§5.5 The Substitution Rule

1. $\displaystyle\int 6(x-4)^5\,dx$

2. $\displaystyle\int \frac{t^2-3}{-t^3+9t+1}\,dt$

3. Suppose f is an odd function. For $a>0$, find $\int_{-a}^{a} f(x)\,dx$.

§6.1 Integration by Parts

4. $\displaystyle\int z^2 \ln z\,dz$

5. $\displaystyle\int e^{2x}\sin x\,dx$

§6.2 Trigonometric Integrals and Substitutions

6. $\displaystyle\int \frac{dx}{x^2\sqrt{9-x^2}}$

7. $\displaystyle\int \frac{dx}{\sqrt{4x^2+1}}$

8. $\displaystyle\int \sin^4 x\cos^3 x\,dx$

9. $\displaystyle\int_{\sqrt{3}}^{2} \frac{\sqrt{x^2-3}}{x}\,dx$

§6.3 Partial Fractions

10. $\displaystyle\int \frac{5x^2+20x+6}{x^3+2x^2+x}\,dx$

11. $\displaystyle\int \frac{8x^3+13x}{(x^2+2)^2}\,dx$

§6.4 Integration with Tables and C.A.S.

12. Use the equation $\displaystyle\int \sqrt{u^2-a^2}\,du = \frac{u}{2}\sqrt{u^2-a^2} - \frac{a^2}{2}\ln\left|u+\sqrt{u^2-a^2}\right| + C$ to evaluate the following integral: $\displaystyle\int x\sqrt{x^4-9}\,dx$

13. Use the equation $\displaystyle\int \frac{du}{1+e^u} = u - \ln(1+e^u) + C$ to evaluate the following integral: $\displaystyle\int \frac{x}{1+e^{-x^2}}\,dx$

§6.5 Approximate Integration

14. Use the Trapezoid Rule with $n=4$ intervals to approximate $\displaystyle\int_0^{\pi} \sin\theta\,d\theta$.

15. Use Simpson's Rule with $n=4$ intervals to approximate $\displaystyle\int_0^{\pi} \sin\theta\,d\theta$.

§6.6 Improper Integrals

Evaluate each integral below if it converges. If it diverges, clearly state that it diverges.

16. $\displaystyle\int_0^{\infty} e^{-x}\,dx$

17. $\displaystyle\int_{-\infty}^{\infty} \frac{e^x}{1+e^{2x}}\,dx$

18. $\displaystyle\int_{-1}^{2} \frac{dx}{x^3}$

19. $\displaystyle\int_0^{\infty} \frac{1}{(x+1)\sqrt{x}}\,dx$

§7.1 Area Between Curves

In each of the following, find the areas between the given curves.

20. $y = x^2 + 2$, $y = -x$, $x = 0$, $x = 1$

21. $y = 2 - x^2$, $y = x$

22. $y = 3x^3 - x^2 - 10x$, $y = -x^2 + 2x$

23. $x = 3 - y^2$, $x = y + 1$

§7.2 Volumes

24. Find the volume of the solid whose base is bounded by $y = 1 - \frac{x}{2}$, $y = -1 + \frac{x}{2}$, $x = 0$, and whose vertical cross-sections are equilateral triangles.

25. Find the volume of the solid generated by rotating the region bounded by $y = 2 - x^2$, $y = 1$ about the line $y = 1$.

26. Find the volume of the solid generated by rotating the region bounded by $y = \sqrt{25 - x^2}$, $y = 3$ about x-axis.

27. Find the volume of the solid generated by rotating the region bounded by $y = x^2 + 1$, $y = 0$, $x = 0$, $x = 1$ about y-axis.

§7.3 Volumes by Cylindrical Shells

Using the method of cylindrical shells, find the volume of the solid generated by rotating the specified region about the specified line.

28. Region bounded by $y = x - x^3$, the x-axis $(0 \le x \le 1)$ about the y-axis.

29. Region bounded by $x = e^{-y^2}$, the y-axis $(0 \le y \le 1)$ about the x-axis.

30. Region bounded by $y = x^3 + x + 1$, $y = 1$, $x = 1$ about the line $x = 2$.

§7.4 Arc Length

Find the arc length for each of the following functions over the specified interval.

31. $y = \dfrac{x^3}{6} + \dfrac{1}{2x}$, $\left[\frac{1}{2}, 2\right]$

32. $(y - 1)^3 = x^2$, $[0, 8]$

33. $y = \ln(\cos x)$, $[0, \frac{\pi}{4}]$

§7.6 Applications to Physics and Engineering

34. A force of 750 pounds compresses a spring 3 inches from its natural length of 15 inches. Find the work done in compressing the spring an additional 3 inches.

35. A tank in the shape of a right circular cone is half full of water. The tank is 6 ft across the top and 8 ft high. How much work is done in pumping all of the water out over the top edge of the tank?

§8.1 Sequences

Determine whether the following sequences converge. If they converge, find the limit.

36. $\left\{\dfrac{n}{\ln n}\right\}_{n=2}^{\infty}$

37. $\left\{\dfrac{7^{n+8}}{9^n}\right\}_{n=0}^{\infty}$

§8.2 Series

Determine the convergence or divergence of the following series. If it converges, find the sum.

38. $\displaystyle\sum_{n=0}^{\infty}\left(\frac{1}{2^n}-\frac{1}{3^n}\right)$

39. $\displaystyle\sum_{n=1}^{\infty}\frac{2^{n+1}}{3^{n-1}}$

40. $\displaystyle\sum_{n=0}^{\infty}\left[\left(\frac{2}{3}\right)^n-\frac{1}{(n+1)(n+2)}\right]$

§8.4 Other Convergence Tests

Determine whether the following series converge. Justify your answer.

41. $\displaystyle\sum_{n=1}^{\infty}\frac{n}{e^n}$

42. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{\sqrt[4]{n^3}}$

43. $\displaystyle\sum_{n=2}^{\infty}\frac{(-1)^n n}{n^2-3}$

44. $\displaystyle\sum_{n=1}^{\infty}\frac{2^n}{n^3}$

§8.5 Power Series

Find the interval and radius of convergence of the following series.

45. $\displaystyle\sum_{n=0}^{\infty}\left(\frac{x}{10}\right)^n$

46. $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n(x-2)^n}{(n+1)^2}$

47. $\displaystyle\sum_{n=0}^{\infty}n!(x-2)^n$

§8.6 Representing Functions as Power Series

48. Find a power series representation for $f(x)=\frac{1}{1+x}$, centered at 0.

49. Find a power series representation for $g(x)=-\frac{1}{(1+x)^2}$, centered at 0.

50. Use power series to evaluate $\displaystyle\int\frac{3}{1-x^7}\,dx$.

§8.7 Taylor and Maclaurin Series

Find a power series representation for the following function, centered at a.

51. $f(x) = 3^x$, $a = 0$

52. $f(x) = \frac{1}{x}$, $a = -1$

Find the second-degree Taylor polynomial, centered at a.

53. $f(x) = e^{-x/2}$, $a = 0$

54. $f(x) = \tan x$, $a = -\frac{\pi}{4}$

§9.1 Parametric Curves

Eliminate the parameter to find a Cartesian equation of the curve. Then sketch the curve and indicate with an arrow the direction in which the curve is traces as the parameter increases.

55. $x = t^2 + 4t$, $y = 2 - t$, $-4 \leq t \leq 1$

56. $x = 1 + e^{2t}$, $y = e^t$, for all t

57. $x = \cos\theta$, $y = \sec\theta$, $0 \leq \theta < \frac{\pi}{2}$

§9.2 Calculus with Parametric Curves

58. Find the equation of the tangent line for the parametric equations $x = t^3 + 25t$, $y = 25t^2 - t^4$ at $t = 5$.

59. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ expressed as a function of t for curve given by $x = t + \sin t$, $y = t - \cos t$.

60. Find the length of the curve parametrized by $x = 3t^2$, $y = 2t^3$, $0 \leq t \leq 2$

61. Find the length of the curve parametrized by $x = \sin\theta + \cos\theta$, $y = \sin\theta - \cos\theta$, $0 \leq \theta \leq \frac{3\pi}{4}$

§9.3 Polar Coordinates

62. Convert the Cartesian equation $x = 7$ into a polar equation of the form $r = f(\theta)$

63. Convert the polar equation $r = 6\cos\theta$ into a Cartesian equation.

64. Find the equation of the tangent line to the polar equation $r = \sin^2\theta + 1$ at the point where $\theta = \frac{\pi}{4}$.

65. Find the y-coordinate of the highest point on the graph of $r = 4\cos\theta$

§9.4 Areas and Lengths in Polar Coordinates

66. Find the area of the region bounded by $r = \tan\theta$ and $\frac{\pi}{6} \leq \theta \leq \frac{\pi}{3}$.

67. Find the area of the region that lies inside $r = 2\cos\theta$ and outside of $r = 1$.

68. Find the area enclosed by the inner loop of the curve $r = 1 + 2\sin\theta$, $0 \leq \theta \leq 2\pi$

69. Find the arc length of the curve $r = 3\sin\theta$, $0 \leq \theta \leq \frac{2\pi}{3}$

1. $(x-4)^6 + C$

2. $-\frac{1}{3}\ln\left|-t^3 + 9t + 1\right| + C$

3. 0

4. $\frac{z^3}{3}\ln z - \frac{z^3}{9} + C$

5. $\frac{1}{5}e^{2x}(2\sin x - \cos x) + C$

6. $-\frac{\sqrt{9-x^2}}{9x} + C$

7. $\frac{1}{2}\ln\left|\sqrt{4x^2+1} + 2x\right| + C$

8. $\frac{1}{5}\sin^5 x - \frac{1}{7}\sin^7 x + C$

9. $1 - \frac{\sqrt{3}}{6}\pi$

10. $6\ln|x| - \ln|x+1| - \frac{9}{x+1} + C$

11. $4\ln(x^2+2) + \frac{3}{2(x^2+2)} + C$

12. $\frac{x^2}{4}\sqrt{x^4-9} - \frac{9}{4}\ln\left|x^2 + \sqrt{x^4-9}\right| + C$

13. $\frac{x^2}{2} + \frac{1}{2}\ln(1 + e^{-x^2} + C$

14. $\frac{\pi}{8}\left(0 + \sqrt{2} + 2 + \sqrt{2} + 0\right) \approx 1.896$

15. $\frac{\pi}{12}\left(0 + 2\sqrt{2} + 2 + 2\sqrt{2} + 0\right) \approx 2.005$

16. 1

17. $\frac{\pi}{2}$

18. The integral diverges.

19. π

20. $\frac{17}{6}$

21. $\frac{9}{2}$

22. 24

23. $\frac{9}{2}$

24. $\frac{2\sqrt{3}}{3}$

25. $\frac{16\pi}{15}$

26. $\frac{256\pi}{3}$

27. $\frac{3\pi}{2}$

28. $\frac{4\pi}{15}$

29. $\pi\left(1 - \frac{1}{e}\right) \approx 1.986$

30. $\frac{29\pi}{15}$

31. $\frac{33}{16}$

32. $\frac{1}{27}\left(40^{3/2} - 4^{3/2}\right) \approx 9.073$

33. $\ln(\sqrt{2}+1) \approx 0.881$

34. 3375 inch-pounds

35. $\frac{1875}{2}\pi \approx 2945.2$ ft-lb

36. Diverges.

37. Converges to 0.

38. $\frac{1}{2}$

39. 12

40. 2

41. Converges by ratio test.

42. Diverges by p-series test.

43. Converges by alternating series test.

44. Diverges by ratio test.

45. Radius: 10. Interval: $(-10, 10)$

46. Radius: 1. Interval: $[1, 3]$

47. Converges only at $x = 2$.

48. $\sum_{n=0}^{\infty}(-x)^n$

49. $\displaystyle\sum_{n=1}^{\infty}(-1)^n n x^{n-1}$

50. $\displaystyle C + \sum_{n=0}^{\infty}\frac{3x^{7n+1}}{7n+1}$

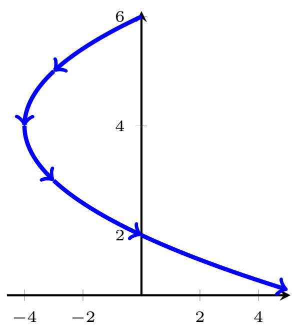
51. $\displaystyle\sum_{n=0}^{\infty}\frac{(x\ln 3)^n}{n!}$

52. $\displaystyle -\sum_{n=0}^{\infty}(x+1)^n$

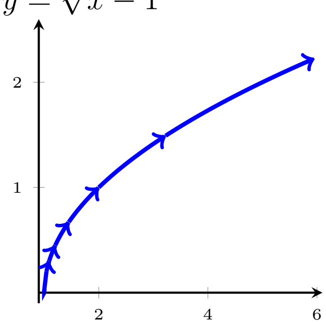
53. $\displaystyle 1 - \frac{x}{2} + \frac{x^2}{8}$

54. $-1 + 2\left(x+\frac{\pi}{4}\right) - 2\left(x+\frac{\pi}{4}\right)^2$

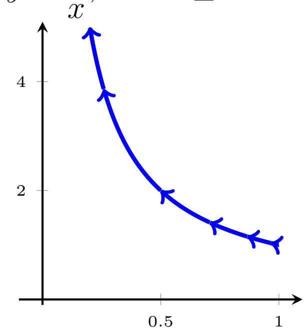
55. $x = y^2 - 8y + 12$



56. $y = \sqrt{x-1}$



57. $\displaystyle y = \frac{1}{x},\ 0 < x \le 1$



58. $\displaystyle y = -\frac{5}{2}x + 625$

59. $\displaystyle \frac{dy}{dx} = \frac{1+\sin t}{1+\cos t}$

$\displaystyle \frac{d^2y}{dx^2} = \frac{1+\cos t+\sin t}{(1+\cos t)^3}$

60. $-2 + 10\sqrt{5} \approx 20.361$

61. $\displaystyle \frac{3}{2\sqrt{2}}\pi \approx 3.332$

62. $\displaystyle r = \frac{7}{\cos\theta}$

63. $(x-3)^2 + y^2 = 9$

64. $\displaystyle y = -5x + \frac{9}{\sqrt{2}}$

65. $y = 2$

66. $\displaystyle \frac{1}{\sqrt{3}} - \frac{\pi}{12} \approx 0.316$

67. $\displaystyle \frac{\sqrt{3}}{2} + \frac{\pi}{3} \approx 1.913$

68. $\displaystyle \pi\frac{3\sqrt{3}}{2} \approx 0.544$

69. $2\pi \approx 6.283$