MAT 210 Exam 3 Review Questions

Indefinite Integral (Section 13.1)

1. Find the indefinite integrals.
   (a) $\int 2x^4 - 4x^{-2} + 5x^{-5} + 3 \ dx$
   (b) $\int \frac{7}{x} + \frac{1}{3x^7} dx$
   (c) $\int \frac{2}{x^2} - 5\sqrt{x} \ dx$
   (d) $\int e^x - x^{-0.3} \ dx$
   (e) $\int (x+3)(x-2) dx$
   (f) $\int \frac{x^2+5x-2}{x} dx$

   ***Answer*:**

   (a) $\frac{2}{5}x^5 + \frac{4}{x} - \frac{5}{4x^4} + 3x + C$

   (b) $7\ln|x| - \frac{1}{18x^6} + C$

   (c) $-\frac{2}{x} - \frac{10}{3}x^{\frac{3}{2}} + C$

   (d) $e^x - \frac{x^{0.7}}{0.7} + C = e^x - \frac{10}{7}x^{0.7} + C$

   (e) $\int x^2 + x - 6 \ dx = \frac{x^3}{3} + \frac{x^2}{2} - 6x + C$

   (f) $\int x + 5 - \frac{2}{x} \ dx = \frac{x^2}{2} + 5x - 2\ln|x| + C$

2. Find $f(x)$ if $f(0) = -1$ and the derivative $f'(x) = 9e^x + 9$.

   Answer: $f(x) = 9e^x + 9x - 10$

3. The velocity of a particle moving in a straight line is $v(t) = t^2 + 6$. Find the expression for the position, $s(t)$, of the particle at time t, if $s(3) = 0$.

   Answer: $s(t) = \frac{1}{3}t^3 + 6t - 27$

4. Suppose the function $C(x)$ gives the total cost (in dollars) of producing x units of a certain product. The marginal cost of producing the xth unit is $C'(x) = 0.5x + \frac{1}{x}$. If the cost to produce the first unit is 5 dollars, find the cost function $C(x)$.

   Answer: $C(x) = 0.25x^2 + \ln|x| + 4.75$ dollars

$$C(x) = \int C'(x)\,dx = \int 0.5x + \frac{1}{x} dx = 0.25x^2 + \ln|x| + K$$

   $C(1) = 5$, So $5 = 0.25 \cdot 1^2 + \ln|1| + K$. Then solve for constant K: $K = 5 - 0.25 = 4.75$.

Substitution (Section 13.2)

5. Use integration by substitution to find the integrals.

   (a) $\int 16e^{-3x}\,dx$ (can also use short-cut formula)

   (b) $\int (5x-2)^3 dx$ (can also use short-cut formula)

   (c) $\int \frac{1}{2x-5} dx$ (can also use short-cut formula)

   (d) $\int 4xe^{x^2-3} dx$

   (e) $\int x(x^2+1)^{10}\,dx$

   (f) $\int 15x\sqrt{-x^2+7}\,dx$

   (g) $\int (3x^2+1)(x^3+x-2)^9 dx$

   ***Answer*:**

   (a) $16\cdot\frac{e^{-3x}}{-3}+C=-\frac{16}{3}e^{-3x}+C$

   (b) $\frac{(5x-2)^4}{4}\cdot\frac{1}{5}+C=\frac{1}{20}(5x-2)^4+C$

   (c) $\frac{1}{2}\ln|2x-5|+C$

   (d) $2e^{x^2-3}+C$

   (e) $\frac{1}{22}(x^2+1)^{11}+C$

   (f) $-5(-x^2+7)^{\frac{3}{2}}+C$

   (g) $\frac{1}{10}(x^3+x-2)^{10}+C$

Fundamental Theorem of Calculus; Definite Integral; Left Riemann Sum (Sections 13.3, 13.4)

6. Evaluate the definite integrals.

   (a) $\int_0^1 (6x^5+15x^4-9x^2+1)\,dx$

   (b) $\int_2^7 \left(x+\frac{5}{x}\right) dx$

   (c) $\int_1^{10} \frac{1}{x^2} dx$

   (d) $\int_0^6 e^{-x+6}\,dx$

   (e) $\int_{-1}^1 5e^{3x} dx$

   (f) $\int_{e^3}^{e^5} \frac{2}{x} dx$

   (g) $\int_{\ln 3}^{\ln 5} e^{2x} dx$

   Answer:

   (a) 2

   (b) $\frac{45}{2}+5\ln\left(\frac{7}{2}\right)$

(c) $\frac{9}{10}$
(d) $-1+e^6$
(e) $\frac{5}{3}(e^3-e^{-3})$
(f) 4
(g) 8

7. Assume that b is a positive number, solve the following equation for b.

$\int_2^b (2x-4)\,dx = 9$

Answer: $b = 5$

8. Calculate the left Riemann sum for the function $f(x) = 3x^2 + 2x - 3$ over the interval $[1,3]$, with $n = 5$.

Answer: 22.56

$\Delta x = \frac{b-a}{n} = \frac{3-1}{5} = 0.4$, $x_0 = a = 1, x_1 = x_0 + \Delta x = 1.4, x_2 = 1.8,\ x_3 = 2.2,\ x_4 = 2.6.$
$\text{LRS} = \Delta x \cdot \big(f(1) + f(1.4) + f(1.8) + f(2.2) + f(2.6)\big) = 0.4(2 + 5.68 + 10.32 + 15.92 + 22.48) = 22.56$

9. Use a left Riemann sum to estimate the definite integral with $n = 4$ subintervals.

$$\int_2^3 \frac{1}{1+2x}\,dx$$

Answer: 0.18

$\Delta x = 0.25$, $\text{LRS} = 0.25\left(\frac{1}{1+2(2)} + \frac{1}{1+2(2.25)} + \frac{1}{1+2(2.5)} + \frac{1}{1+2(2.75)}\right) = 0.25(0.2 + 0.18 + 0.17 + 0.15) = 0.18$

Applications of Definite Integrals (Section 13.4)

10. A particle moves in a straight line with velocity $v(t) = -t^2 + 8$ meters per second, where t is time in seconds. Find the displacement of the particle between $t = 2$ and $t = 6$ seconds.

Answer: -37 meters

Displacement $= s(6) - s(2) = \int_2^6 v(t)\,dt = \int_2^6 (-t^2+8)\,dt = -\frac{112}{3} \approx -37$ meters.

11. The marginal revenue of the xth box of flash cards sold is $500e^{-0.001x}$ dollars. Find the revenue generated by selling box 101 through 5,000.

Answer: 448,598 dollars

Total revenue generated $= R(5000) - R(101) = \int_{101}^{5000} MR\,dx = \int_{101}^{5000} 500e^{-0.001x}\,dx \approx 448597.54$ dollars

12. Since YouTube first became available to the public in mid-2005, the rate at which video has been uploaded to this site can be approximated by $f(t) = 1.1t^2 - 2.6t + 2.3$ million hours of videos per year ($0 \le t \le 9$), where t is time in years since June 2005. Use a definite integral to estimate the total number of hours of video uploaded from June 2007 to June 2010.

Answer: 23 million hours of video

Total number of hours $= \int_2^5 f(t)\,dt = \int_2^5 (1.1t^2 - 2.6t + 2.3)\ dt \approx 23$ million hours of video

13. Calculate the area of the region bounded by $y = \sqrt{x}$, the x-axis, and the lines $x = 0$ and $x = 16$.

Answer: $\frac{128}{3}$

Area under curve $= \int_0^{16} \sqrt{x}\,dx = \frac{128}{3}$

Integration by Parts (Section 14.1)

Integration by parts formula: $\int u dv = uv - \int v du$

14. Use integration by parts to find the integrals.

(a) $\int 2xe^x dx$

(b) $\int (3x+4)e^{-5x} dx$

(c) $\int \ln x\, dx$

(d) $\int x^2 \ln x\, dx$

***Answer*:**

(a) Let $u = 2x, dv = e^x dx$. Then $du = 2dx$ and $v = e^x$.
Using the formula: $\int u dv = uv - \int v du$ to get

$$\int 2xe^x dx = 2xe^x - \int e^x\, 2dx = 2xe^x - 2e^x + C$$

(b) $-\frac{1}{5}(3x+4)e^{-5x} - \frac{3}{25}e^{-5x} + C = \left(-\frac{3}{5}x - \frac{23}{25}\right)e^{-5x} + C$
(Let $u = 3x + 4, dv = e^{-5x}dx$. Then $du = 3dx$ and $v = -\frac{1}{5}e^{-5x}$)

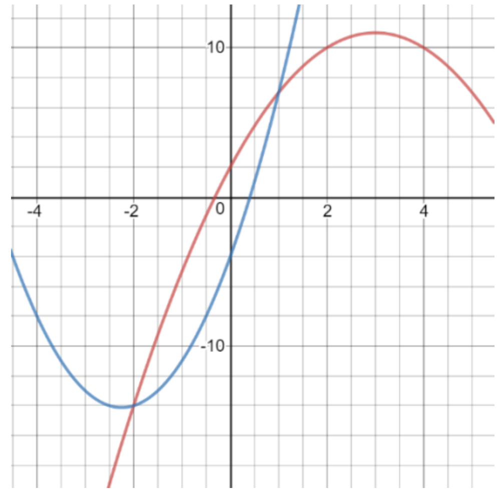
(c) $x \ln x - x + C$ (Let $u = \ln x, dv = dx$)

(d) $\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C$ (Let $u = \ln x, dv = x^2 dx$)

Area between Curves (Section 14.2)

15. Find the area of the region enclosed by the curves of $y = -x^2 + 6x + 2$ and $y = 2x^2 + 9x - 4$.

***Answer*:** 13.5



Find the intersection points: $-x^2 + 6x + 2 = 2x^2 + 9x - 4$

$$0 = 3x^2 + 3x - 6$$

$$0 = 3(x + 2)(x - 1)$$

So $x = -2$ and $x = 1$.

The area enclosed by the curves from -2 to 1 is

$$\int_{-2}^{1} (\text{top} - \text{bottom})\, dx = \int_{-2}^{1} [(-x^2 + 6x + 2) - (2x^2 + 9x - 4)]\, dx$$

$$= \int_{-2}^{1} (-3x^2 - 3x + 6)\, dx$$

$$= -x^3 - \tfrac{3}{2}x^2 + 6x\Big|_{-2}^{1}$$

$$= \left(-1^3 - \tfrac{3}{2}1^2 + 6(1)\right) - \left(-(-2)^3 - \tfrac{3}{2}(-2)^2 + 6(-2)\right) = 13.5$$

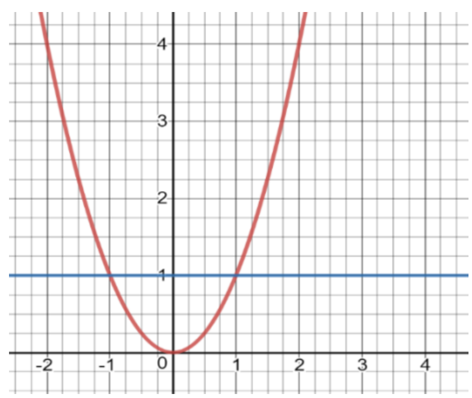
16. Find the area of the region enclosed by the curves of $f(x) = x^2 - x + 5$ and $g(x) = x + 8$.

***Answer*:** $\frac{32}{3}$

17. Find the area of the region between $y = x^2$ and $y = -1$ from $x = -1$ and $x = 1$.

***Answer*:** $\frac{8}{3}$

18. Which of the following calculates the area of the region(s) between the curves $y = x^2$ and $y = 1$ from $x = -1$ to $x = 2$?



A. $\int_{-1}^{2} (x^2 - 1) dx$

B. $\int_{-1}^{2} (1 - x^2) dx$

C. $\int_{-1}^{1} (1 - x^2) dx + \int_{1}^{2} (x^2 - 1) dx$

D. $\int_{-1}^{1} (x^2 - 1) dx + \int_{1}^{2} (1 - x^2) dx$

E. None of the above.

***Answer*:** C

Average Value (Section 14.3)

19. Find the average value of $f(x) = 6e^{0.5x}$ over the interval $[-1, 3]$.
***Answer*:** $3(e^{1.5} - e^{-0.5})$

The average value of a continuous function $f(x)$ over interval $[a, b]$ is $\frac{1}{b-a}\int_a^b f(x)\,dx$.

$$\frac{1}{3-(-1)}\int_{-1}^{3} 6e^{0.5x}\,dx = \frac{1}{4}\cdot 6\cdot \frac{e^{0.5x}}{0.5}\,|_{-1}^{3} = 3e^{0.5x}|_{-1}^{3} = 3(e^{1.5} - e^{-0.5})$$

20. Find the average of the function $f(x) = x^3 - x$ over the interval $[0, 2]$.

***Answer*:** 1

21. Find the average value of the function $f(x) = 6x^2 - 4x + 7$ over the interval $[-2, 2]$.

***Answer*:** 15