LIST OF CONCEPTS AND SKILLS FOR TEST 1
The test covers sections 1.1-1.3, 2.1-2.5, 2.7, 3.1, 3.2

Chapter 1

- Know how to sketch (BY HAND) a direction field for a first order ODE, and how to sketch integral curves.
- Know how to find Equilibrium (constant) Solutions of ODEs of the form $y' = f(y)$ and how to classify them as stable, unstable or semi-stable.
- Given the slope field for a differential equation, be able to read the long term behavior of the solutions for different initial conditions.
- Know how to verify by substitution that a given function is a solution of a given ODE.
- Given the general solution to an ODE, know how to use initial conditions to find the value of the constant C.
- Know how to classify differential equations by their order and linearity.
- Know how to derive differential equations that model simple applied problems.

Chapter 2

- Know how to use the method of integrating factor to integrate linear first order equations of the form $y' + p(t)y = g(t)$. The integrating factor is given by
$$\mu(t) = e^{\int p(t)dt}.$$
- Know how to solve separable equations. Whenever possible write the solution in explicit form and be able to determine the interval in which the solution is defined.
- Know the Theorems for existence and uniqueness of solutions to first-order initial value problems (Theorem 2.4.1 and Theorem 2.4.2) and how to apply them.
- Be able to solve applied problems such as "mixture" problems and problems involving Newtons's Law of Cooling.
- Know the formula for Euler's method and how to use it to derive recursive approximations for a given IVP.

Chapter 3 (3.1-3.2)

- A second order linear differential equation in standard form has the form:
$$y'' + p(t)y' + q(t)y = g(t).$$
- If $g(t) = 0$, it is called homogeneous; otherwise nonhomogeneous.

Linear Homogeneous DEs with constant coefficients. Given the homogeneous ODE
$$ay'' + by' + cy = 0$$
with a, b and c constant:

- The characteristic equation is $ar^2 + br + c = 0$.
- If the roots of the characteristic equation are real and distinct, r_1 and r_2, then the general solution of the homogeneous ODE is $y = c_1 e^{r_1 t} + c_2 e^{r_2 t}$.
- Know how to determine the exact solution (solve for c_1 and c_2) of the above when given two initial conditions.

The Wronskian:

- Know Theorem 3.2.1: Existence and Uniqueness. Consider the Initial Value Problem
$$y'' + p(t)y' + q(t)y = g(t), \qquad y(t_0) = y_0, \qquad y'(t_0) = y_0'$$
where $p(t)$, $q(t)$ and $g(t)$ are continuous on an open interval I that contains t_0. Then there is exactly one solution of this problem, and the solution exists throughout the interval I.

- Know Theorem 3.2.2: **Principle of Superposition**. If y_1 and y_2 are solutions of the homogeneous differential equation $y'' + p(t)y' + q(t)y = 0$, so is $c_1 y_1 + c_2 y_2$ for any constants c_1 and c_2.

- Know Theorem 3.2.4: **Wronskian of Solutions**: Given two solutions y_1 and y_2 of a homogeneous ODE, always check whether the Wronskian of the two solutions is not everywhere zero. If this is case the two solutions are linearly independent and we say they form a Fudamental Set of Solutions for the ODE. The general solution is given by $c_1 y_1 + c_2 y_2$ with c_1 and c_2 arbitrary constants.