Do you wish to take this as a qualifying exam? YES or NO.
Please write each answer on a separate paper. Make sure your name is on every sheet.

(1) (5 points each)
   (a) Find all non-isomorphic abelian groups of order 450. Be sure to state accurately any result you use to know your list is complete.
   (b) Let $G$ be the group of $n \times n$ matrices where all diagonal entries are $\pm 1$ and off-diagonal entries are 0. Find the isomorphism class of $G$.

(2) (10 points) Prove that for any subgroup $H$ of a group $G$, the quotient group $N_G(H)/C_G(H)$ is isomorphic to a subgroup of $\text{Aut}(H)$.

(3) (10 points) Show that there are no simple groups of order $5103 = 3^6 \cdot 7$.

(4) (7 points) Explicitly construct a non-abelian group of order 155.

(5) Prove or disprove the following statements (6 points each).
   (a) In an integral domain, if an element has a factorization into primes, then the factorization is unique up to order and units.
   (b) If $R$ is a unique factorization domain and $d \neq 0$ is in $R$, then there are only finitely many distinct principal ideals containing the principal ideal $(d)$.

(6) Prove the following statements (5 points each).
   (a) If $R$ is an integral domain such that $R/I$ is finite for all ideals $I \neq \{0\}$, then every nonzero prime ideal of $R$ is maximal.
   (b) If $F$ is a field, the polynomial ring $R = F[x]$ has the property that every nonzero prime ideal of $R$ is maximal.

(7) (6 points) Prove that $\mathbb{R}[x]/(x^2 + 1)$ is a field which is isomorphic to $\mathbb{C}$, the field of complex numbers.

(8) (3 and 4 points) Let $R = \mathbb{Z}/4\mathbb{Z}$ and let $D = \{1, 3\}$.
   (a) Show that $D$ is a multiplicatively closed subset of $R$.
   (b) Write out the multiplication table for the ring $D^{-1}R$. 