Limits and Continuity

1. Calculate the limit: \( \lim_{x \to \infty} \frac{5-8x^2}{7-9x-4x^2} \). Answer: 2

2. Calculate the limit: \( \lim_{x \to -\infty} \frac{4-8x^2}{2x+6} \). Answer: \( \infty \)

3. Calculate the limit: \( \lim_{x \to \infty} \frac{3-2x^2}{2-4x-x^2+x^3} \). Answer: 0

4. Calculate the limit: \( \lim_{x \to 1} \frac{x^2-3x+2}{7x-7} \). Answer: -1/7

5. Calculate the limit: \( \lim_{x \to 3} \frac{x^2-9}{2x-6} \). Answer: 3

6. Calculate the limit: \( \lim_{x \to 3} \frac{x^2-9}{6-2x} \). Answer: -3

7. Calculate the limit: \( \lim_{x \to -7} \frac{x^2+3x-2}{x^2+8x+7} \). Answer: 11/6

8. Calculate the limit: \( \lim_{x \to 2} \frac{2x^2-3}{x-2} \). Answer: DNE

9. Calculate the limit: \( \lim_{x \to 4} \frac{7x-1}{x-4} \). Answer: \( -\infty \)

10. Calculate the limit: \( \lim_{x \to 4^+} \frac{7x-1}{x-4} \). Answer: \( \infty \)

11. Calculate the limit: \( \lim_{x \to 2} \frac{2x^2-3}{x^2-4x+4} \). Answer: \( \infty \)

12. Let \( f(x) = \begin{cases} x^2 - 4, & x < 3 \\ 2x - 1, & x \geq 3 \end{cases} \)

   Is \( f \) continuous at \( x = 3 \)? Justify your answer. Answer: Yes

13. Let \( f(x) = \begin{cases} x^2 - 4, & x < 4 \\ 5x + 1, & x \geq 4 \end{cases} \)

   Is \( f \) continuous at \( x = 4 \)? Justify your answer. Answer: No

14. Let \( f(x) = \begin{cases} 2x, & x \leq 5 \\ \frac{x^2-25}{x-5}, & x > 5 \end{cases} \)

   Is \( f \) continuous at \( x = 5 \)? Justify your answer. Answer: Yes
15. Let \( f(x) = \frac{5x-1}{x^2-5x+6} \). Find the \( x \)-values where \( f \) is not continuous. \[ \text{Answer: } x = 2, 3 \]

16. Use the graph of function \( f(x) \) answer the questions. Write DNE if a limit does not exist.

(a) \( \lim_{x \to 6^+} f(x) \) \[ \text{Answer: 5} \]

(b) \( \lim_{x \to 6^-} f(x) \) \[ \text{Answer: 2} \]

(c) \( \lim_{x \to 6} f(x) \) \[ \text{Answer: DNE} \]

(d) \( f(6) \) \[ \text{Answer: 5} \]

(e) \( \lim_{x \to -8^+} f(x) \) \[ \text{Answer: -6} \]

(f) \( \lim_{x \to -8^-} f(x) \) \[ \text{Answer: -6} \]

(g) \( \lim_{x \to -8} f(x) \) \[ \text{Answer: -6} \]

(h) \( f(-8) \) \[ \text{Answer: -3} \]

(i) \( \lim_{x \to -2^+} f(x) \) \[ \text{Answer: \infty} \]

(j) \( \lim_{x \to -2^-} f(x) \) \[ \text{Answer: 3} \]

(k) \( \lim_{x \to -2} f(x) \) \[ \text{Answer: DNE} \]

(l) \( f(-2) \) \[ \text{Answer: 3} \]
(m) \( \lim_{x \to 10^+} f(x) \)  \hspace{1cm} \text{Answer: 0}

(n) \( \lim_{x \to 10^-} f(x) \)  \hspace{1cm} \text{Answer: 0}

(o) \( \lim_{x \to 10} f(x) \)  \hspace{1cm} \text{Answer: 0}

(p) \( f(10) \)  \hspace{1cm} \text{Answer: 0}

(q) List the \( x \)-values of discontinuous point(s) of the function \( f(x) \). \hspace{1cm} \text{Answer: -8, -2, 6}

Rates of Change

1. Let \( f(x) = x^3 + 2 \). Find the average rate of change of \( f \) over the interval \([1,4]\).
   \hspace{1cm} \text{Answer: 21}

2. Calculate the average rate of change of the given function over the interval \([2,6]\) and specify the unit of measurements.

\[
\begin{array}{c|cccccccc}
 x \text{ (days)} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
\hline
 f(x) \text{ (dollars)} & 6 & 20 & -3 & 8 & 4.6 & 12 & 1.5 & 4.9 & -9 \\
\end{array}
\]

Answer: -2 dollars/day

3. Suppose the following table shows U.S. daily oil imports from a certain country, for 1991–1999 (\( t = 1 \) represents the start of 1991). Use the data in the table to compute the average rate of change of \( I(t) \) over the period 1991–1999 and interpret the meaning of the result.

\[
\begin{array}{c|cccccccc}
 t \text{ (year since 2000)} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
\hline
 I(t) \text{ (million barrels)} & 5.6 & 2.5 & 3.8 & 9.4 & 8.2 & 3.7 & 9.6 & 7.2 & 8.1 \\
\end{array}
\]

Answer: 0.3125 million barrels per year.

U.S. daily oil imports from a certain country increased by an average rate of 0.3125 million barrels per year over the period 1991 to 1999.
4. The graph below shows the population of beetles in a greenhouse $t$ weeks after the season's flowers were planted.

![Graph showing population of beetles](image)

a) Calculate the average rate of change over the interval [1,10].

**Answer:** $425/9$

b) Circle **T** for True or **F** for False next to each statement.

- **T** F  a) During weeks [4,12] the instantaneous rate of change of the population is increasing
- **T** F  b) During weeks [4,12] the instantaneous rate of change of the population is decreasing
- **T** F  c) The average rate of change of the population on [4,12] is less than the instantaneous rate of change of the population at $t = 4$
- **T** F  d) The average rate of change of the population on [0,12] is greater than the instantaneous rate of change of the population at $t = 2$
- **T** F  e) The instantaneous rate of change of the population first increased then decreased

5. Based on data from 1982 to 2017, the number of students taking the AP Calculus exam may be modeled by the function $S(t) = 157.8t^2 - 770.6t + 10268$, where $t$ is the number of years since 1982. Interpret the meaning of $S(16) = 156,682$ and $S'(16) = 9,644$.

- **A.** In 1998 the number of students who took the AP Calculus exam is 156,682 and this number is increasing at a rate of 9,644 student per year.
- **B.** In 1998 the number of students who took the AP Calculus exam is 156,682 and this number is increasing at a rate of 9,644 student in every 16 years after 1982.
- **C.** In 1998 the number of students who took the AP Calculus exam is 156,682 and the test score is increasing at a rate of 9,644 points in every 16 years after 1982.
- **D.** In 1998 the number of students who took the AP Calculus exam is 156,682 and between 1982 and 2017 the number students who took the AP Calculus exam increased by an average of 9,644 students per year.
- **E.** None of these
Differentiation (Power Rule, Product Rule, Quotient Rule, Contant Multiple Rule)

1. Use the limit definition of derivative \( \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} \) to find \( f'(x) \).
   a) \( f(x) = 4x^2 + 2 \)  
      Answer: \( f'(x) = 8x \)
   b) \( f(x) = 8 - 2x \)  
      Answer: \( f'(x) = -2 \)
   c) \( f(x) = 2x^2 + 6x - 7 \)  
      Answer: \( f'(x) = 4x + 6 \)

2. Find the derivative of each function using differentiation rules.
   a) \( f(x) = \frac{8}{\sqrt{x}} - \frac{4}{x} \)  
      Answer: \( f'(x) = -4x^{-3/2} + 4x^{-2} \)
   b) \( g(x) = 3x^2 + 8x - \frac{2}{x^6} - 5x^{2.3} + 7.95 \)  
      Answer: \( g'(x) = 6x + 8 + 12x^{-7} - 11.5x^{1.3} \)
   c) \( h(x) = 8x(3x^2 - 2x + 9) \)  
      Answer: \( h'(x) = 72x^2 - 24x + 72 \)
   d) \( k(x) = \frac{3x+2}{4x} \)  
      Answer: \( k'(x) = -0.5x^{-2} \)
   e) \( m(x) = (-5x^2 + 7x^{-1})(x^{-2} - 7) \)  
      Answer: \( m'(x) = 70x - 21x^{-4} + 49x^{-2} \)
   f) \( v(x) = \frac{2x-1}{4x+8} \)  
      Answer: \( v'(x) = \frac{20}{(4x+8)^2} \)
   g) \( p(x) = \frac{x^2+1}{x^3-2x} \)  
      Answer: \( p'(x) = \frac{2x(x^3-2x)-(x^2+1)(3x^2-2)}{(x^3-2x)^2} = \frac{-x^4-5x^2+2}{(x^3-2x)^2} \)
Applications to Derivatives and Rates of Change

Tangent Lines

1. Find the equation of the tangent line to the graph of \( f(x) = 3x^2 - 4x + 6 \) at \( x = 2 \).
   Answer: \( y = 8x - 6 \)

2. Find the equation of the line tangent to the graph of \( f(x) = x^2 - 2 \) at \( (3, 7) \).
   Answer: \( y = 6x - 11 \)

3. Find the slope of the tangent line to the graph of \( f(x) = 5x^3 + 2x + 4 \) at \( x = 1 \)
   Answer: 17

Marginal Analysis

1. Assume that your monthly profit (in dollars) from selling homemade cookies is given by \( P(x) = 8x - 2\sqrt{x} \), where \( x \) is the number of boxes of cookies you sell in a month.
   a) Determine the marginal profit function, \( MP(x) \).
      Answer: \( MP(x) = 8 - \frac{1}{\sqrt{x}} \)
   b) Determine value of marginal profit if you are selling 25 boxes of cookies per month and interpret.
      Answer: 7.8 dollars per box. After 25 boxes of cookies have been sold, the total profit will increase by about 7.8 dollars per additional box sold, or the profit from selling the 26th box is about 7.8 dollars.

2. Find the marginal cost, the marginal revenue, and the marginal profit functions, where the cost and revenue functions, respectively, are \( C(x) = 8x^2 \) and \( R(x) = 4x^3 + 2x + 10 \).
   Answer: \( MC(x) = 16x \)
   \( MR(x) = 12x^2 + 2 \)
   \( MP(x) = 12x^2 - 16x + 2 \)

3. Assume that your monthly profit (in dollars) from selling books is given by \( P(x) = 5x^2 + 6x - 2 \), where \( x \) is the number of books you sell in a month. If you are currently selling \( x = 50 \) books per month, find your profit and your marginal profit.
   Answer: profit = 12798 dollars
   Marginal profit = 506 dollars per book
4. Your monthly cost (in dollars) from selling homemade candles is given by 
\[ C(x) = 150 + 0.1x + 0.002x^2, \] where \( x \) is the number of candles you sell in a month. The revenue from selling \( x \) candles is \( R(x) = 7x \).

a) Write a function \( P(x) \) for your monthly profit of producing and selling \( x \) candles.
   Answer: \( P(x) = -150 + 6.9x - 0.002x^2 \)

b) Calculate \( P(100) \). Include units.
   Answer: 520 dollars

c) Write a function for your marginal profit.
   Answer: \( MP(x) = 6.9 - 0.004x \)

d) Calculate your marginal profit if you produce and sell 100 candles. Include units and interpret your answer.
   Answer: 6.5 dollars per candle. The profit from selling the 101st candle is about 6.5 dollars. Or the total profit will increase by 6.5 dollars per candle sold, after 100 candles are sold.

Average Velocity and Instantaneous Velocity

1. Assume that the distance, \( s \) (in meters), traveled by a car moving in a straight line is given by the function 
   \[ s(t) = t^2 - 3t + 5, \] where \( t \) is measured in seconds.
   a) Find the average velocity of the car during the time period from \( t = 1 \) to \( t = 4 \).
      Answer: 2 m/s

   b) Find the instantaneous velocity of the car at time \( t = 3 \) seconds.
      Answer: 3 m/s