MAT 266
TEST 3 ANSWERS/SOLUTIONS

SoMSS, ASU

Directions:

1. **There are 14 questions worth a total of 60 points.**
2. **Questions 1 - 10 are Multiple Choice worth 4 points each to be answered on the supplied SCANTRONS.**
3. **Questions 11 - 14 are Free Responses worth 5 points each and are to be answered in the space provided on the test.**
4. **Read all the questions carefully.**
5. **For the Free Response, you must show all work in order to receive credit!!**
6. **When possible, box your answer, which must be complete, organized, and exact unless otherwise directed.**
7. **Always indicate how a calculator was used (i.e. sketch graph, etc. …).**
8. **No calculators with QWERTY keyboards or ones like TI-89 or TI-92 that do symbolic algebra may be used.**

Honor Statement:

By signing below you confirm that you have neither given nor received any unauthorized assistance on this exam. This includes any use of a graphing calculator beyond those uses specifically authorized by the Mathematics Department and your instructor. Furthermore, you agree not to discuss this exam with anyone until the exam testing period is over. In addition, your calculator's program memory and menus may be checked at any time and cleared by any testing center proctor or Mathematics Department instructor.

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Signature Date

1. Find the interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n+4}$

Select the correct answer.

a. (0, 2) b. (-1, 2) c. (-1, 1) d. diverges everywhere e. None of these

2. Suppose that the radius of convergence of the power series $\sum_{n=0}^{\infty} c_n x^n$ is 16. What is the radius of convergence of the power series $\sum_{n=0}^{\infty} c_n x^{2n}$?

Select the correct answer.

a. 256 b. 4 c. 1 d. 16 e. None of these

3. Find the radius of convergence of the series. $\sum_{n=1}^{\infty} \frac{8^n x^n}{(n+5)^2}$

Select the correct answer.

a. $R = 8$ b. $R = 1/8$ c. $R = 1$ d. $R = 5$ e. None of these

4. Find the Taylor polynomial T_3 for the function $f(x) = \ln x$ at the number $a = 1$.

Select the correct answer.

a. $(x+1) - \frac{1}{3}(x+1)^2 + \frac{1}{2}(x+1)^3$

b. $(x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3$

c. $(x-1) + \frac{1}{4}(x-1)^2 + \frac{1}{2}(x-1)^3$

d. $(x-1) + \frac{1}{4}(x-1)^2 + \frac{1}{2}(x-1)^3$

e. None of the above

5. Find the sum of the series: $\sum_{n=0}^{\infty} \frac{1}{2^n n!}$

Select the correct answer.

a. $1/\sqrt{e}$ b. $\sqrt{e}$ c. e^2 d. e^{-2} e. None of these

6. Evaluate the indefinite integral $\int \tan^{-1}(t^2)\ dt$ as a power series.

[The MacLaurin series for $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$]

Select the correct answer.

a. $C + \sum_{n=0}^{\infty} \frac{(-1)^n t^{4n+3}}{(2n+1)(4n+3)}$

b. $C + \sum_{n=0}^{\infty} \frac{(-1)^n t^{4n+3}}{(4n+3)}$

c. $C + \sum_{n=0}^{\infty} \frac{(-1)^n t^{4n+2}}{(2n+1)(4n+3)}$

d. $C + \sum_{n=0}^{\infty} \frac{(-1)^n t^{2n+2}}{(2n+1)}$

e. None of the above

7. Find the first three nonzero terms in the MacLaurin series for the function $f(x) = e^{-x^2} \cos 4x$

Select the correct answer.

a. $1 - 17x^2 + 19.17x^4$

b. $1 - 9x^2 + 11.17x^4$

c. $1 - 9x^2 + 19.17x^4$

d. $1 - 9x + 19.17x^2$

e. None of the above.

8. Eliminate the parameter to find the Cartesian equation of the curve

$x = \cos\theta,\ y = \sec\theta,\ 0 \le \theta < \pi/2$

Select the correct answer.

a. $y = x^2$ b. $y = x$ c. $y = 2x$ d. $y = \frac{1}{x}$ e. None of these

9. Find the radius of convergence of the binomial series expansion of the $f(x) = \frac{1}{(2+x)^2}$

Select the correct answer.

a. $R = 1$ b. $R = 1/2$ c. $R = 2$ d. $R = 4$ e. None of these

10. Find parametric equations to represent the line segment from (-1, 2) to (10, -6).

Select the correct answer.

a. $x = 10 - 8t,\ y = -1 + 11t,\ 0 \le t \le 1$

b. $x = -1 + 11t,\ y = 2 - 8t,\ 0 \le t \le 1$

c. $x = -1 + 11t,\ y = -6 - 8t,\ 0 \le t \le 1$

d. $x = -1 - 11t,\ y = -8t,\ 0 \le t \le 1$

e. None of the above

11. Find the polar equation for the curve represented by the given Cartesian equation $x + y = 4$

Select the correct answer.

a. $r = \dfrac{2}{\cos\theta - \sin\theta}$

b. $r = \dfrac{4}{\cos\theta - \sin\theta}$

c. $r = 4(\cos\theta + \sin\theta)$

d. $r = 2(\cos\theta + \sin\theta)$

e. $r = \dfrac{4}{\cos\theta + \sin\theta}$

12. **Set up the integral for the length of the curve** $x = 3t^2,\ y = 2t^3,\ 0 \le t \le 1.$
Select the correct answer.

a. $\int_0^1 \sqrt{(6t)^2 - (6t^2)^2}\ dt$

b. $\int_0^1 \sqrt{(6t)^2 + (6t^2)^2}\ dt$

c. $\int_0^1 \sqrt{6t + 6t^2}\ dt$

d. $\int_0^1 \sqrt{(6t + 6t^2)^2}\ dt$

e. $\int_0^1 \sqrt{(36t)^2 + (36t^2)^2}\ dt$

f. none of these

FREE RESPONSE

13. Use series to evaluate: $\lim_{x \to 0} \dfrac{(\sin x) - x}{x^3}$

14. Use the power series representation of $f(x) = \sin(x)$ to find a power series representation for $g(x) = x\sin(x^2)$.

15. Test the series for convergence or divergence: $\sum_{m=1}^{\infty} \frac{m^5}{3^m}$

16. Use the Binomial Theorem to find the first 3 terms of the MacLaurin Series for $f(x) = \sqrt[3]{5+x}$ and use it to estimate $\sqrt[3]{5.08}$.

17. **[5 pts]** Find the area bounded by the curve $x = t - \frac{1}{t}$, $y = t + \frac{2}{t}$ and the line $y = 4.5$

18. **[5 pts]** Find the slope of the tangent line to the polar curve $r = 2\theta$ when $\theta = \pi$

19. **[5 pts]** Find the area of the region that lies inside the curve $r = 3\cos\theta$ and outside the curve $r = 1 + \cos(\theta)$.

20. **[5 pts]** Find the exact coordinates of the highest point on the curve $x = 15te^{t}$, $y = 3te^{-t}$.