{\sqrt{\theta}}, 0 < \theta \leq 2\pi$, that is in quadrant II.

VII. Slope of the tangent line to a given polar curve.

1. Find the points on the curve where the tangent line is horizontal: $r = 5(1 - \cos(\theta))$.

2. Find the slope of the tangent to the given polar curve at the point specified by the value of θ:

(a) $r = \frac{1}{\theta}, \theta = \pi$ **(b)** $r = \sin(3\theta)\,, \theta = \frac{\pi}{3}$ **(c)** $r = 2 - \sin(\theta), \ \theta = \frac{\pi}{3}$

VIII. The Maclaurin/Taylor series for $f(x)$ centered at a

1. Find the first four nonzero terms of the Taylor series for $f(x) = \ln(x)$ at $a = 1$. Use that result to approximate $\ln(0.9)$.

2. Find the first four nonzero terms of the Taylor series for $f(x) = \sqrt{x}$ at $a = 4$. Use that result to approximate $\sqrt{4.5}$.

3. Find the first four non-zero terms of the Maclaurin series for $f(x) = (16 + x)^{1/4}$. Use that result to approximate $\sqrt[4]{14}$.

4. Find the first four non-zero terms of the Maclaurin series for $f(x) = \frac{1}{\sqrt{4+x}}$.

5. Use a known Maclaurin series to find the Maclaurin series for the following functions:

(a) $f(x) = \cos\left(\frac{2x}{3}\right)$ **(b)** $f(x) = e^{-x}$

(c) $f(x) = \begin{cases} \frac{\sin(x)}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$ **(d)** $f(x) = \frac{1}{1+2x^2}$

IX. The radius of convergence and interval of convergence of a power series

1. Find the radius of convergence and interval of convergence of each power series:

(a) $\sum_{n=1}^{\infty} \frac{x^n}{2^{2n} n^5}$ **(b)** $\sum_{n=1}^{\infty} (-1)^n \frac{(x-2)^n}{n5^n}$ **(c)** $\sum_{n=0}^{\infty} \frac{(-3)^n x^n}{n!}$

X. Definite integral for a volume (washers & shells).

1. Set up and evaluate an integral for finding the volume of the solid obtained by rotating the region bounded by the given curves about **(i)** the x-axis and **(ii)** the y-axis:

(a) $y = e^x, y = 0, x = 0,$ and $x = 1.$

(b) $y = 8 - x^3, y = 0,$ and $x = 0.$

XI. The area enclosed by two curves

1. Sketch the region enclosed by the given curves, and then find the area of the region;

(a) $y = x^2,\ y = 4x - x^2$

(b) $y = \cos(x),\ y = \sin(2x)$ with $0 \le x \le \frac{\pi}{2}$

(c) $x = 2y^2,\ x + y = 1$

(d) $y = x^3,\ 12x - y = 16$ and the x-axis

XII. Use Partial Fraction Decomposition to evaluate the indefinite integral

1. $\int \frac{3x+2}{(x-1)(x+2)}\,dx$ **2.** $\int \frac{4}{x^2(3x+2)}\,dx$ **3.** $\int \frac{x-2}{(x-1)(x^2+1)}\,dx$

XIII. Find the antiderivative to evaluate the definite integral

1. $\int_0^{\pi/3} \sin^3(\theta)\cos^2(\theta)\,d\theta$ **2.** $\int_2^3 \frac{2}{(3t-4)^{2/3}}\,dt$

3. Use integration by parts to find the following improper integrals:

(a) $\int_0^\infty 6xe^{-2x}dx$ **(b)** $\int_0^4 \frac{1}{\sqrt{x}}\ln(2x)\,dx$

XIV. Evaluate the indefinite integral

1. $\int \frac{\sin(x)}{1+\cos^2(x)}\,dx$ **2.** $\int \frac{\sin(x)}{1+\cos(x)}\,dx$

3. Use integration by parts to find

(a) $\int 4xe^{x/3}\,dx$ **(b)** $\int 5t\cos(\pi t)\,dt$

ANSWERS:

Trig Review

1.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin(\theta)$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos(\theta)$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

2. (a) $\theta = \frac{\pi}{3}, \frac{5\pi}{3}$ (b) $\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$ (c) $\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$

I.

1. **(a)** $y = -\frac{16}{e}(x-e)+1$ **(b)** $y = \frac{x}{6\pi} + 6\pi$
2. $\frac{\sqrt{3}-2}{1+2\sqrt{3}}$
3. horizontal tangent at $(-\sqrt{2},-1), (-\sqrt{2},1), (\sqrt{2},-1), (\sqrt{2},1)$; vertical tangent at $(2,0), (-2,0)$.

II.

1. Using $x = 4\cos(t), y = 3\sin(t),\ 0 \le t \le \frac{\pi}{2}$; we get $A = 3\pi$. **2.** $6 + \frac{20}{7}\sqrt{2}$

III.

1. $4\sqrt{2} - 2$ **2.** $\sqrt{2}\left(e^{\pi/5} - 1\right)$ **3.** $\int_1^2 \sqrt{\frac{1}{t^4} + \frac{1}{t^2}}\, dt = \int_1^2 \frac{\sqrt{1+t^2}}{t^2}\, dt$

IV.

1. **(a)** $(0,-1)$ **(b)** $\left(-\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$ **(c)** $(-2,-2)$

2. **(a)(i)** $\left(\sqrt{2}, \frac{3\pi}{4}\right)$ **(a)(ii)** $\left(-\sqrt{2}, \frac{7\pi}{4}\right)$
(b)(i) $\left(2, \frac{7\pi}{6}\right)$ **(b)(ii)** $\left(-2, \frac{\pi}{6}\right)$
(c) (i) $\left(\sqrt{13},\ \pi + \tan^{-1}\left(\frac{3}{2}\right)\right)$ **(c)(ii)** $\left(-\sqrt{13},\ \tan^{-1}\left(\frac{3}{2}\right)\right)$

V.

1. $x^2 + \left(y - \frac{3}{2}\right)^2 = \frac{9}{4}$ **2.** $r = \frac{4}{\cos(\theta) + \sqrt{3}\sin(\theta)}$ **3.** $y = x$ **4.** $y = 2x + 6$

V I.

1. $\pi - \frac{3\sqrt{3}}{2}$ **2.** $2\pi - 4$ **3.** $\frac{\pi}{8}$ **4.** $\frac{1}{2}\ln(\pi) - \frac{1}{2}\ln\left(\frac{\pi}{2}\right) = \frac{1}{2}\ln(2)$

VII.

1. $(0,0), \left(-\frac{15}{4}, -\frac{15\sqrt{3}}{4}\right), \left(-\frac{15}{4}, \frac{15\sqrt{3}}{4}\right)$ **2. (a)** $-\pi$ **(b)** $-\sqrt{3}$ **(c)** $\frac{2-\sqrt{3}}{1-2\sqrt{3}}$

VIII.

1. $f(x) \approx (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4$ so $\ln(0.9) = f(0.9) \approx -0.1053583$
2. $f(x) \approx 2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2 + \frac{1}{512}(x-4)^3$ so $\sqrt{4.5} = f(4.5) \approx 2.121338$
3. $f(x) \approx 2 + \frac{1}{32}x - \frac{3}{4096}x^2 + \frac{7}{262144}x^3$ so $\sqrt[4]{14} = (16-2)^{1/4} = f(-2) \approx 1.934357$
4. $f(x) \approx \frac{1}{2} - \frac{1}{16}x + \frac{3}{256}x^2 - \frac{5}{2048}x^3$
5. **(a)** $f(x) \approx 1 - \frac{2}{9}x^2 + \frac{2}{243}x^4 - \frac{4}{32805}x^6$

   (b) $f(x) \approx 1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3$

   (c) $f(x) \approx 1 - \frac{1}{6}x^2 + \frac{1}{120}x^4 - \frac{1}{5040}x^6$

   (d) $f(x) \approx 1 - 2x^2 + 4x^4 - 8x^6$

IX.

1 (a) $R = 4,\ I = (-4,4)$ **(b)** $R = 5,\ I = (-3,7)$ **(c)** $R = \infty,\ I = (-\infty,\infty)$

X.

1 (a) (i) Disk: $\pi \int_0^1 e^{2x}\,dx = \frac{e^2-1}{2}\pi$ **(ii)** Shell: $2\pi \int_0^1 xe^x\,dx = 2\pi$

(b) (i) Disk: $\pi \int_0^2 (8-x^3)^2\,dx = \frac{576}{7}\pi$ **(ii)** Shell: $2\pi \int_0^2 x(8-x^3)\,dx = \frac{96}{5}\pi$

XI.

1. 8/3 **2.** 1/2 **3.** 9/8 **4.** 4/3

XII.

1. $\frac{4}{3}\ln|x+2| + \frac{5}{3}\ln|x-1| + \mathrm{C}$
2. $-\frac{2}{x} + 3\ln|3x+2| - 3\ln|x| + \mathrm{C} = -\frac{2}{x} + 3\ln\left|\frac{3x+2}{x}\right| + \mathrm{C}$
3. $\frac{1}{4}\ln|x^2+1| - \frac{1}{2}\ln|x-1| + \tan^{-1}(x) + \mathrm{C}$

XIII.

1. $47/480$ **2.** $2\left(5^{1/3} - 2^{1/3}\right)$ **3. (a)** $3/2$ **(b)** $4\ln(8) - 8$

XIV.

1. $-\tan^{-1}(\cos(x)) + \mathrm{C}$ **2.** $-\ln(1+\cos(x)) + \mathrm{C}$

3. (a) $12xe^{x/3} - 36e^{\frac{x}{3}} + \mathrm{C}$ **(b)** $\frac{5t}{\pi}\sin(\pi t) + \frac{5}{\pi^2}\cos(\pi t) + \mathrm{C}$