MAT266 EXAM 1 REVIEW

1. (5.5) A weather balloon is being filled at a rate of $r(t) = 18(1+2t)^2$ liters per second, starting with a volume of 0 liters at time $t = 0$. What is the volume of the balloon after 1 minute?

2. (5.5) A balloon is being filled at a rate of $r(t)$ liters per second, starting with a volume of 0 liters at time $t = 0$. What is an expression that represents the volume of the balloon after 2 minutes in liters?

3. (5.5) The temperature of a city t hours after 9:00 am is modeled (in degrees Fahrenheit)by the function $T(t) = 80 + 19\sin(\pi t/12)$.

   (a) According to the model, what is the temperature at 9:00am?

   (b) According to the model, what is the temperature at 3:00pm?

   (c) According to the model, what is the average temperature over the time period from 9:00am to 9:00pm?

   Provide exact answers. Do not approximate your answers.

4. (5.5) Evaluate $\displaystyle\int \frac{e^x}{6e^x + 2}\,dx$.

5. (5.5) Evaluate $\displaystyle\int (6x+3)\cos(x^2+x)\,dx$.

6. (5.5) A bacteria colony starts with 200 bacteria and grows at a rate of $r(t) = 450.268e^{2.25134t}$ bacteria per hour. How many bacteria will there be after half an hour? *Round your answer to the nearest whole number.*

7. (5.5) Evaluate $\displaystyle\int (2x+7)\sin(x^2+7x)\,dx$.

8. (5.5) Evaluate $\displaystyle\int \sin^3 x\cos x\,dx$.

9. The function $f(t) = \dfrac{1}{2}\sin\left(\dfrac{2\pi t}{5}\right)$ has often been used to model the rate of air flow into the lungs t seconds after inhaling in liters per second. Use this model to find the volume of inhaled lungs at time t.

10. (5.5) Evaluate $\displaystyle\int \sec^2 x\tan^4 x\,dx$.

11. (5.5) A model for the basal metabolism rate, in kcal/h, of a young person is $R(t) = 85 - 0.17\cos\left(\dfrac{\pi t}{12}\right)$, where t is the time in hours measured from 5:00 AM. What is the total basal metabolism of this person for a 24-hour period?

12. (5.5) Evaluate $\displaystyle\int \frac{\sin(\sqrt{x})}{\sqrt{x}}\,dx$.

13. (5.5) Evaluate $\int \frac{\sin(\ln x)}{x}\,dx$.

14. (6.1) Evaluate $\int 4x \ln x\,dx$.

15. (6.1) A patient is given an injection of a drug at a rate of $r(t) = 2te^{-2t}$ ml/sec, where t is the number of seconds since the injection started. What is the amount of the drug injected during the first 5 seconds? *Round your answer to the nearest tenth of a milliliter.*

16. (6.1) Evaluate $\int \ln(9x+1)\,dx$.

17. (6.1) Suppose a particle travels along a straight line with velocity $v(t) = t^2e^{-3t}$ meters per second after t seconds. How far does the particle travel during the first 3 seconds? *Round your answer to the nearest hundredth of a meter.*

18. (6.1) Evaluate $\int_0^{\pi} 3x \sin x\,dx$.

19. (6.2) Evaluate $\int \frac{1}{\sqrt{9-x^2}}\,dx$.

20. (6.2) Evaluate $\int \frac{1}{x^2\sqrt{x^2+25}}\,dx$.

21. (6.2) Evaluate $\int_0^{\pi/2} \sin^2 x\,dx$.

22. (6.2) Evaluate $\int \sin^3 x \cos^2 x\,dx$.

23. (6.2) Evaluate $\int \sec^4\left(\frac{x}{4}\right)\,dx$.

24. (6.2) Evaluate $\int \sec^5 x \tan^3 x\,dx$.

25. (6.3) Evaluate $\int \frac{11x-46}{x^2+2x-8}\,dx$.

26. (6.3) What is the partial fraction decomposition of $\frac{x^2+1}{(x-4)^2(x^2+9x+45)}$?

27. (6.3) Evaluate $\int \frac{3x}{x^2+6x+9}\,dx$.

28. (6.4) Evaluate $\int \frac{x}{1+e^{-x^2}}\,dx$ using the formula

$$\int \frac{1}{1+e^u}\,du = u - \ln(1+e^u) + C$$

29. (6.4) Evaluate $\displaystyle\int \frac{3x^2}{\sqrt{x^6 - 16}}\,dx$ using the formula

$$\int \frac{du}{\sqrt{u^2 - a^2}} = \ln|u + \sqrt{u^2 - a^2}| + C.$$

30. (6.4) Evaluate $\displaystyle\int (\ln x)^2\,dx$ using the formula

$$\int (\ln x)^n\,dx = x(\ln x)^n - n\int (\ln x)^{n-1}\,dx$$

31. (6.5) The record of the speed of a runner during the first 3 seconds of a race is given in the following table:

t (s)	0	0.5	1.0	1.5	2.0	2.5	3.0
v (m/s)	0	2.1	5.3	5.6	6.4	7.45	8.5

   (a) Use the Trapezoidal Rule with $n = 6$ to approximate the distance the runner covered during these 3 seconds. *Round your answer to the nearest hundredth of a meter.*

   (b) Use Simpson's Rule with $n = 6$ to approximate the distance the runner covered during these 3 seconds. *Round your answer to the nearest hundredth of a meter.*

32. (6.5) Approximate $\displaystyle\int_0^1 \cos(x^2)\,dx$ to 3 decimal places,

   (a) using the Trapezoidal Rule with $n = 4$.

   (b) using Simpson's Rule with $n = 4$.

33. (6.5) The time, T, to complete one swing of a pendulum can be modeled by

$$T = 4\sqrt{\frac{L}{g}}\int_0^{\pi/2} \frac{dx}{\sqrt{1 - k^2\sin^2 x}}$$

where L is the length of the pendulum, g is the acceleration due to gravity, and k is a constant that depends on the maximum angle the pendulum swings. If you use Simpson's rule to approximate T with $n = 8$, what is Δx (the length of each subinterval)?

34. (6.5) A population of ladybugs increases at a rate of $r(t)$ ladybugs per week beginning at time $t = 0$. What information would you need to use Simpson's Rule with 6 subintervals to approximate the increase in the ladybug population during the first 24 weeks?

35. The velocity of a rocket at time t can be modeled by

$$v(t) = -9.8t - v_e \ln\left(\frac{m - rt}{m}\right)$$

where m is the initial mass of the rocket at liftoff (including its fuel), r is the constant rate at which the fuel is consumed, and v_e is the constant velocity at which the exhaust gases are ejected (relative to the rocket). Which of the following integration techniques, if any, is best suited to find the height of the rocket one minute after liftoff?

A. Integration by substitution
B. Integration by parts
C. Integration by partial fractions
D. Trigonometric substitution
E. None of these can be used.

ANSWERS

1. $5,314,680$ liters
2. $\int_0^{120} r(t)\,dt$
3. $T(0) = 80°\text{F}$, $T(6) = 99°\text{F}$, $\frac{1}{12}\int_0^{12} \left(80 + 19\sin\left(\pi t/12\right)\right)\,dt = 80 + \frac{38}{\pi}^{\circ} F$
4. $\frac{1}{6}\ln(6e^x + 2) + C$
5. $3\sin(x^2 + x) + C$
6. 616 bacteria
7. $-\cos(x^2 + 7x) + C$
8. $\frac{1}{4}\sin^4 x + C$
9. $\frac{5}{4\pi}\left(1 - \cos\left(\frac{2\pi t}{5}\right)\right)$ liters
10. $\frac{1}{5}\tan^5 x + C$
11. 2,040 kcal
12. $-2\cos(\sqrt{x}) + C$
13. $-\cos(\ln x) + C$
14. $2x^2\ln x - x^2 + C$
15. $0.5\,\text{ml}$

16. $\dfrac{(9x+1)\ln(9x+1)}{9} - x + C$

17. $0.07\,\mathrm{m}$

18. 3π

19. $\arcsin\left(\dfrac{x}{3}\right) + C$

20. $-\dfrac{\sqrt{x^2+25}}{25x} + C$

21. $\dfrac{\pi}{4}$

22. $\dfrac{\cos^5 x}{5} - \dfrac{\cos^3 x}{3} + C$

23. $\dfrac{4}{3}\tan^3\left(\dfrac{x}{4}\right) + 4\tan\left(\dfrac{x}{4}\right) + C$

24. $\dfrac{\sec^7 x}{7} - \dfrac{\sec^5 x}{5} + C$

25. $15\ln|x+4| - 4\ln|x-2| + C$

26. $\dfrac{A}{x-4} + \dfrac{B}{(x-4)^2} + \dfrac{Cx+D}{x^2+9x+45}$, where A, B, C, D are constants.

27. $3\ln|x+3| + \dfrac{9}{x+3} + C$

28. $\dfrac{1}{2}\left(x^2 + \ln(1+e^{-x^2})\right) + C$

29. $\ln|x^3 + \sqrt{x^6-16}| + C$

30. $x(\ln x)^2 - 2x\ln x + 2x + C$

31. 15.55 meters, 15.42 meters.

32. 0.896, 0.905

33. $\pi/16$.

34. $r(0), r(4), r(8), r(12), r(16), r(20), r(24)$

35. Integration by parts