Practice for Test 2, MAT 266

Algebraically, evaluate the following integrals.

1. $\int_0^{\infty} \frac{1}{x^2+7x+12}\,dx$ 2. $\int_{-3}^{2} \frac{dx}{x^5}$ 3. $\int_{-\infty}^{\infty} x^2 e^{-x^3}\,dx$

4. $\int_2^{\infty} \frac{3}{(x+4)^{3/2}}\,dx$ 5. $\int_{-\infty}^{-3} \frac{1}{\sqrt{8-x}}\,dx$

6. Determine whether the sequence converges or diverges. If it converges, find the limit:
$a_n = e^{1/n}$

7. Determine whether the sequence converges or diverges. If it converges, find the limit:
$a_n = \sqrt[n]{3^{2n+1}}$

8. Test the series for convergence or divergence. If convergent, find the sum

(a) $\sum_{n=3}^{\infty} \frac{1}{4}^n$ (b) $\sum_{k=1}^{\infty} \frac{(-3)^{k+1}}{4^{2k}}$

Find the area of the specified region **exactly** by sketching the region, labeling the intersection points, clearly showing the integral(s) as well as the antiderivatives.

9. $y = x$ and $y = \frac{x^2}{9}$ 10. $x = y^2$ and $x = y + 2$ 11. $x = y^2$ and $x = -2y^2 + 3$

12. $y = \frac{7}{x}, y = 9x$ and $y = \frac{1}{9}x$ 13. $y = \sin\frac{\pi x}{2}$ and $y = x$

Set up the integral and find the volume of the bounded specified region with the **washer/disc** method. Include limits of integration. Include a sketch.

14. $y = 2\sqrt{x}, x = 0$ and $y = 10$, rotated about the x-axis.

15. Below by $y = 3x^2 + 1$ and above by $y = 4$, rotated about the line $y = 4$.

16. $y = \frac{1}{x}, y = 0, x = 1, x = 3$, rotated about the line $y = -1$.

Set up the integral to find the volume of the bounded specified region with the **washer/disc** method. Include limits of integration. Include a sketch. **Do not evaluate.**

17. $2x + 3y = 6$, $(y-1)^2 = 4 - x$, rotated about the line $x = -5$.

18. $y = \sqrt[3]{x}, x = 4y$ in Quadrant I, rotated about the y-axis.

Set up the integral and find the volume of the bounded specified region with the **cylindrical shells** method. Include limits of integration. Include a sketch.

19. $x = \sqrt{y}, x = 0, y = 1$ about x-axis. 20. $x + 3 = 4y - y^2, x = 0$ about x-axis

21. $y = x^3, y = 8, x = 0$, about the y-axis

Set up the integral to find the volume of the bounded specified region. Include limits of integration. Include a sketch. **Do not evaluate.**

22. $y = \sqrt{x-1}, y = 0, x = 5$ about $y = 3$.
23. $y = x^4, y = \sin\frac{\pi x}{2}$, about $x = -1$.

24. Find the length of the arc formed by $y = \frac{4\sqrt{2}x^{3/2}}{3} - 1$ from $x = 0$ to $x = 1$. Clearly show the integral(s) as well as the antiderivatives.

25. Find the length of the arc formed by $y = \frac{x^3}{3} + \frac{1}{4x}$ from $x = 1$ to $x = 3$. Clearly show the integral(s) as well as the antiderivative.

26. Find the length of the arc formed by $y = \frac{1}{3}(x^2 + 2)^{3/2}$ from $x = 0$ to $x = 3$. Clearly show the integral(s) as well as the antiderivative.

27. Find the length of the arc formed by $9x^2 = 4y^3$ from (0,0) to $(2\sqrt{3}, 3)$. Clearly show the integral(s) as well as the antiderivative.

28. A spring has a natural length of 0.2m. If the force required to keep it stretched to a length of 0.6m is 6 Newtons, how much work is required to stretch it from 0.2m. to 0.4m? Clearly show the integral(s) as well as the antiderivative.

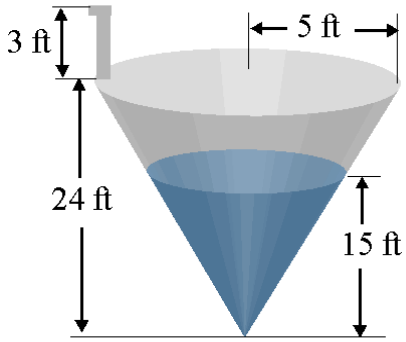
29. An aquarium is 2m long, 1m wide and 1m deep. It is full of water. Find the work needed to pump out half of the water from the aquarium. [Use the fact that the density of water is 1000kg/m^3]

30. A circular swimming pool has a diameter of 24ft. The sides are 5ft high and the depth of the water is 4ft. How much work is required to pump all of the water over the side. Water weighs 62.5 lbs/ft^3

31. A spherical tank of radius 10 feet is filled with water. Find the work done in pumping all of the water through the top. The weight of the water is 62.5 lbs/ft^3

32. Find the volume of a frustum of a cone of height 5, lower base radius 30 and top radius 20.

33. The tank shown contains water to a depth of 15 ft.
Find the work required to pump the water out of the spout (the spout is 3 ft high).
[Use 62.5 lb/ft^3 as the weight density of the water].



ANSWERS:

1. $\ln\frac{4}{3}$ 2. Divergent 3. Divergent 4. $\sqrt{6}$ 5. Diverges

6. 1 7. 9 8. (a) Convergent to 1/48. (b) Convergent to 9/19

9. 13.5 10. 4.5 11. 4 12. 28 ln3 13. $\frac{4}{\pi}-1$

14. 3926.9908 15. $\frac{48\pi}{5}$ 16. $2\pi\ln(3)+\frac{1}{3}$

17. $\pi\int_0^{\frac{7}{2}}\left[9-(y-1)^{2^2}-8-\frac{3}{2}y^2\right]dy$ 18. $\pi\int_0^2[(4y)^2-(y^3)^2]dy$

19. $\frac{4\pi}{5}$ 20. $\frac{16\pi}{3}$ 21. $\frac{96\pi}{5}$

22. $2\pi\int_0^2[(3-y)(5-y^2-1)]dy$ 23. $2\pi\int_0^1\left[(x+1)\sin\frac{\pi x}{2}-x^4\right]dx$

24. $\frac{13}{6}$ 25. $\frac{53}{6}$ 26. 12 27. $\frac{14}{3}$

28. 0.3 newtons-meters 29. 2450 Joules 30 108,000π ft-lb

31. 2.61×10^6ft-lbs 32. $\frac{9500}{3}\pi$ 33. 48065π

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Additional Problems

1. For the sequence $\{a_n\}=\left\{\frac{1}{2},\frac{-2}{3},\frac{3}{4},\frac{-4}{5},\frac{5}{6}\ \ldots\right\}$ the limit is _____.
2. Determine if the series converges or diverges:

$$\sum_{n=1}^{\infty}\frac{3n}{2n+3}$$

3. Find the sum if it exists

$$\sum_{k=0}^{\infty}7\left(\frac{3}{4}\right)^k$$

4. For the sequence

$$\{a_n\}=\left\{\frac{3n}{2n+3}\right\}$$

the limit is _____.

5. Set up, but do not evaluate, an integral to find the length of the curve $y=\sin x$ for $0\le x\le1$.
6. Set up, but do not evaluate, an integral to find the length of the curve $y^2=x$ from $0,\ 0$ to $9,\ 3$.

7. Calculate the work required to lift a 10 m chain over the side of a building, assuming that the chain has a density of 8 kg/m.
8. A tank has the shape of a right circular cylinder with height 10 m and radius 4 m. It is filled to a height of 8 m. Find the work required to empty the tank by pumping all of the water to the top of the tank (the density of water is 1000 kg/m^3).
9. Find the volume of the solid generated by revolving the region $D = \{(x, y) \mid 0 \le y \le -x \ln x,\ 0 \le x \le 1\}$ about the x-axis.
10. Find the volume of the solid generated by revolving the region bounded by $y = x - 1,\ y = 1,\ \&\ x = 1$ about the y-axis.
11. Find the area enclosed by the curves $y = x^2,\ \&\ y = 4x - x^2$
12. Find the area enclosed by the curves $y = \sin(\pi x / 2),\ \&\ y = x$
13. Determine if the improper integral is convergent or divergent. If convergent, evaluate it: $\int_{-3}^{2} (1/x^5)\,dx$
14. Determine if the improper integral is convergent or divergent. If convergent, evaluate it: $\int_{0}^{\infty} e^{-6x}\,dx$

Answers:

1. Does not exist
2. Diverges
3. 28
4. 3/2
5. $\int_0^1 \sqrt{1+\cos^2 x}\,dx$
6. $\int_0^3 \sqrt{1+4y^2}\,dy$
7. 3920 J
8. 7526400π J
9. $2\pi / 27$
10. $2\pi / 3$
11. 8/3
12. $(2/\pi) - (1/2)$
13. diverges
14. Converges to 1/6