Mat 171 - Final Exam Formulas

The following formulas plus a blank circle will be provided on the Final Exam.

\[
A = P\left(1 + \frac{r}{n}\right)^n
\]

\[A = Pe^{rt}\]

\[
\cos^2(x) + \sin^2(x) = 1 \quad 1 + \tan^2(x) = \sec^2(x) \quad \cot^2(x) + 1 = \csc^2(x)
\]

\[
\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)
\]

\[
\cos(x - y) = \cos(x)\cos(y) + \sin(x)\sin(y)
\]

\[
\tan(x + y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}
\]

\[
\tan(x - y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}
\]

\[
\sin(2x) = 2\sin(x)\cos(x)
\]

\[
\cos(2x) = \begin{cases} 
\cos^2(x) - \sin^2(x) \\
2\cos^2(x) - 1 \\
1 - 2\sin^2(x)
\end{cases}
\]

\[
\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}
\]

\[
\cos\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 + \cos(x)}{2}}
\]

\[
\sin\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 - \cos(x)}{2}}
\]

\[
\tan\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 - \cos(x)}{1 + \cos(x)}}
\]

\[
c^2 = a^2 + b^2 - 2ab\cos(C)
\]

\[
\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}
\]
Mat 171 – Final Exam Review Problems

1.2 – Find the domain

(a) \( f(x) = \frac{1-x}{x^2-9} \)  
(b) \( f(x) = \frac{x+1}{3x-2} \)

(c) \( f(x) = \log(2x + 1) \)  
(d) \( f(x) = \sqrt{8 - 2x} \)

1.3 – Evaluate the difference quotient

(a) \( f(x) = \frac{1}{3x} \)  
(b) \( f(x) = -x^2 + 5x + 9 \)  
(c) \( f(x) = 3x^2 + 4x - 8 \)

1.6 – Find the function \( g(x) \) after applying the following transformations to \( x^2 \):

reflect about the \( x \)-axis, shift left 5 units, shift up 3 units

1.7 – Find compositions

(a) Find \((f \circ g)(x)\) and \((g \circ f)(x)\) where \( f(x) = x^2 - x + 4 \) and \( g(x) = 2x - 3 \).

(b) Find \((g \circ f)(x)\) where \( f(x) = e^{2x} - 1 \) and \( g(x) = \ln(x + 1) \).

1.8 – Find inverse functions

(a) \( f(x) = \frac{4x}{3+x} \)  
(b) \( f(x) = x^3 - 10 \)

2.2 – Quadratic function application

An astronaut on the moon throws a baseball upward. The height of the ball is approximately by the function \( h(t) = -2.7t^2 + 30t + 6.5 \) feet, \( t \) is the time in seconds after the ball was thrown. When does the baseball reach its maximum height? What is the maximum height of the baseball?

2.3-2.5 – Find all zeros

(a) \( f(x) = -x^3 + x^2 + 2x \)  
(b) \( f(x) = x^3 - x^2 + 9x - 9 \)
2.6 – Rational function applications

(a) Suppose that the insect population in millions is modeled by \( f(x) = \frac{10x+1}{0.2x+1} \), where \( x \geq 0 \) is in months. What happens to the insect population after a long time?

(b) A company that manufactures calculators has determined that the average cost for producing \( x \) calculators is \( \bar{C} = \frac{15000+20x}{x} \) dollars. In the long run, what value does the average cost approach?

3.4 – Solve exponential and logarithmic equations

(a) \( 3^{2x} - 3^x - 42 = 0 \)

(b) \( 5^x = 3^{x-1} \)

(c) \( \log_2(x) + \log_2(x - 7) = 3 \)

(d) \( \ln(x) - \ln(x - 2) = 1 \)

4.5 – Find the amplitude, period and phase shift for the function: \( y = -3 \cos(2x + \pi) \)

5.1 – Verify the trigonometric identities

(a) \( \cos x \cot x + \sin x = \csc x \)

(b) \( \frac{\cos x + \sin x - \sin^3 x}{\sin x} = \cot x + \cos^2 x \)

4.2, 5.2, 5.3 – Given \( \sin \alpha = -\frac{3}{8} \), \( \pi < \alpha < \frac{3\pi}{2} \) and \( \cos \beta = \frac{3}{5} \), \( 0 < \beta < \frac{\pi}{2} \), find the following

(a) \( \cos \alpha \)

(b) \( \sec \alpha \)

(c) \( \tan \alpha \)

(d) \( \cot \alpha \)

(e) \( \csc \alpha \)

(f) \( \cos(2\alpha) \)

(g) \( \sin(2\alpha) \)

(h) \( \tan(2\alpha) \)

(i) \( \cos \left( \frac{\alpha}{2} \right) \)

(j) \( \sin \left( \frac{\alpha}{2} \right) \)

(k) \( \sin(\alpha - \beta) \)

(l) \( \cos(\alpha + \beta) \)

5.5 – Solve trigonometric equations on \([0, 2\pi]\)

(a) \( \sin(2x) + \sqrt{2} \cos x = 0 \)

(b) \( 2 \sin^2 x - 5 \sin x + 2 = 0 \)
6.1 – Solve an application using Law of Sines

An aircraft is spotted by two observers who are 5000 meters apart. As the airplane passes over the line joining the observers, each observer takes a sighting of the angle of elevation of the airplane. The first observer sights the plane at 40° and the second observer sights the plane at 35°. How far away is the airplane from the first observer?

6.2 – Solve an application using Law of Cosines

A tourist stands 100 feet from the base of the Leaning Tower of Pisa. With the tower leaving away from the observer, the observer looking up at an angle of 52° finds that the distance from the top of the tower to where he is standing is 228 feet. Find the angle the Leaning Tower makes with the ground.