# MAT 266
# TEST 2 - ANSWERS
***SoMSS, ASU***

**Directions:**
1. **There are 14 questions worth a total of 60 points.**
2. **Questions 1 - 10 are Multiple Choice worth 4 points each to be answered on the supplied SCANTRONS.**
3. **Questions 11 - 14 are Free Responses worth 5 points each and are to be answered in the space provided on the test.**
4. **Read all the questions carefully.**
5. **For the Free Response, you must show all work in order to receive credit!!**
6. **When possible, box your answer, which must be complete, organized, and exact unless otherwise directed.**
7. **Always indicate how a calculator was used (i.e. sketch graph, etc. …).**
8. **No calculators with QWERTY keyboards or ones like TI-89 or TI-92 that do symbolic algebra may be used.**

**Honor Statement:**

By signing below you confirm that you have neither given nor received any unauthorized assistance on this exam. This includes any use of a graphing calculator beyond those uses specifically authorized by the Mathematics Department and your instructor. Furthermore, you agree not to discuss this exam with anyone until the exam testing period is over. In addition, your calculator's program memory and menus may be checked at any time and cleared by any testing center proctor or Mathematics Department instructor.

______________________________ ______________

Signature Date

Answer: d

1. Find the area of the region bounded by the curves $y = x^2 - 2x$ & $y = x + 4$.
Select the correct answer.
a. $\frac{125}{3}$ b. $\frac{25}{3}$ c. 20 d. $\frac{125}{6}$ e. None of these

Answer: b

2. Find the volume of the solid obtained by rotating the region bounded by $y = x^2$ and $x = y^2$ about the $x$-axis.
Select the correct answer.
a. $\frac{6\pi}{5}$ b. $\frac{3\pi}{10}$ c. $\frac{2}{5}$ d. $\frac{\pi}{5}$ e. None of these

Answer: c

3. Which of the following integrals is equal to 1.25?
Select the correct answer.
a. $\int_0^1 \frac{1}{x^{0.8}}\,dx$ b. $\int_0^1 \frac{1}{x^{0.3}}\,dx$ c. $\int_0^1 \frac{1}{x^{0.2}}\,dx$ d. $\int_0^1 \frac{1}{x^{2}}\,dx$ e. none of these

Answer: c

4. A spring has a natural length of 22 cm. If a force of 15 N is required to keep it stretched to a length of 32 cm, how much work is required to stretch it from 22 cm to 40 cm?
Select the correct answer.
a. 3.43 J b. 1.93 J c. 2.43 J d. 3.93 J e. None of these

Answer: a

5. Find $\lim_{n\to\infty}(3 + e^{-2n})$
Select the correct answer
a. 3 b. 0 c. 4 d. 2 e. None of these

Answer: a

6. A rope, 40 ft long, weighs 0.8 lb/ft and hangs over the edge of a building 110 ft high. How much work is done in pulling the rope to the top of the building?
Select the correct answer.
a. 640 ft-lb b. 590 ft-lb c. 641 ft-lb d. 489 ft-lb e. None of these

Answer: b

7. Find the sum of the series $\sum_{n=4}^{\infty}\left(\frac{1}{3}\right)^n$
Select the correct answer
a. $\frac{1}{81}$ b. $\frac{1}{54}$ c. $\frac{1}{3}$ d. $\frac{2}{27}$ e. None of these

8. Set up, but do not evaluate, an integral for the length of the curve $y = e^x \sin x,\ 0 \le x \le 3\pi/2$

Select the correct answer.

Answer: b

a. $L = \int_0^{3\pi/2} \sqrt{1 - e^{2x}(1 + \sin 2x)}\ dx$

b. $L = \int_0^{3\pi/2} \sqrt{1 + e^{2x}(1 + \sin 2x)}\ dx$

c. $L = \int_0^{3\pi/2} \sqrt{1 - e^{2x}(1 - \sin 2x)}\ dx$

d. $L = \int_0^{3\pi/2} \sqrt{1 + e^{2x}(1 - \sin 2x)}\ dx$

e. None of the above

9. The integral representing the volume of the solid obtained by rotating the region bounded by the curves $y^2 = x$, $x = 2y$ about the *y*-axis is:

Answer: b

a. $\int_0^2 \pi[2y - y^2]dy$

b. $\int_0^2 \pi[4y^2 - y^4]\ dy$

c. $\int_0^2 2\pi y[y^2 - 2y]dy$

d. $\int_0^4 2\pi x\ [\sqrt{x} + \frac{x}{2}]\ dx$

e. None of the above

10. The integral $\int_0^4 \sqrt{x} - \frac{1}{2}x\ dx$ defines the area between two curves. Which of the following integrals calculates the area of the same region using integration with respect to *y*?

Answer: d

Select the correct answer

a. $\int_0^4 (y^2 - 2y)\ dy$

b. $\int_0^4 (2y - y^2)\ dy$

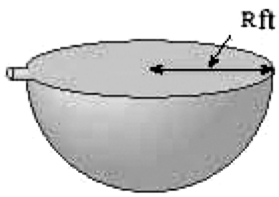
c. $\int_0^2 (y^2 - 2y)\ dy$

d. $\int_0^2 (2y - y^2)\ dy$

e. None of the above

## FREE RESPONSE

11. The tank shown is full of water. Given that water weighs $62.5\ \text{lb/ft}^3$ and R = 5 ft, find the work required to pump the water out of the tank.




hemisphere

Solution:

Volume of the $i^{th}$ element = $\pi(25-x_i^2)\Delta x$

Weight of the $i^{th}$ element = $62.5\pi(25-x_i^2)\Delta x$

Work done to pump out the $i^{th}$ element = $62.5\pi(25-x_i^2)x_i\Delta x$

Work done to pump out all the water = $i^{th}$ element = $\int_0^5 62.5\pi(25-x^2)x\,dx = 9765.625\pi\, ft-lb$

12. Evaluate the integral **or** show that it is divergent: $\int_2^\infty \frac{1}{x(\ln x)^4}\,dx$.

Solution:

First observe

$$\int \frac{1}{x(\ln x)^4}\,dx = \int u^{-4}\,du \qquad (u=\ln x)$$
$$= -\frac{1}{3[\ln x]^3}$$

So,

$$\int_2^\infty \frac{1}{x(\ln x)^4}\,dx = \lim_{t\to\infty}\int_2^t \frac{1}{x(\ln x)^4}\,dx$$
$$= \lim_{t\to\infty}\left[\frac{-1}{3(\ln x)^3}\right]_{x=2}^{x=t}$$
$$= \lim_{t\to\infty}\left[\frac{-1}{3(\ln t)^3} + \frac{1}{3(\ln 2)^3}\right] = \frac{1}{3(\ln 2)^3}$$

13. Set up the integral for the volume of the solid obtained by rotating the region bounded by $y = x^3$ and $x = y^3$ in the first quadrant, about the line $x = -1$. (**Do not evaluate the integral**)

Solution:
Outer radius of the cross-section = $y^{1/3} + 1$
Inner radius of the cross-section = $y^3 + 1$
Volume = $\pi\int_0^1 [(y^{1/3}+1)^2 - (y^3+1)^2]\ dy$

14. The height of a monument is 20 m. A horizontal cross-section at a distance *x* meters from the top is an equilateral triangle with side *x*/4 meters. Find the volume of the monument.

Solution:
The height of an equilateral triangle with side-length equal to s units = $\frac{\sqrt{3}}{2}s$

So, area of the equilateral triangle = $\frac{1}{2}\cdot s\cdot\frac{\sqrt{3}}{2}s = \frac{\sqrt{3}}{4}s^2$

For our problem, area of the equilateral triangle = $\frac{\sqrt{3}}{64}x^2$

Hence, volume of the monument = $\int_0^{20}\frac{\sqrt{3}}{64}x^2\ dx = \frac{125\sqrt{3}}{3}$